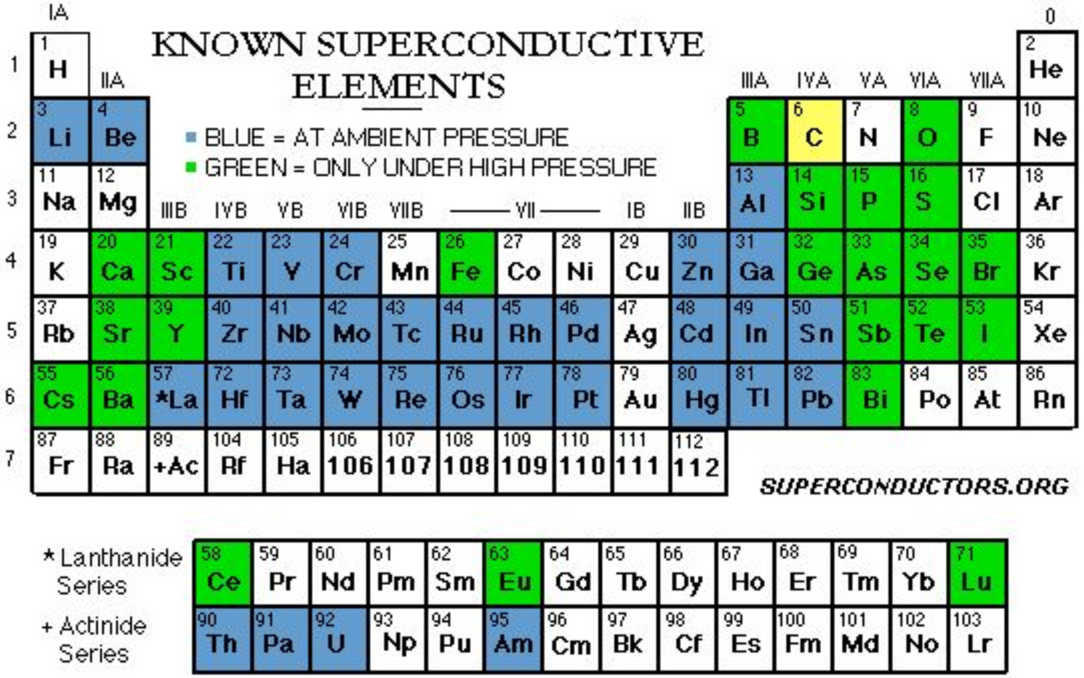
Super Conductivity

- Super conductivity is the phenomenon observed in some metals and materials.
- Kammerlingh Onnes in 1911 observed that the electrical resistivity of pure mercury drops abruptly to zero at about 4.2K .
- This state is called super conducting state. The material is called superconductor .
- The temperature at which they attain superconductivity is called critical temperature T_c.



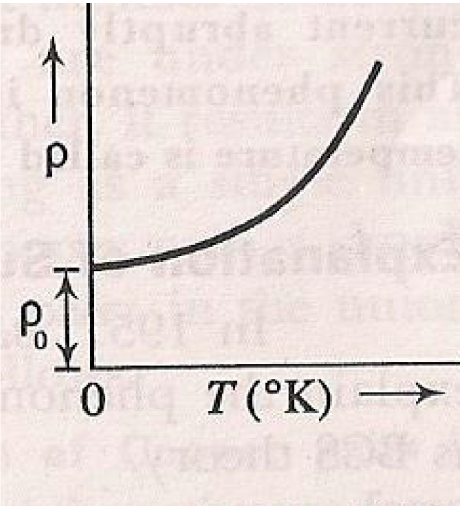
2. General properties of Superconductors:-

Properties of superconductors:-

1. It is a low temperature phenomenon.
2. The transition temperature is different for different substances.
3. Materials having high normal resistivities exhibit superconductivity.
4. Materials for which $Z\rho = 10^6$ (where Z is a atomic number and ρ is resistivity) show superconductivity.
5. For chemically pure and structurally perfect specimen, the superconductivity is very sharp.
6. Ferro magnetic and Anti ferromagnetic materials are not superconductors.
7. Below the transition temperature the magnetic flux lines are rejected out of the superconductors.
8. Superconducting elements, in general, lie in the inner columns of the periodic table.
9. Those metallic elements having their valence electrons lies between 2 to 8 exhibit superconductivity.
10. Below the transition temperature the specific heat curve is discontinuous.

Temperature dependence of resistivity of a metal:

All metals are good conductors of electricity. The electrical conductivity of metal varies with the temperature. The electrical resistance of a metal, to the flow of current, is due to scattering of conduction electrons by lattice vibrations. When the temperature increases the amplitude of lattice vibrations also increases, thereby increasing the resistance. The dependence of resistance of metal (non-superconducting state) is shown in figure. The resistance decreases with temperature and reaches a minimum value at T = 0K. The residual resistance at T = 0K is due to impurities in the metal.



DEPENDENCE OF ρ ON T

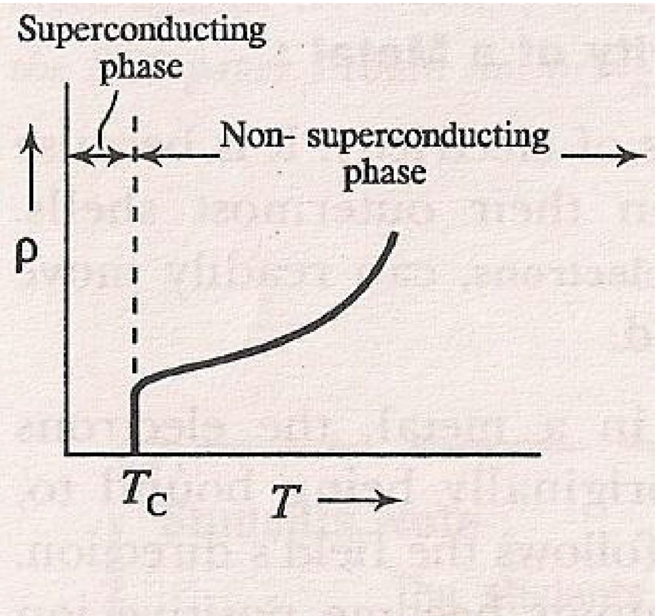
By Matthiessen's rule

$\rho = \rho_o + \rho(T)$

Where 'ρ' is the resistivity of the given material, 'ρ_o' is the residual resistivity and 'ρ(T)' is the temperature dependent part of resistivity.

