

BMS COLLEGE OF ENGINEERING

Bull Temple Road, Bangalore-560019

ELEMENTS OF ELECTRONICS ENGINEERING

SCHEME: UNIT 4

DIGITAL ELECTRONICS

PART-I

1. State and prove De Morgan's law. (5 marks)

Ans:

DeMorgan's Theorems:

1. $\overline{A \cdot B} = \overline{A} + \overline{B}$

A	B	$A \cdot B$	$\overline{A \cdot B}$	\overline{A}	\overline{B}	$\overline{A} + \overline{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

2. $\overline{A + B} = \overline{A} \cdot \overline{B}$

A	B	$A + B$	$\overline{A + B}$	\overline{A}	\overline{B}	$\overline{A} \cdot \overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

2. Discuss the universality of NAND gate. (5 marks)

Ans:

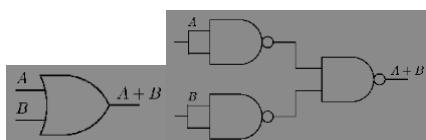
NOT gate



AND gate



OR gate



3. Write the logic symbol and truth table of Basic Gates

(5 marks)

AND



Inputs	Output	
A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

Ans:

OR



Inputs	Output	
A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

NOT



Input	Output
A	C
0	1
1	0

4. State and prove duality theorem

(5 marks)

Ans:

Duality Principle: The duality property of Boolean algebra state that all binary expressions remain valid when following two steps are performed:

Step 1 : Interchange OR and AND operators.

Step 2 : Replace all 1's by 0's and 0's by 1's.

Ex: $A(B+C)=AB+AC$

- Dual of the expression is $A+(BC)=(A+B)+(A+C)$
- $1 \cdot 0 = 0$ and its Dual is $0+1=1$

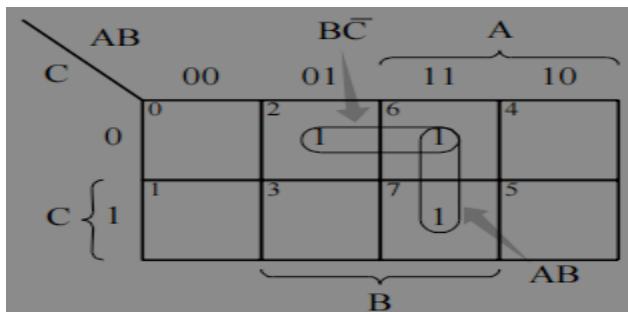
5. Write the steps to reduce expression using K maps for SOP.

(5 marks)

Ans:

- An n -variable K-map has 2^n cells with each cell corresponding to an n -variable truth table value.
- K-map cells are labeled with the corresponding truth-table row.
- K-map cells are arranged such that adjacent cells correspond to truth rows that differ in only one bit position
- K-map cells that are physically adjacent are also logically adjacent. Also, cells on an edge of a K-map are logically adjacent to cells on the opposite edge of the map.
- If two logically adjacent cells both contain logical 1s, the two cells can be combined to eliminate the variable that has value 1 in one cell's label and value 0 in the other. This is equivalent to the algebraic operation, $aP + a'P = P$ where P is a product term not containing a or a' .
- A group of cells can be combined only if all cells in the group have the same value for some set of variables.

Example: Simplify $f = A'BC' + ABC' + ABC$

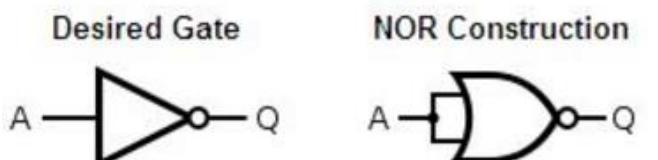


After Simplification: $C'B + AB$

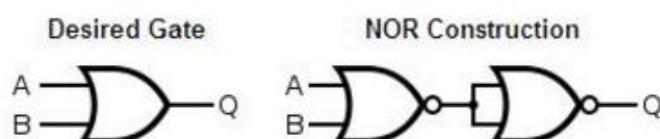
6. Discuss the universality of NOR gate. (5 marks)

Ans:

