Applied Physics Unit-IV

PART-I- Semiconductor

Metals have large number of free electrons ~ 10^{28} m^{-3} and hence are good conductors of heat and electricity. In the case of semiconductors, the electron densities are considerably low ~ 10^{6} to 10^{13} m^{-3} leading to lower electrical conductivities and the electron densities for few metals and semiconductors are listed in the table

Element	σ $(\Omega m)^{-1}$	ρ (Ωm)	n m^{-3}
Metals			
Cu	5.98×10^{7}	1.67×10^{-8}	8.47×10^{28}
Ag	6.30×10^{7}	1.58×10^{-8}	$5.86 \times .10^{28}$
Au	4.25×10^{7}	2.35×10^{-8}	5.90×10^{28}
Zn	1.68×10^{7}	5.95×10^{-8}	$13.2 \times .10^{28}$
Al	3.50×10^{7}	2.86×10^{-8}	$18.1 \times .10^{28}$
Semiconductors			
Ge	2.17	0.47	2.2×10^{13}
Si	1.56×10^{-3}	640	8.72×10^{9}
GaAs	5×10^{-8} to 10^{3}		2.03×10^{6}

In metals only free electrons contribute to both electrical and thermal conduction, whereas in the case of semiconductors in addition to electrons, holes also contribute. Thus, one needs to consider both electrons and holes while studying the transport properties in semiconductors.

Expression for concentration of electrons in conduction band:

For a semiconductor in a conduction band, some lower energy states are filled with electrons at room temperature. The concentration of electrons in conduction band is given by

$$n = \int N(E)f(E)dE \quad \text{----------------(1)}$$

Where

N(E) = No. of electrons/unit energy/unit volume and f(E) =Fermi factor

We have

$$N(E) = \left(\frac{4\pi}{h^3}\right)(2m_e^*)^{3/2}\, E^{1/2} \quad \text{----------------(2)}$$

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_f}{k_B T}\right)} \quad \text{----------------(3)}$$

At room temperature, (E – E_f) > k_BT

$f(E) = \frac{1}{\exp\left(\frac{E-E_f}{k_bT}\right)} = \exp\left[-\left(\frac{E-E_f}{k_BT}\right)\right]$ ----------------(4)

Hence eqn(1) becomes

$n = \int_{E_C}^{E_C+\varphi} \left(\frac{4\pi}{h^3}\right)(2m_e^*)^{3/2}\ (E-E_C)^{1/2} \exp\left[-\left(\frac{E-E_f}{k_BT}\right)\right] dE$

Here $E^{1/2}$ has been replaced by $(E\text{-}E_c)^{1/2}$ since Ec is lower energy level in conduction band and Φ is the work function of the metal. The combination of Ec + Φ turns out to be huge and we can replace it by ∞**.**

$n = \left(\frac{4\pi}{h^3}\right)(2m_e^*)^{3/2}\ \int_{E_C}^{\infty}(E-E_C)^{1/2} \exp\left[-\left(\frac{E-E_f}{k_BT}\right)\right] dE$----------------(5)

Evaluating the above integral by taking E = Ec +x and dE = dx and limit of the integration as 0 to ∞ **(assuming the lowest energy almost to be equal to zero).**

$$n = \left(\frac{4\pi}{h^3}\right)(2m_e^*)^{3/2} \int_0^{\infty} (x)^{1/2} \exp\left(\frac{E_f - E_c - x}{k_BT}\right) dx$$

$$n = \left(\frac{4\pi}{h^3}\right)(2m_e^*)^{3/2} \int_0^{\infty} (x)^{1/2} \exp\left(\frac{E_f - E_c}{k_BT}\right) \exp\left(\frac{-x}{k_BT}\right) dx$$

$$n = \left(\frac{4\pi}{h^3}\right)(2m_e^*)^{3/2}\ \exp\left(\frac{E_f - E_c}{k_BT}\right) \int_0^{\infty} (x)^{1/2} \exp\left(\frac{-x}{k_BT}\right) dx$$

Let $y = \frac{x}{k_BT}\ \ then\ \ dy = \frac{dx}{k_BT}$

$n = \left(\frac{4\pi}{h^3}\right)(2m_e^*)^{3/2}\ \exp\left(\frac{E_f-E_c}{k_BT}\right)(k_BT)^{\frac{3}{2}} \int_0^{\infty}(y)^{1/2} \exp(-y)\, dy$ ----------------(6)