Temperature dependence of resistivity of a superconductor:

One of the most interesting properties of solid at low temperature is that electrical resistivity of metals and alloys vanish entirely below a certain temperature. This zero resistivity or infinite conductivity is known as superconductivity. Temperature at which transition takes place is known as *transition temperature or critical temperature (T_c)*. Above the transition temperature, the substance is in the normal state and below it will be in superconducting state. T_c value is different for different materials.

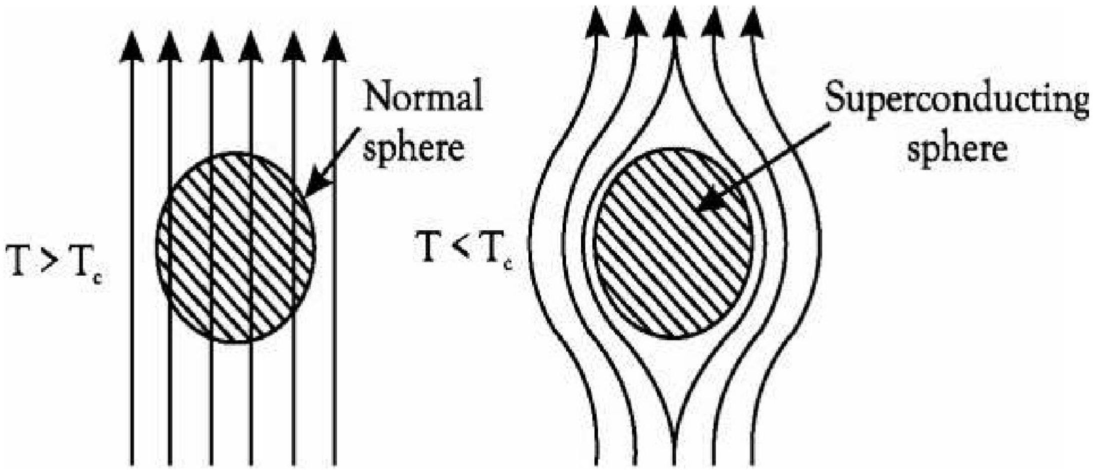


DEPENDENCE OF ρ ON *T*

"The resistance offered by certain materials to the flow of electric current abruptly drop to zero below a threshold temperature. This phenomenon is called *superconductivity* and threshold temperature is called "*critical temperature.*"

Meissner effect:

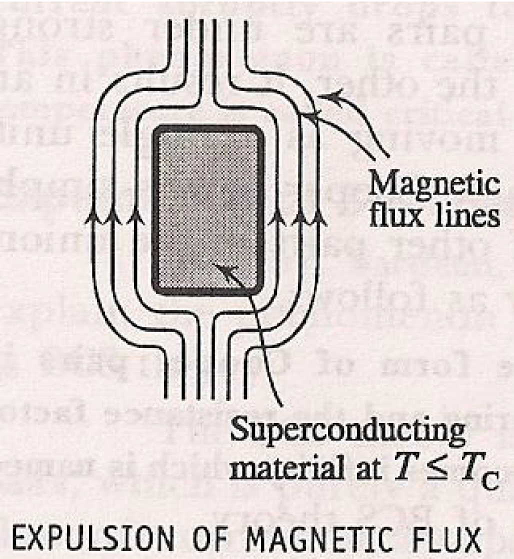
A superconducting material kept in a magnetic field expels the magnetic flux out of its body when it is cooled below the critical temperature and thus becomes perfect diamagnet. This effect is called Meissner effect.



When the temperature is lowered to T_c, the flux is suddenly and completely expelled, as the specimen becomes superconducting. The Meissner effect is reversible. When the temperature is raised the flux penetrates the material, after it reaches T_c. Then the substance will be in the normal state.

The magnetic induction inside the specimen

$B = \mu_o (H + M)$



EXPULSION OF MAGNETIC FLUX

Where 'H' is the intensity of the magnetizing field and 'M' is the magnetization produced within the material.

For $T < Tc$, $B = 0$

$\mu0 (H + M) = 0$

$M = -H$

$M/H = -1 = \chi$

Susceptibility is -1 i.e. it is perfect diamagnetism.

Hence superconducting material do not allow the magnetic flux to exist inside the material.

Effect of magnetic field:

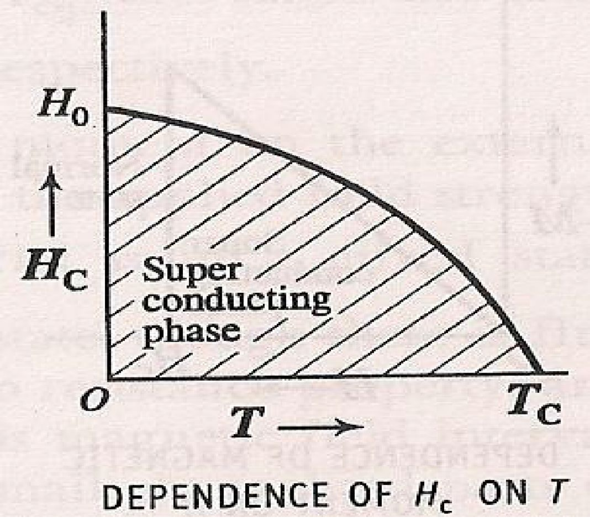
Superconductivity can be destroyed by applying magnetic field. The strength of the magnetic field required to destroy the superconductivity below the T_c is called critical field. It is denoted by $H_c(T)$.

