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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February 2025 Semester End Main Examinations

Programme: B.E.

Semester: IV

Branch: Artificial Intelligence and Machine Learning

Duration: 3 hrs.

Course Code: 22AM4PCPSM

Max Marks: 100

Course: Probability and Statistics for Machine Learning

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I			CO	PO	Marks
1	a)	<p>In a community, 70% of the people use social media platform X, 50% use social media platform Y, and 30% use both platforms.</p> <p>i) What percentage of people use either platform X or platform Y or both?</p> <p>ii) What percentage of people use neither platform X nor platform Y?</p>	CO1	PO1	06
	b)	<p>At a college, the probability that a student enrolls in a science class is 0.4, and the probability that a student enrolls in a history class is 0.6. The probability that a student enrolls in a science class given that they are enrolled in a history class is 0.3.</p> <p>i) What is the probability that a student enrolls in both science and history classes?</p> <p>ii) What is the probability that a student enrolls in either science or history classes?</p> <p>iii) Is enrollment in science and history classes independent?</p> <p>iv) Is enrollment in science and history classes mutually exclusive?</p>	CO1	PO1	08
	c)	Define sample space and event. Prove that sample space of N possible outcomes yields 2^N possible events with an example.	CO1	PO2	06
OR					
2	a)	A computer program is tested by 3 independent tests. When there is an error, these tests will discover it with probabilities 0.2, 0.3, and 0.5, respectively. Suppose that the program contains an error. What is the probability that it will be found by at least one test?	CO1	PO2	06
	b)	Discuss the various definitions of probability, including the Classical, Frequentist and Axiomatic approaches. Highlight the merits and demerits of each definition with suitable examples.	CO2	PO1	08
	c)	In random arrangement of the letters of the word 'ENGINEERING', what is the probability that vowels always occur together.	CO2	PO2	06

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

UNIT - II						
3	a)	<p>In a factory, the probability that a machine produces a defective item is 0.1. If 10 items are produced by the machine, what is the probability that:</p> <ul style="list-style-type: none"> i) Exactly 2 items are defective? ii) At most 2 items are defective? iii) More than 2 items are defective? 	CO1	PO2	06	
	b)	<p>Time between arrival of customers at a coffee shop is to be modelled, and the average time between arrivals of customers is 8 minutes ($\lambda = 1/8$).</p> <ul style="list-style-type: none"> i) Calculate the probability that the next customer will arrive within 4 minutes of the previous one? ii) Compute the probability that the next customers will arrive in the interval of 4 to 10 minutes? 	CO1	PO2	08	
	c)	<p>For the Bernoulli distribution derive mean, variance and establish relationship between them.</p>	CO1	PO2	06	
	OR					
4	a)	<p>A program consists of two modules. The number of errors, X_1, in the 1st module and the number of errors, X_2, in the 2nd module have the joint distributions $P(0,0) = P(0,1) = P(1,0) = 0.2$, $P(1,1) = P(1,2) = P(1,3) = 0.1$, $P(0,2) = P(0,3) = 0.05$. The maximum number errors in X_1 are 1 and X_2 is 3</p> <p>Find:</p> <ul style="list-style-type: none"> i) Marginal Distributions of total number of errors. ii) The Probability of number of errors in the first module. iii) The distribution of the total number of errors in the program. 	CO2	PO4	06	
	b)	<p>Obtain the mean and variance of Exponential distribution.</p>	CO1	PO2	08	
	c)	<p>The score on a quiz is uniformly distributed between 40 and 80. A student is randomly selected and is known to have scored above 60. What is the probability that the student scored between 65 and 75?</p>	CO1	PO2	06	
	UNIT - III					
5	a)	<p>Prove unbiasedness, consistency and asymptotic normality of sample mean.</p>	CO2	PO3	08	
	b)	<p>Given Data 8,12,15,18,19,22,25,30,31,150</p> <p>The Mean and Standard Deviation of the dataset are 33 and 41.76, respectively.</p> <ul style="list-style-type: none"> i) Identify the quartiles. ii) Identify outliers using the $1.5 * \text{IQR}$ rule. iii) Delete the detected outliers and identify the mean, quartiles, and standard deviation again. iv) Make a conclusion about the effect of outliers on basic descriptive statistics. 	CO2	PO3	08	
	c)	<p>Illustrate the relationship between Mean, Median and Mode.</p>	CO2	PO2	04	
	OR					
6	a)	<p>A researcher analyses test scores of 200 students. She calculates the mean, median, and standard deviation and creates histograms.</p>	CO3	PO3	06	