Using gamma integral function we can write $\int_0^{\infty}(y)^{1/2} \exp(-y)\, dy = \frac{\pi^{1/2}}{2}$

$$n = \left(\frac{4\pi}{h^3}\right)(2m_e^*)^{3/2}\ \exp\left(\frac{E_f - E_c}{k_BT}\right)(k_BT)^{\frac{3}{2}} \left(\frac{\pi^{1/2}}{2}\right)$$

$\boldsymbol{n = 2\left(\frac{2\pi m_e^* k_BT}{h^2}\right)^{3/2}\ \exp\left(\frac{E_f-E_c}{k_BT}\right) = N_c \exp\left(\frac{E_f-E_c}{k_BT}\right)}$ ----------------(7)

Where $\boldsymbol{N_c = 2\left(\frac{2\pi m_e^* k_BT}{h^2}\right)^{3/2}}$ and equation (7) represents concentration of electrons in conduction band

Expression for concentration of holes in valence band:

The concentration of holes in valence band can be calculated as follows

$n = \int N(E)[1 - f(E)]\, dE$ ----------------(1)

By substituting the values for N(E) & f(E) and solving the integral as we did for the concentration of electrons in conduction band we can arrive at the following equation

$\boldsymbol{n = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2} \exp\left(\frac{E_v - E_f}{k_B T}\right) = N_v \exp\left(\frac{E_v - E_f}{k_B T}\right)}$ ----------------(2)

Where $\boldsymbol{N_v = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2}}$ $\textit{represents the concentration of holes in valence band}$

and equation (2) represents concentration of holes in valence band

Expression for intrinsic carrier concentration:

In an intrinsic semiconductor, the concentration of holes and electrons are equal, i.e.,. n = p. The product of hole and electron concentrations for a given material is constant at a given temperature. If impurity is added to increase n, there will be corresponding decrease in p so that the product n x p remains constant and is sometimes called law of mass action. Therefore in an intrinsic semiconductor

$$n \times p = n_i \times p_i = n_i^2$$

$$n_i^2 = \left[2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \exp\left(\frac{E_f - E_c}{k_B T}\right)\right]\left[2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2} \exp\left(\frac{E_v - E_f}{k_B T}\right)\right]$$

$$n_i^2 = 4\left(\frac{2\pi k_B T}{h^2}\right)^{3} (m_h^* m_e^*)^{3/2} \exp\left[-\left(\frac{E_c - E_v}{k_B T}\right)\right]$$

Multiply and divide on right side by **m³**

$$n_i^2 = 4\left(\frac{2\pi m k_B T}{h^2}\right)^{3} \left(\frac{m_h^* m_e^*}{m^2}\right)^{3/2} \exp\left[-\left(\frac{E_c - E_v}{k_B T}\right)\right]$$

Substituting (E_c – E_v) = E_g, we get

$$n_i^2 = 4\left(\frac{2\pi m k_B T}{h^2}\right)^{3} \left(\frac{m_h^* m_e^*}{m^2}\right)^{3/2} \exp\left[-\left(\frac{E_g}{k_B T}\right)\right]$$

$$n_i = 2\left(\frac{2\pi\, m\, k_B T}{h^2}\right)^{3/2} \left(\frac{m_h^* m_e^*}{m^2}\right)^{3/4} \exp\left[-\left(\frac{E_g}{2k_B T}\right)\right]$$

$$n_i = 2\left(\frac{2\pi\, m\, k_B}{h^2}\right)^{3/2} \left(\frac{m_h^* m_e^*}{m^2}\right)^{3/4} T^{3/2} \exp\left[-\left(\frac{E_g}{2k_B T}\right)\right]$$

$$\boldsymbol{n_i = A_o T^{3/2}\ exp\left[-\left(\frac{E_g}{2k_B T}\right)\right]}$$

Where A₀ = $2\left(\frac{2\pi\, m\, k_B}{h^2}\right)^{3/2}\ \left(\frac{m_h^* m_e^*}{m^2}\right)^{3/4}$

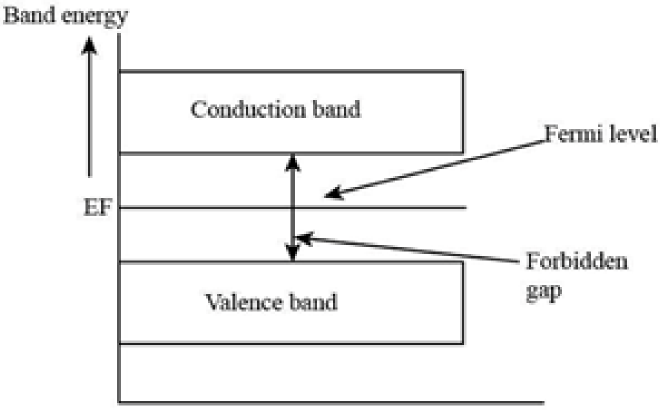
Substituting for constants we get

$$A_o\ = 2\left(\frac{2\pi\ x\ 1.38\ x\ 10^{-23}\ x\ \ 9.1\ x\ 10^{-31}}{(6.634\ x\ 10^{-34})^2}\right)^{3/2}\ \left(\frac{m_h^* m_e^*}{m^2}\right)^{3/4}$$