If 'T' is the temperature of the superconducting material, 'T_c' is the critical temperature, 'H_c' is the critical field and 'H_o' is the critical field at 0_oK.

They are related by

$H_c = H_o[1-(T/T_c)_2]$

By applying magnetic field greater than H_o, the material can never become superconductor whatever may be the low temperature. The critical field need not be external but large current flowing in superconducting ring produce critical field and destroys superconductivity.



As temperature decreases, the critical field increases generally to a maximum at absolute zero.

Material	T_c(K)
Zn	0.88
Al	1.19
Sn	3.72
Hg	4.15
Pb	7.18
Nb	9.46
Nb_3Sn	18.05
Nb_3Ge	23.2
YBa_2Cu_3O7	92
Bi–Sr-Ca–Cu–O	105
Tl–Ba–Ca–Cu–O	125
$HgBa_2Ca_2Cu_3O_8$	134

Isotopic effect

In superconducting materials the transition temperature varies with the average isotopic mass of their constituents. The variation is found to be in general form

$$T_C \propto M^{-\alpha}$$

Or $T_C M^{+\alpha} = \text{constant}$

Where α is the isotopic effect coefficient and is defined as

$$\alpha = \frac{\partial \ln T_C}{\partial \ln M}$$

The value of α is approximately 0.5. For example, the average mass varies from 199.5 to 203.4 atomic mass units and accordingly the transition temperature varies from 4.185K to 4.146K.

Types of superconductors:

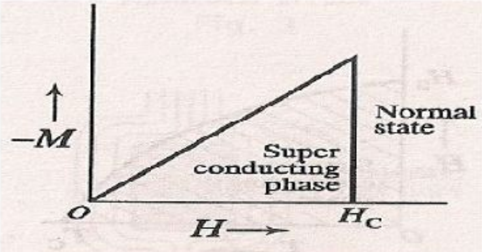
There are two types of superconductors. They are type-I superconductors and type-II superconductors.

Type of superconductors

1.Type I superconductors:

These materials exhibit complete Meissner effect and have well defined critical field H_c. These are perfect diamagnetic in the superconducting state and possesses negative magnetic moment.

The material remains in the superconducting when the field is less than the critical field. It expels the magnetic lines of force from the body of the mater immediately after H_c the material transits to normal state and the flux penetrates the material i.e. Meissner effect is absent. H_c is of the order of 0.1 T or less. Since H_c very low, even weak magnetic field can destroy the phenomenon. As weak magnetic field can penetrate the material more easily and they are also called *soft superconductor*.



DEPENDENCE OF MAGNETIC MOMENT ON H

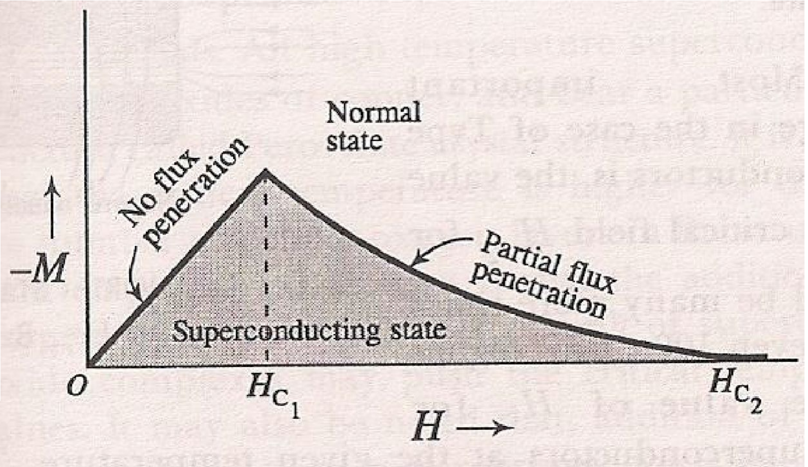
Element	T_c (K)
Aluminium	1.196
Cadmium	0.52
Gallium	1.09
Indium	3.40
Tin	3.72
Mercury	4.12
Lead	7.175

Type-II superconductors

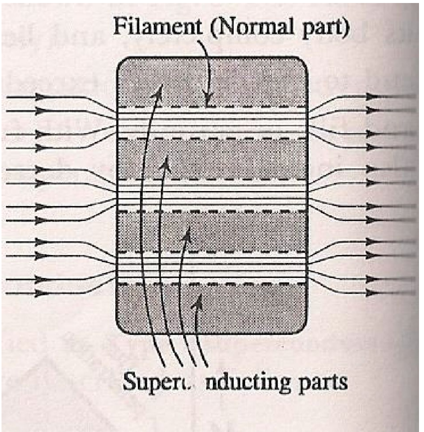
Type-II superconductors are hard superconductors. They exist in three states 1) Superconducting state 2) Mixed state 3) Normal state

These materials are having two critical fields H_{c1} and H_{c2}. For the field less then H_{c1}(lower critical field), it expels the magnetic field completely and there is no flux penetration. It becomes a perfect diamagnetic and the material is in the super conducting state. After Hc_1 the flux penetrates and partially fills the body of the material through channels called filaments. As the field is increased these filaments broaden and by Hc_2 (the material possesses both normal and superconducting state, hence the state is called *'Mixed State'*. This is also referred to as *'vortex state'* where the material is in a magnetically mixed state but electrically it is a superconductor. After Hc_2 the material transits to normal state and the resistance is finite. Type II superconductors can carry larger currents when the magnetic field is between Hc_1 and Hc_2.

