

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

May 2023 Semester End Make-Up Examinations

Programme: B.E.

Branch: Civil Engineering

Course Code: 20CV5PETOE

Course: Theory of Elasticity

Semester: V

Duration: 3 hrs.

Max Marks: 100

Date: 17.05.2023

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

- 1 a) The components of stress at a point are: $\sigma_x = 20 \text{ N/mm}^2$, $\sigma_y = -40 \text{ N/mm}^2$, $\sigma_z = 80 \text{ N/mm}^2$, $\tau_{xy} = 40 \text{ N/mm}^2$, $\tau_{yz} = -60 \text{ N/mm}^2$, $\tau_{zx} = 20 \text{ N/mm}^2$. Determine (a) The principal stresses at the point (b) Deviatoric and spherical stress tensors. **10**
- b) When the stress tensor at a point with reference to axes (x, y, z) is given by the array,
$$\begin{bmatrix} 9 & 6 & 3 \\ 6 & 5 & 2 \\ 3 & 2 & 4 \end{bmatrix} \text{ MPa}$$
 show that by transformation of the axes by 45° about the z-axis, the stress invariants remain unchanged. **10**

UNIT - II

- 2 a) The following strains were measured in a structure during the test by means of 45° strain rosette, $\epsilon_0 = 700 \times 10^{-6}$, $\epsilon_{45} = -100 \times 10^{-6}$ and $\epsilon_{90} = 200 \times 10^{-6}$. Determine the magnitude and direction of principal strains. **10**
- b) The displacement field imposed at a point in a body is given by $u = 0.01xy + 0.02y^2$, $v = 0.02x^2 + 0.01z^3y$, $w = 0.01xy^2 + 0.05z^2$. Obtain the strain components at the point (3, 2, -5). What is the octahedral normal and shearing strains. **10**

UNIT - III

- 3 a) Explain plane-stress and plane-strain conditions. Give examples. Derive stress strain relation for plane-stress condition. **10**
- b) A square plate with 800 mm sides parallel to the x and y-axes has a uniform thickness of 10 mm and is made of an isotropic steel ($E = 82.6 \text{ GPa}$ and $\mu = 0.29$). The plate is subjected to plane strain state. If $\sigma_x = \sigma_1 = 500 \text{ MPa}$ and $\epsilon_x = 2\epsilon_y$, determine the magnitude of $\sigma_y = \sigma_2$ and $\sigma_z = \sigma_3$, assuming linearly elastic conditions. **10**

UNIT - IV

- 4 a) Show that for a cantilever beam, length L , depth h and unit width, loaded by a concentrated load at free end, the Airy's stress function $\phi = A (xy^3 - \frac{3}{4} xyh^2)$ is satisfying the loading condition. Find the stresses in the beam. **10**
- b) Derive the biharmonic equation in Cartesian coordinates for a two-dimensional plane stress state neglecting body forces. Use Airy's stress function. **10**

OR

- 5 a) The following are the Airy's stress functions, figure out what problems they solve by calculating the stresses. Plot the stress variations along the x and y directions. **08**
- i. $\phi = axy^3$
ii. $\phi = bxy$
iii. $\phi = axy^3 + bxy$
iv. $\phi = ay^3$
- b) Show that the Airy's stress function, $\phi = \frac{a}{6} xy^3 + cxy$ represents stress distribution in a simply supported beam subjected to central concentrated load. The cross-section of the beam is $b \times 2h$. where ' $2h$ ' is the depth of the beam. **12**

UNIT - V

- 6 a) A thick cylinder of inner radius 10 cm and outer radius 15 cm is subjected to an internal pressure of 12 MPa. Determine the radial and hoop stresses in the cylinder at the inner and outer surfaces. **10**
- b) Given the stress function in polar coordinates determine the stress components and check for compatibility. **10**

$$\phi = \frac{P}{\pi} r \theta \cos \theta$$

OR

- 7 Show that the maximum stress concentration for an infinite plate with a circular hole $\sigma_{\theta} = 3\sigma_0$ when the plate is subjected to uniform tension far away from the hole. **20**