$$A_o\ = 4.819\ x\ 10^{21}\left(\frac{m_h^* m_e^*}{m^2}\right)^{3/4}$$

Fermi level in an intrinsic semiconductor:

The Fermi-level in an intrinsic semiconductor is nearly midway between the conduction and valence band.



In an intrinsic semiconductor, the concentrations of electrons and holes are equal. Therefore we can write

$$N_c \exp\left(\frac{E_f - E_c}{k_B T}\right) = N_v\ \exp\left(\frac{E_v - E_f}{k_B T}\right)$$

$$\frac{N_c}{N_v} = \exp\left[-\left(\frac{E_f - E_c}{k_B T}\right)\right]\ \exp\left(\frac{E_v - E_f}{k_B T}\right)$$

$$\frac{N_c}{N_v} = \exp\left[\left(\frac{E_v + E_c\ - 2E_f}{k_B T}\right)\right]$$

$$ln\left[\frac{N_c}{N_v}\right] = \frac{E_v + E_c\ - 2E_f}{k_B T}$$

$$E_v + E_c\ - 2E_f\ =\ k_B T\ ln\left[\frac{N_c}{N_v}\right]$$

$$E_f\ =\ \frac{E_v + E_c}{2} - \frac{k_B T}{2}\ ln\left[\frac{N_c}{N_v}\right]$$

If the effective masses of the holes and electrons are equal, then

$$E_f = \frac{E_v + E_c}{2}$$

Hence it is clear from the above equation that for an intrinsic semiconductor, the Fermi level lies at the middle of the energy gap, when the effective masses of the holes and electrons are equal.

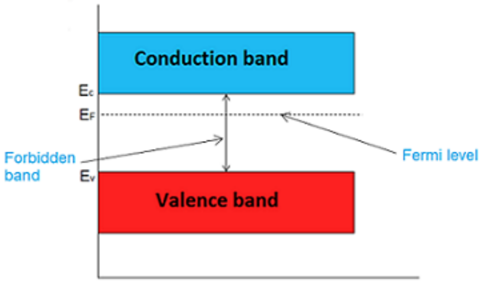
Extrinsic semiconductor

Fermi level in extrinsic semiconductor

In extrinsic semiconductor, the number of electrons in the conduction band and the number of holes in the valence band are not equal. Hence, the probability of occupation of energy levels in conduction band and valence band are not equal. Therefore, the Fermi level for the extrinsic semiconductor lies close to the conduction or valence band.

1. Fermi level in n-type semiconductor

In n-type semiconductor pentavalent impurity is added. Each pentavalent impurity donates a free electron. The addition of pentavalent impurity creates large number of free electrons in the conduction band.



At room temperature, the number of electrons in the conduction band is greater than the number of holes in the valence band. **Hence, the probability of occupation of energy levels by the electrons in the conduction band is greater than the probability of occupation of energy levels by the holes in the valence band**. This probability of occupation of energy levels is represented in terms of Fermi level. Therefore, the Fermi level in the n-type semiconductor lies close to the conduction band.

The Fermi level for n-type semiconductor is given as

$$E_F = E_C - K_B T \log \frac{N_C}{N_D}$$

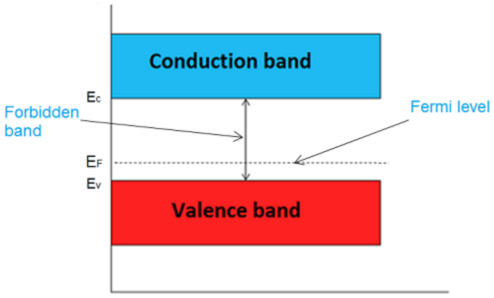
Where E_F is the fermi level, E_C is the conduction band, K_B is the Boltzmann constant.

T is the absolute temperature; N_C is the effective density of states in the conduction band.

N_D is the concentration of donor atoms.