Hc_2 the upper critical field is many a folds greater than Hc_1 the lower critical field. Only strong magnetic field of the order of 10T can penetrate the material hence these are called hard super conductor. Type-II superconductors are used in the manufacturing of the superconducting magnets of high magnetic fields above 10 Tesla.



DEPENDENCE OF MAGNETIC MOMENT ON *H*



VORTE STATE

Element	T_c (K)
Tantalum	4.5
Thallium	2.4
Niobium	9.3

5. Differences between type I and Type II superconductor

Type I superconductor	Type II superconductor
1. It follows complete Meissner effect.	1. It does not follow the complete Meissner effect
2. It has single critical field value H_C	2. It has two critical field values H_{C_1} and H_{C_2}
3. There no mixed state.	3. There is a mixed state
4. They are soft superconductors	4. They are hard superconductors
5. Materials with pure form are type I superconductors	5. Materials with impurities or alloys are type II superconductors
6. Examples; Zn, Al, Hg and Sn	6. Examples: Zr, Nb

Critical current

An electric current is passing through the superconducting material it self may gives rise to necessary magnetic field. For example, when the current is passing a superconducting ring, it gives rise to its own magnetic field. As the current increases to critical value I_c, the associated magnetic field becomes H_C. And the superconductivity disappears.

$I_C = 2\pi r H_C$ r → radius of the ring

The current in a superconductive material above which the material is normal and below which the material is superconducting, at a specified temperature and in the absence of external magnetic fields is known as critical current.

BCS Theory:

- Bardeen, Cooper and Schrieffer (BCS) in 1957 explained the phenomenon of superconductivity based on the formation of cooper pairs.

- When a current flow in a superconductor, electrons come near a positive ion core of lattice, due to attractive force. The ion core also gets displaced from its position, which is called *lattice distortion*. The lattice vibrations are quantized in a term called *Phonons.*

- Now an electron which comes near that place will interact with the distorted lattice. This tends to reduce the energy of the electron. It is equivalent to interaction between the two electrons through the lattice. This leads to the formation of cooper pairs. "*Cooper pairs* are a bound pair of electrons formed by the interaction between the electrons with opposite spin and momenta in a phonon field".

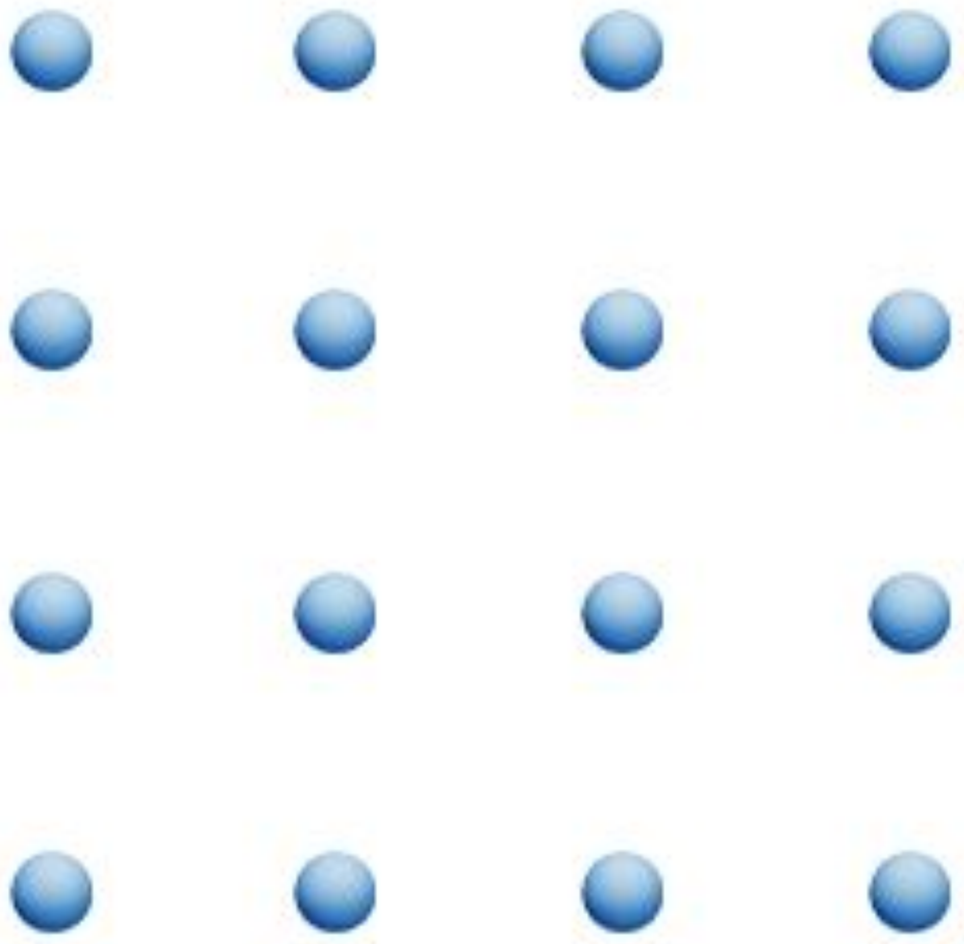
- According to quantum mechanics a cooper pair is treated as single entity. A wave function is associated with each cooper pair. This holds good over a large volume with finite value for its amplitude. The wave function of similar cooper pairs overlaps. For one cooper pair overlapping may extend over 106 other pairs.

BCS Theory:

- Thus it covers entire volume of the superconductor. It leads to union of large number of cooper pairs. The resistance encountered by any single cooper pair is overcome by combined action of other pairs in the union.