		<p>She then uses the data to predict overall student performance and tests score differences between two departments.</p> <ol style="list-style-type: none"> Identify the descriptive statistics used. Identify the inferential statistics used. Explain the difference between descriptive and inferential statistics using this example. 																					
	b)	<p>An incomplete frequency distribution is given as follows:</p> <table border="1"> <tr> <td>x</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60-70</td><td>70-80</td><td>Total</td> </tr> <tr> <td>f</td><td>12</td><td>30</td><td>?</td><td>65</td><td>?</td><td>25</td><td>19</td><td>230</td> </tr> </table> <p>Calculate the missing values by considering mean to be 70.</p>	x	10-20	20-30	30-40	40-50	50-60	60-70	70-80	Total	f	12	30	?	65	?	25	19	230	CO3	PO2	10
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f	12	30	?	65	?	25	19	230															
	c)	<p>Explain why standard deviation is used to measure the dispersion of data, even though variance is also available as a measure.</p>	CO1	PO1	04																		
UNIT - IV																							
7	a)	<p>The lifetime of a certain type of electronic component follows a Gamma distribution with unknown shape parameter α and scale parameter β. Assume that five components are tested independently, and their lifetimes are recorded as follows: 75, 85, 90, 80, and 95 hours.</p> <ol style="list-style-type: none"> Calculate the moment estimators for α and β based on the given data. Solve α and β using the measured lifetimes. 	CO2	PO3	10																		
	b)	<p>The weights of a particular type of package are normally distributed with a known population standard deviation of 4.2 pounds. A random sample of 50 packages has an average weight of 25.3 pounds. Estimate the population mean with:</p> <ol style="list-style-type: none"> 85% confidence. 95% confidence. 99% confidence. <p>Also conclude what effect an increase in confidence level has on the critical value, margin of error, and width of the confidence interval?</p>	CO2	PO3	10																		
		OR																					
8	a)	<p>A study was conducted to assess the preference for two different online learning platforms X and Y. 300 random users were surveyed about their preference for Platform X, and 165 users reported a preference for Platform X. Another 300 random users were surveyed about their preference for Platform Y, and 180 users reported a preference for Platform Y. At 1% level of significance, determine if there is a difference in the proportion of users who prefer the two platforms.</p>	CO2	PO3	10																		
	b)	<p>Derive the maximum likelihood estimator of a geometric distribution function $f(x,p) = p(1-p)^{x-1}$.</p>	CO2	PO3	10																		

		UNIT - V																			
9	a)	Given a logistic regression model $\text{logit}(p) = b_0 + b_1 * \text{study hours}$. The threshold for classification is 0.5. Based on the following study hours, classify each student as "Pass" (1) or "Fail" (0):			CO3	PO3	10														
		<table border="1"> <thead> <tr> <th>Student name</th> <th>Study Hours</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>10</td> </tr> <tr> <td>B</td> <td>15</td> </tr> <tr> <td>C</td> <td>20</td> </tr> <tr> <td>D</td> <td>25</td> </tr> <tr> <td>E</td> <td>30</td> </tr> </tbody> </table>	Student name	Study Hours	A	10	B	15	C	20	D	25	E	30							
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	b)	Given the following data:			CO3	PO3	10														
		<table border="1"> <thead> <tr> <th></th> <th>Attribute 1</th> <th>Attribute 2</th> </tr> </thead> <tbody> <tr> <td>Observation 1</td> <td>4</td> <td>5</td> </tr> <tr> <td>Observation 2</td> <td>3</td> <td>6</td> </tr> <tr> <td>Observation 3</td> <td>2</td> <td>7</td> </tr> <tr> <td>Observation 4</td> <td>1</td> <td>8</td> </tr> </tbody> </table>		Attribute 1	Attribute 2	Observation 1	4	5	Observation 2	3	6	Observation 3	2	7	Observation 4	1	8				
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Observation 3	2	7																			
Observation 4	1	8																			
		With the help of PCA, reduce its dimensionality.																			
		OR																			
10	a)	At a tire company, 200 customers were asked to record the tire pressure of their vehicles and the number of years they have used the tires. The results are summarized in the table:			CO3	PO3	10														
		<table border="1"> <thead> <tr> <th></th> <th>Sample Mean</th> <th>Standard Deviation</th> </tr> </thead> <tbody> <tr> <td>Tire Pressure (psi)</td> <td>35</td> <td>5</td> </tr> <tr> <td>Years Used</td> <td>6</td> <td>2</td> </tr> </tbody> </table>		Sample Mean	Standard Deviation	Tire Pressure (psi)	35	5	Years Used	6	2										
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Tire Pressure (psi)	35	5																			
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		The sample correlation coefficient between tire pressure and years of use is $r = -0.25$.																			
		i) Compute the least squares regression line which describes how the number of miles per gallon depends on the mileage. What does the obtained slope and intercept mean in this situation?																			
		ii) Use R^2 to evaluate its goodness of fit. Is this a good model?																			
	b)	The following statistics were obtained from a sample of size $n=60$			CO3	PO3	10														
		<ul style="list-style-type: none"> The predictor variable X has a mean of 40 and a variance of 10; The response variable Y has a mean of 15 and a variance of 5; The sample covariance between X and Y is 7. 																			
		i) Estimate the linear regression equation for predicting Y based on X.																			
		ii) Compute the ANOVA table.																			