2. Fermi level in p-type semiconductor:

In p-type semiconductor trivalent impurity is added. Each trivalent impurity creates a hole in the valence band and ready to accept an electron. The addition of trivalent impurity creates large number of holes in the valence band.



At room temperature, the number of holes in the valence band is greater than the number of electrons in the conduction band. **Hence, the probability of occupation of energy levels by the holes in the valence band is greater than the probability of occupation of energy levels by the electrons in the conduction band.** This probability of occupation of energy levels is represented in terms of Fermi level. Therefore, the Fermi level in the p-type semiconductor lies close to the valence band.

The Fermi level for p-type semiconductor is given as $E_F = E_V + K_B T \log \frac{N_V}{N_A}$

Where N_V is the effective density of states in the valence band. N_A is the concentration of acceptor atoms.

Expression for conductivity of semiconductors:

Consider a semiconductor of area of cross section 'A', in which a current 'I' is flowing. Let 'v' be the velocity of electrons.

The volume swept by the electrons/second=Av

If 'n' is the number of electrons/unit volume and 'e' is the magnitude of electric charge on the electron, then the current is

I=neAv

Current density, J=I/A=nev ------- (1)

Electron mobility, μ_e=v/E ------ (2) where E is Electric field

Substituting v from Eq(2) in Eq(1) we get

$J=ne\mu_e E$ -------- (3)

But from Ohm's law

$J=\sigma_e E$ ------- (4) where σ_e---conductivity is due to electrons in semiconductor materials

Comparing Eq(3) and (4), we get

$\sigma_e = ne\mu_e$ -------- (5)

Similarly, conductivity due to holes in semiconductor is

$\sigma_h = pe\mu_h$ ------- (6) where p----number of holes/unit volume and μ_h-----mobility of holes

Total conductivity, $\sigma = \sigma_e + \sigma_h = ne\mu_e + pe\mu_h = e(n\mu_e + p\mu_h)$ ------------------- (7)

For an intrinsic semiconductor, the concentrations of electrons & holes are equal, hence Eq.(7) can be written as

$\sigma = n_i e(\mu_e + \mu_h)$ ------------------------- (8)

Substituting the value for $\mathbf{n_i}$ in eqn. (8) we get

$$\sigma = 2\left(\frac{2\pi\, m\, k_B T}{h^2}\right)^{3/2} \left(\frac{m_h^* m_e^*}{m^2}\right)^{3/4} \exp\left[-\left(\frac{E_g}{2k_B T}\right)\right] e(\mu_e + \mu_h) \text{ ----------------(9)}$$

Eqn. (9) can be written as

$$\sigma = A \exp\left[-\left(\frac{E_g}{2k_B T}\right)\right] \text{ ---------------- (10)}$$

Where $A = 2\left(\frac{2\pi\, m\, k_B T}{h^2}\right)^{3/2} \left(\frac{m_h^* m_e^*}{m^2}\right)^{3/4} e(\mu_e + \mu_h)$ ---------------- (11)

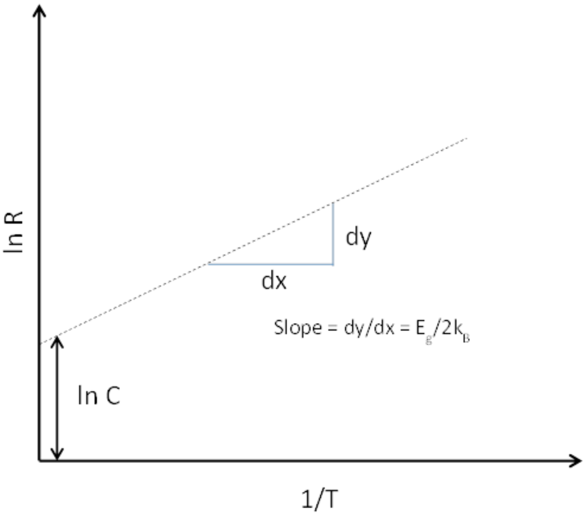
Hence eqn. (10) can be rewritten as

$$\frac{1}{\rho} = A\ \exp\left[-\left(\frac{E_g}{2k_B T}\right)\right] = \frac{l}{Ra} \quad \text{---------------- (12)}$$

$$R = \frac{l}{Aa}\exp\left[\frac{E_g}{k_B T}\right] = C\ \exp\left[\frac{E_g}{2k_B T}\right] \text{---------------- (13)}$$

Here C = l/Aa, where a = area of cross section ; l = length of the specimen. Taking natural log on both sides of eqn. (9) we get