- When the electrons flow in the form of cooper pairs in materials, they do not encounter any scattering and the resistance factor vanishes or in other words conductivity becomes infinity which is called as superconductivity.

- In superconducting state electron-phonon interaction is stronger than the coulomb force of attraction of electrons. Cooper pairs are not scattered by the lattice points. They travel freely without slow down as their energy is not transferred. Due to this they do not posses any electrical resistivity.

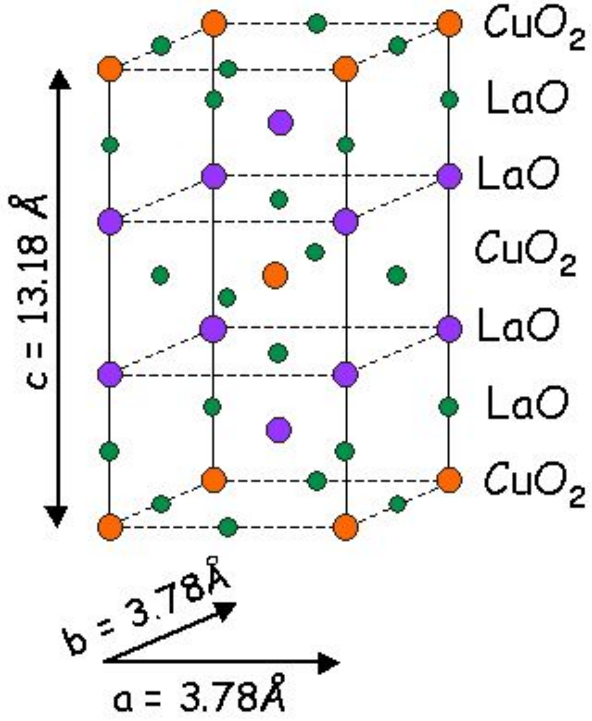


High temperature superconductors:

- The term high-temperature superconductor was first used to designate the new family of cuprate-perovskite ceramic materials discovered by Bednorz and Müller in 1986.

- The first high-temperature superconductor, LaBaCuO (Lanthanum Barium Copper Oxide) with a transition temperature of 30 K and in the same year LSCO (La2-xSrxCuO4) discovered with TC of 40K.

- In 1987 it was shown that superconductors with Tc greater than 77K could be prepared, this temperature is greater than the liquid helium temperature. YBa2Cu3O7 was discovered to have a Tc of 92 K. Bismuth/lead strontium Calcium Copper (Bi Pb)2Sr2 Ca2Cu3Ox (x<0.1) with Tc=105K. Thallium barium Calcium copper oxide (Tl Ba2Ca2Cu3O4) of Tc=115K. Mercury barium calcium copper oxide(Hg Ba2Ca2Cu3O4) with Tc=135K.



$La_{2-x}Sr_xCuO_2$

High temperature superconductors:

- All high temperature superconductors are different types of oxides of copper, and bear a particular type of crystal structure called Perovskite crystal structure.

- The current in the high T_C materials is direction dependent. It is strong in parallel to copper-oxygen planes and weak in perpendicular to copper-oxygen planes.

- High Tc materials are Type-II superconductors and they are brittle and don't carry enough current.

- The high temperature superconductors are useful in high field applications. It can carry high currents of 105 to 106 amps in moderate magnetic fields.

- They are used in military applications, Josephson junction in SQUIDS, under sea communication, submarines.

Applications of Superconductors:

Superconductors are mainly used in

(1) Superconducting Magnets

(2) Magnetic levitation and permanent magnets

(3) Wires and cables for motors and power distribution systems

(4) Microwave filters for cellular telephone networks

(5) SQUIDS as sensors of foetal heart beats and brain signals, and

(6) Single flux quantum logic in quantum computing system

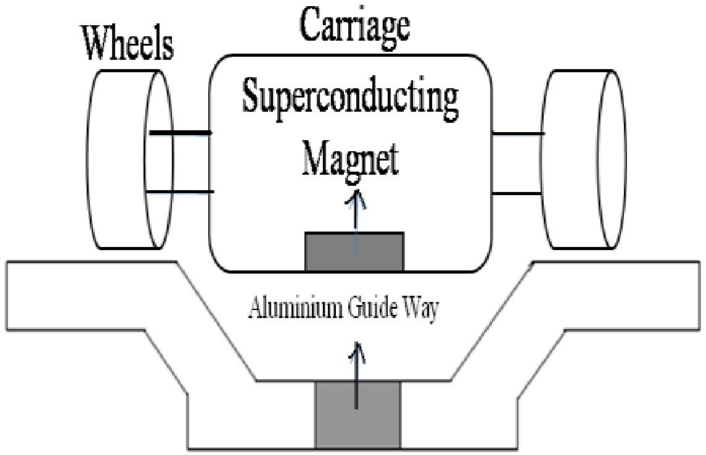
Magnetically Leviated Vehicles (Maglev vehicles):

- Magnetically levitated vehicles are called Maglev vehicles. The magnetic levitation is based on the principle of Meissner effect.

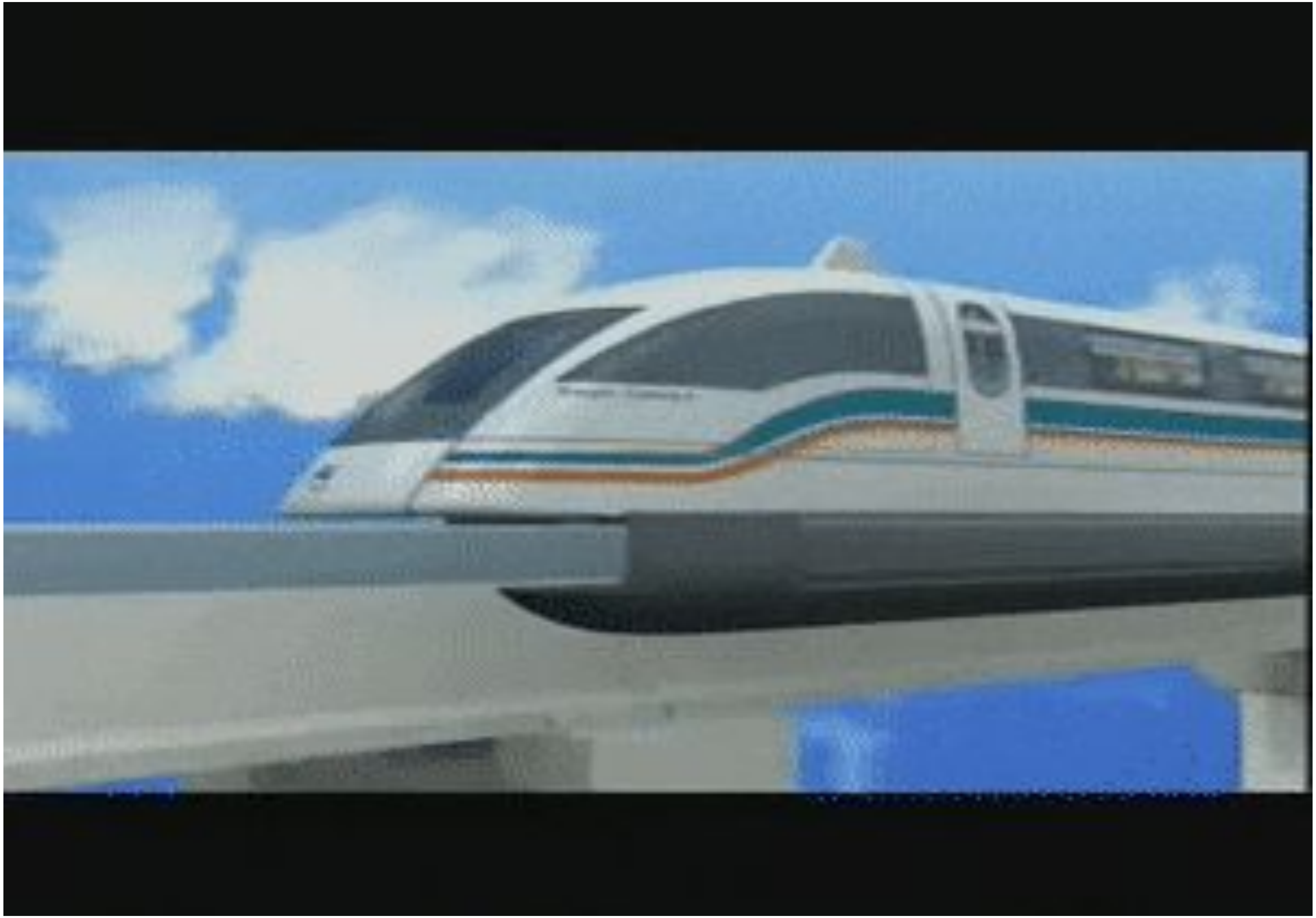
- The magnetic field is produced by the superconducting magnet and electric current. The superconducting magnet is kept inside the vehicle and the electric current is in the Aluminum guide way.

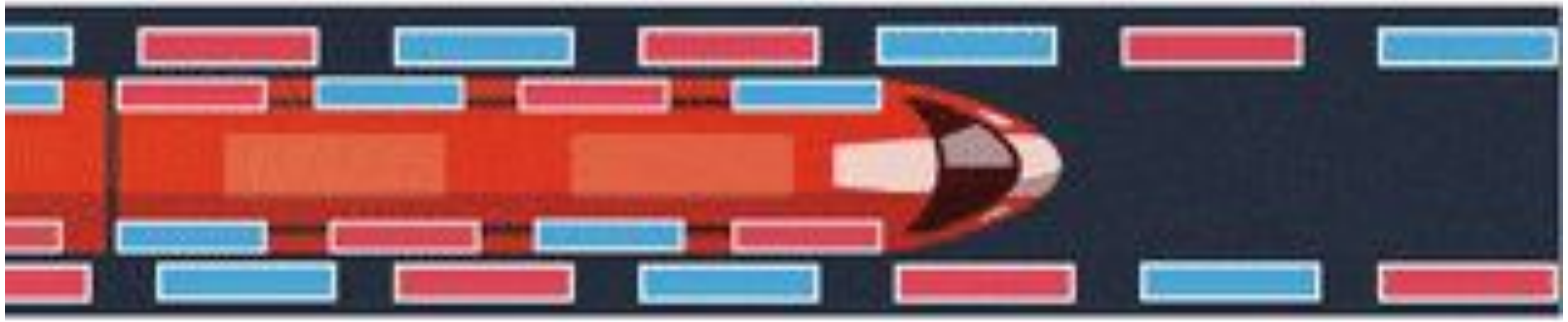
- The vehicle is on the Aluminum guide way. The vehicle is provided with retractable wheels. The vehicle runs on the guide way, once it is levitated in air the wheels are retracted into the body.

- The height to which the vehicle is levitated above guideway is about 10 to 15cm. While stopping, the wheels are drawn out and the vehicle slowly settles on the guide way by running a distance.