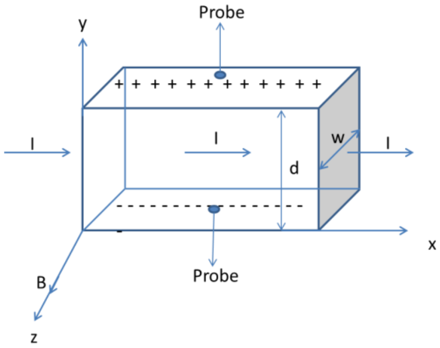
ln R = ln C + $E_g/2k_BT$ ---------------- (14)



Eq.(13) is similar to equation of straight line y = mx + c, where y = ln R, x = 1/T, m = $E_g/2k_B$ and c = ln C **.** If a plot is drawn between ***<u>ln R verses 1/T</u>*** **.** The value of E_g can be determined from the slope of the straight line. Hence $E_g = 2k_B$ x slope.

<u>Hall Effect :</u>

In a conductor, the flow of electric current is the movement of charges due to the presence of an electric field. If a magnetic field is applied in a direction perpendicular to the direction of motion of the charges, the moving charges accumulate such that opposite charges lie on opposite faces of the conductor. This distribution of charges produces a potential difference across the material that opposes the migration of further charge. This creates a steady electrical potential as long as the charges are flowing in the material and the magnetic field is on. This is the Hall effect.



Consider a rectangular slab of a semiconductor material in which a current I is flowing in the positive x-direction. Let the semiconducting material be of n-type, which means that the charge carriers are electrons. Let a magnetic field B be applied along the z-direction as shown in the figure. Under the influence of the magnetic field, the electrons experience the Lorentz force F_L given by

$$F_L = -Bev \quad \text{------------ (1)}$$

Applying the Flemming's left hand rule, we see that the force is exerted on the electrons in the negative y-direction. The electrons are therefore deflected downwards. As a result, the density of the electrons increases in the lower end of the material, due to which its bottom edge becomes negatively charged. On the other hand, the loss of electrons from the upper end causes the top edge of the metrical to become positively charged. Hence a potential V_H, called the Hall voltage appears between the upper and lower surfaces of the semiconductor material which establishes an electric field E_H called the Hall field across the conductor in the negative y-direction. The field E_H, exerts an upward force F_H on the electrons given by

$$F_H = -eE_H \text{ ------------ (2)}$$

Now as the deflection of electrons continues in the downward direction due to the Lorentz force FL, it also contributes to the growth of Hall field. As a result , the force FH which acts on the electron in the upward direction also increases. These two opposing forces reach equilibrium at which stage,

$$F_L = F_H$$

Using eqns. (1) & (2), above equation becomes

$-Bev = -eE_H$ or $E_H = Bv$ ------------ (3)

If d is the distance between the upper and lower surfaces of the slab, then,

$E_H = V_H/d$ or $V_H = E_H d = Bvd$ ------------ (4)

Let w be the thickness of the material in the z-direction

Its area of cross section normal to the direction of I is = wd.

The current density $J = I/wd$ ------------ (5)

But, we know that $J= nev = \rho v$ ------------ (6)

where n is carrier concentration and ρ is charge density

Therefore, $\rho v = I/wd$ or $v = I/wd\rho$ ------------ (7)

Comparing equations (4) & (7) we get $\mathbf{V_H = BI/ \rho w}$ $\Rightarrow$ $\rho=BI/V_H w$

Using known I, B and w and measuring V_H developed across the slab, ρ can be determined. By knowing the polarity on the top and bottom surfaces, the type of semiconductor can be determined.

<u>Hall coefficient (R_H):</u>

For a given semiconductor, the Hall field E_H depends upon the current density J and the applied field B.

$E_H \propto JB$

$E_H = R_H JB$, where R_H is called the Hall Coefficient

Therefore,

$R_H = E_H /JB = Bv/ nevB = 1/ne$

$\mathbf{R_H = 1/ne=1/\rho}$

Note:

1. Positive value of $\mathbf{R_H}$ indicates that the charge carriers are holes.
2. Negative value of $\mathbf{R_H}$ indicates that the charge carriers are electrons.

Application of Hall Effect

1. Hall Effect is used to find whether a semiconductor is N-type or P-type.

2. Hall Effect is used to find carrier concentration.

3. Hall Effect is used to calculate the mobility of charge carriers (free electrons and holes).
4. Hall Effect is used to measure conductivity.
5. Hall Effect is used to measure a.c. power and the strength of magnetic field.
6. Hall Effect is used in an instrument called Hall Effect multiplier which gives the output proportional to the product of two input signals.

***************** All the best *****************