浮いた
土佐ノ海
TOSANOUMI
(Sumo Wrestler)
Height of Tosanoumi 186cm
Weight of Tosanoumi 142kg
Weight of disk 60kg
Total weight 202kg
As of February
協力：財団法人　日本相撲協会





Difference between conductor, perfect conductor and superconductor:

Term for comparision	**Conductor**	**Perfect conductor**	**superconductor**
-----------	Conductivity is due to electrons	Conductivity is due to electrons	Conductivity is due to cooper pairs
-----------	A potential difference is required to drive the current	A potential difference is required to drive the current	A potential difference is not required to drive the current
Conductivity (σ)	σ is large	σ tends to infinity	σ is equal to infinity
Mean Collision time (τ)	τ is large	τ tends to infinity	τ is equal to infinity
Temperature dependence of σ	σ decreases with increase of temperature	σ decreases with increase of temperature	σ ∞ at all temperatures below T_c.
Effect of magnetic field	Magnetic flux lines exist within the body and do not effect the flow of current.	Magnetic flux lines exist within the body and do not effect the flow of current	Expels all the magnetic flux lines out of its body and effect the flow of current.
-----------	There is loss of energy due to current flow	There is loss of energy due to current flow	There is no loss of energy due to current flow
Attainability of state	Realizable in practice	Theoretical concept	Realizable in practice
Theory to explain conductivity	Free electron theory	Free electron theory	BCS theory

A superconducting tin has a critical temperature of 3.7 K at zero magnetic field and a critical field of 0.0306 tesla at 0 K. Find the critical field at 2 K.

Given : $T_C = 3.7$ K $\quad\quad H_0 = 0.0306$ tesla at $T = 0$ K

$H_C(T) = ?$ at $T = 2$ K

$$H_C(T) = H_0\left(1 - \left(\frac{T}{T_C}\right)^2\right)$$

$$= 0.0306 \times \left(1 - \left(\frac{2}{3.7}\right)^2\right) = 0.0216 \text{ tesla}$$

The superconducting transition temperature of Lead is 7.26 K. The initial field at 0 K is 64×10^3 Amp/m. Calculate the critical field at 5 K.

Given : $T_C = 7.26\ \mathrm{K}$ $\quad H_0 = 64 \times 10^3\ \frac{\mathrm{Amp}}{\mathrm{m}}$ at $T = 0\ \mathrm{K}$

$H_C(T) = ?$ at $T = 5\ \mathrm{K}$

$$H_C(T) = H_0\left(1 - \left(\frac{T}{T_C}\right)^2\right)$$

$$= 64 \times 10^3 \times \left(1 - \left(\frac{5}{7.26}\right)^2\right) = 33.64 \times 10^3\ \mathrm{Amp/m}$$

Critical field of Niobium is 10^5 A/m at 8 K and 2 x 10^5 at 0 K. Calculate critical temperature of Niobium.

Given : $H_C(T) = 10^5 \frac{A}{m}$ at $T = 8$ K $\quad H_0 = 2 \times 10^5 \frac{A}{m}$ at $T = 0$ K

$T_C = ?$

$$H_C(T) = H_0\left(1 - \left(\frac{T}{T_C}\right)^2\right)$$

$$\therefore 1 - \left(\frac{T}{T_C}\right)^2 = \frac{H_C(T)}{H_0}$$

$$\therefore \left(\frac{T}{T_C}\right)^2 = 1 - \frac{H_C(T)}{H_0}$$

$$\therefore \left(\frac{T}{T_C}\right)^2 = 1 - \frac{H_C(T)}{H_0}$$

$$\therefore T^2 = \left(1 - \frac{H_C(T)}{H_0}\right)(T_C)^2$$

$$\therefore T_C = \frac{T}{\sqrt{1 - \frac{H_C(T)}{H_0}}} = \frac{8}{\sqrt{1 - \frac{10^5}{2 \times 10^5}}} = 11.31\ K$$

Critical temperature of a superconductor when no magnetic field is applied is T_C. Find the temperature at which the critical field becomes half of its value at 0 K.

Given : $H_C(T) = \frac{H_0}{2}$ at T

$$H_C(T) = H_0\left(1 - \left(\frac{T}{T_C}\right)^2\right)$$

$$\therefore 1 - \left(\frac{T}{T_C}\right)^2 = \frac{H_C(T)}{H_0}$$

$$\therefore \left(\frac{T}{T_C}\right)^2 = 1 - \frac{H_C(T)}{H_0}$$

$$\therefore \left(\frac{T}{T_C}\right)^2 = 1 - \frac{H_C(T)}{H_0}$$

$$\therefore T^2 = \left(1 - \frac{H_C(T)}{H_0}\right)(T_C)^2$$

$$\therefore T = \sqrt{\left(1 - \frac{H_C(T)}{H_0}\right)}\,T_C = \sqrt{\left(1 - \frac{H_0/2}{H_0}\right)}\,T_C$$

$$= \sqrt{0.5}\,T_C = 0.707\,T_C$$

The transition temperature for Pb is 7.2K. However, at 5K it loses the superconducting property if subjected to a magnetic field of 3.3×10^4 A/m. Find the maximum value of H which will allow the metal to retain its superconductivity at 0K.

Given : $T_C = 7.2\ K$ $\qquad H_C(T) = 3.3 \times 10^4\ A/m$ at $T = 5\ K$

$H_0 = ?\ A/m$ at $T = 0\ K$

$$H_C(T) = H_0\left(1 - \left(\frac{T}{T_C}\right)^2\right)$$

$$\therefore H_0 = \frac{H_C(T)}{\left(1 - \left(\frac{T}{T_C}\right)^2\right)} = \frac{3.3 \times 10^4}{\left(1 - \left(\frac{5}{7.2}\right)^2\right)} = 6.37 \times 10^4\ A/m$$

A long thin superconducting wire of a metal produces a magnetic field 105×10^3 A/m on its surface due to the current through it at a certain temperature T. The critical field of the metal is 150×10^3 A/m at absolute zero. The critical temperature Tc of the metal is 9.2 K. What is the value of T?

Given : $H_C(T) = 105 \times 10^3 \frac{A}{m}$ at T

$T_C = 9.2$ K

$H_0 = 150 \times 10^3 \frac{A}{m}$ at T = 0 K

T = ?

$$H_C(T) = H_0\left(1 - \left(\frac{T}{T_C}\right)^2\right)$$

$$\therefore 1 - \left(\frac{T}{T_C}\right)^2 = \frac{H_C(T)}{H_0}$$

$$\therefore \left(\frac{T}{T_C}\right)^2 = 1 - \frac{H_C(T)}{H_0}$$

$$\therefore (T)^2 = \left(1 - \frac{H_C(T)}{H_0}\right)(T_C)^2$$

$$\therefore T = \sqrt{\left(1 - \frac{H_C(T)}{H_0}\right)}\, T_C$$

$$= \sqrt{\left(1 - \frac{105\times10^3}{150\times10^3}\right)} \times 9.2 = 5.06 \text{ K}$$

Calculate the critical current which can flow through a long thin super conducting wire of diameter 1 mm. The critical magnetic field is 7.9×10^3 Amp m^{-1}.

Soution:

Given data:

$$\text{Diameter of the wire } d = 1 \text{ mm} = 1 \times 10^{-3} \text{m}$$

$$\text{radius of the wire } r = \frac{d}{2} = \frac{1 \times 10^{-3}}{2} \text{ m}$$

$$\text{The critical magnetic field } H_c = 7.9 \times 10^3 \text{ Amp m}^{-1}$$

Critical current flowing through the wire

$$I_c = 2\pi r H_c$$

$$= 2 \times 3.14 \left(\frac{1 \times 10^{-3}}{2}\right)(7.9 \times 10^3)$$

$$\boxed{I_c = 24.81 \text{ Amp}}$$

The superconducting transition temperature of lead of 7.26 K. The initial field at 0 K is 64×10^3 Amp m^{-1}. Calculate the critical field at 5 K.

Solution:

Given data:

Critical temperature $T_C = 7.26$ K

Critical field $H_0 = 64 \times 10^3$ Amp m^{-1}

Temperature $T = 5K$

The critical field $H_C = H_0 [1 - (T/T_C)^2]$

$$= 64 \times 10^3 [1 - (5/7.26)^2]$$

$$= 64 \times 10^3 \times 0.5257$$

$$H_C = 33.644 \times 10^3 \text{ Amp m}^{-1}$$

Find the critical current which can pass through a long thin superconducting wire of aluminum of diameter 2 mm, the critical magnetic field for aluminum is 7.9×10^3 A m^{-1}.

Solution:

Given data:

The critical magnetic field $H_C = 7.9 \times 10^3 \text{ A m}^{-1}$

$$\text{Radius } r = \frac{\text{Diameter}}{2}$$

$$= \frac{2}{2} = 1 \times 10^{-3}$$

$$\text{Critical current } I_C = 2\pi r H_C$$

$$= 2 \times 3.14 \times 1 \times 10^{-3} \times 7.9 \times 10^3 \text{ A m}^{-1}$$

$$I_C = 49.65 \text{ A}$$

$$\text{Critical current } I_C = 49.65 \text{ A}$$

Prove that susceptibility of superconductor is -1 and relative permeability is zero.

Solution:

Given data:

We know, the induced magnetic field $B = \mu_0(M+H)$ ...(1)

In superconductor, $B = 0$

Therefore, $0 = \mu_0(M+H)$

Since $\mu_0 \# 0$, $M = -H$

$$\frac{M}{H} = \chi = -1 \quad ...(2)$$

Also, $\chi = \mu_r - 1$

$$-1 + 1 = \mu_r = 0$$

Therefore the susceptibility $x = -1$

and Relative permeability $\mu_r = 0$

A superconducting tin has a critical temperature of 3.7 K at zero magnetic field and a critical field of 0.0306 Tesla at 0 K. Find the critical field at 2 K.

Solution:

Given data:

Critical temperature $T_c = 3.7$ K

Critical field $H_c = 0.0306$ Tesla

Temperature $T = 2$ K

The critical magnetic field

$$H_c = H_0\left[1-\left[\frac{T^2}{T_c^2}\right]\right]$$

$$= 0.0306\left[1-\left[\frac{2}{3.7}\right]^2\right] = 0.0216 \text{ Tesla}$$

The critical magnetic field $H_c = 0.0216$ Tesla.

Calculate the critical current and current density for a wire of a lead having a diameter of 1 mm at 4.2 K. The critical temperature for lead is 7.18 K and $H = 6.5 \times 10^4$ A m^{-1}.

Solution:

Given data:

Critical temperature	$T_C = 7.18$ K 7.18 K
Critical field	$H_0 = 6.5 \times 10^4$ A m^{-1}
Temperature	$T = 4.2$ K
Radius of the wire	$r = 0.5 \times 10^{-3}$ m

The critical magnetic field $H_C = H_0\left[1-\left[\frac{T^2}{T_C^2}\right]\right]$

$$= 6.5 \times 10^4\left[1-\left[\frac{4.2}{7.18}\right]^2\right]$$

$$= 4.276 \times 10^4 \text{ A m}^{-1}$$

i) Critical current $I_C = 2\pi r\, H_C$

$$= 2 \times 3.14 \times 0.5 \times 10^{-3} \times 4.276 \times 10^4$$

$$= 134.39 \text{ A}$$

ii) Critical density $$J_C = \frac{I_C}{\pi r^2}$$

$$= \frac{134.39}{3.15 \times (0.5 \times 10^{-3})^2}$$

$$= 1.71 \times 10^8 \text{ A m}^{-2}$$

Critical current $I_C = 134.39$ A

Critical density $J_C = 1.71 \times 10^8 \text{ A m}^{-2}$