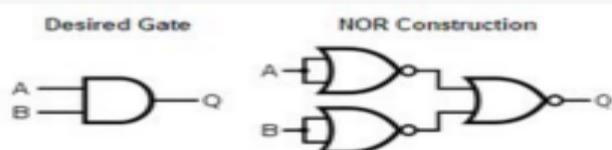
NOT Gate:



OR Gate:



AND Gate:



7. Explain the working of 4x1 multiplexer?

8. Explain the working of 3 to 8 decoder?

9. With truth table explain the working of SR flip-flop?

10. With truth table explain the working of JK flip-flop?

PART-II (APPLY)

1. Implement the expression $F=A(B+CD)+B\bar{C}$ using (10 marks)

a) AND -OR -NOT gates

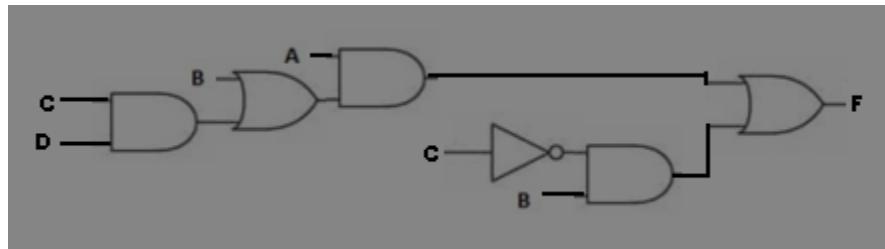
b) Only NAND gates

c) Only NOR gates.

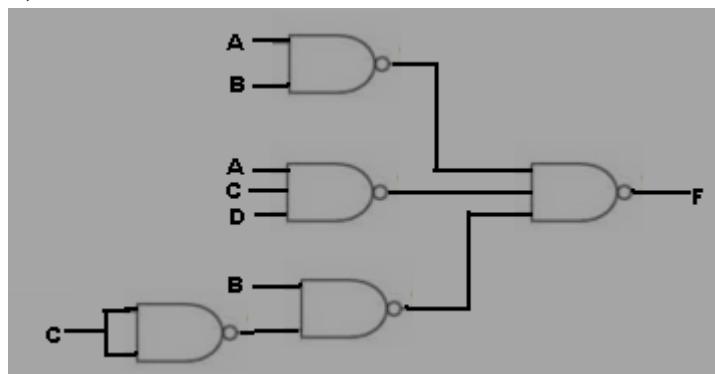
Compare which circuit uses minimum number of gates.

Ans:

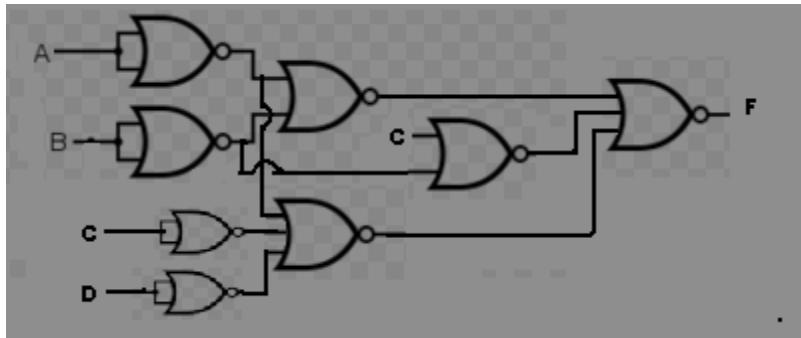
a)



b)



c)



The expression using only NAND gates uses minimum number of gates-5

2. Simplify the following Boolean expression using Boolean Laws and Realize using only NAND gates. (10 marks)

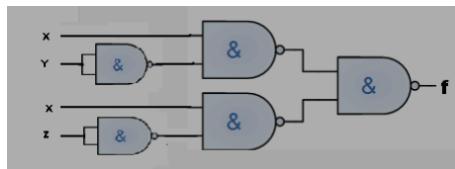
i) $f(W, X, Y, Z) = \overline{WXY\bar{Z}} + XY\bar{Z} + X\bar{Y}\bar{Z} + X\bar{Y}Z$

ii) $f(A, B, C) = AB + \overline{ABC} + A\bar{C} + \overline{A}\bar{B}C$

Ans:

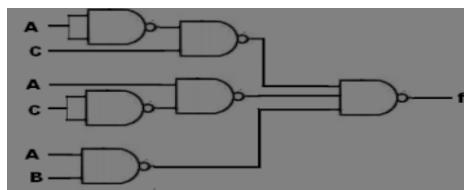
(i) $f(W, X, Y, Z) = \overline{WXY\bar{Z}} + XY\bar{Z} + X\bar{Y}\bar{Z} + X\bar{Y}Z$

$f(W, X, Y, Z) = X\bar{Y} + X\bar{Z}$



(ii) $f(A, B, C) = AB + \overline{ABC} + A\bar{C} + \overline{A}\bar{B}C$

$f(A, B, C) = AB + \overline{AC} + A\bar{C}$



3. Simplify the following Boolean expression using Boolean Laws and Realize using only NOR gates. (10 marks)

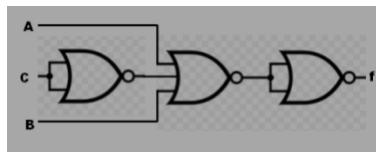
i) $f(A, B, C) = A\bar{B}C + \bar{C} + BC$

ii) $f(A, B, C) = \overline{AB} + \bar{B} + \overline{CA}$

Ans:

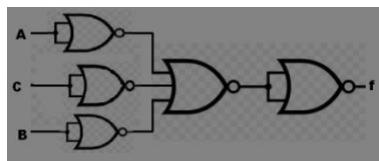
(i) $f(A, B, C) = A\bar{B}C + \bar{C} + BC$

$$f(A, B, C) = A + \bar{C} + B$$



$$(ii) \quad f(A, B, C) = \bar{A}\bar{B} + \bar{B} + \bar{C}A$$

$$f(A, B, C) = \bar{A} + \bar{B} + \bar{C}$$



4. Simplify the following using K-map and realize using minimum number of Basic gates.

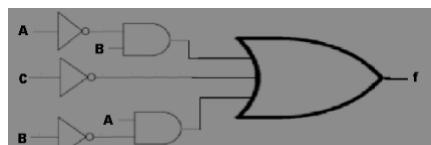
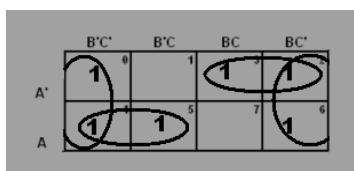
(i) $f(A, B, C) = \sum m(0, 2, 3, 4, 5, 6)$ (10 marks)

(ii) $Y(A, B, C) = \sum m(0, 1, 5, 7)$

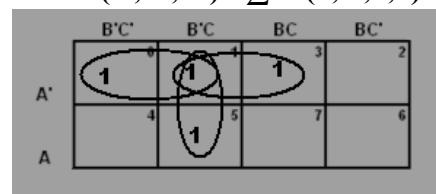
(iii) $f(x, y, z) = \sum m(0, 1, 3, 4, 5, 7)$ using K-map

Ans:

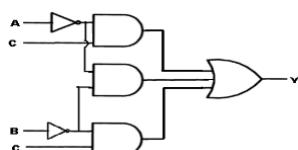
(i) $f(A, B, C) = \sum m(0, 2, 3, 4, 5, 6) \quad f(A, B, C) = A\bar{B} + \bar{C} + B\bar{A}$



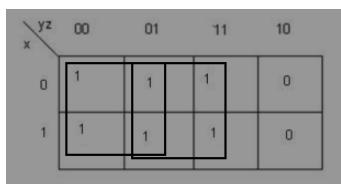
ii) $Y(A, B, C) = \sum m(0, 1, 5, 7)$



$$f(A, B, C) = \bar{B}C + \bar{A}C + \bar{B}\bar{A}$$



iii) $f(x, y, z) = \sum m(0, 1, 3, 4, 5, 7)$ using K-map



$$f(X, Y, Z) = \bar{Y} + Z$$

5. Simplify and realize using only 2 input NOR gates. (10 marks)

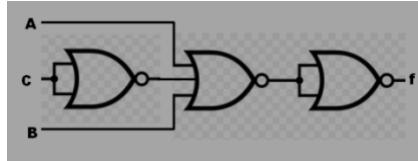
$$(i) f(A, B, C) = A\bar{B}C + \bar{C} + BCA + BC$$

$$(ii) f(A, B, C) = \bar{A}\bar{B} + \bar{B} + \bar{C}A + \bar{B}\bar{C}$$

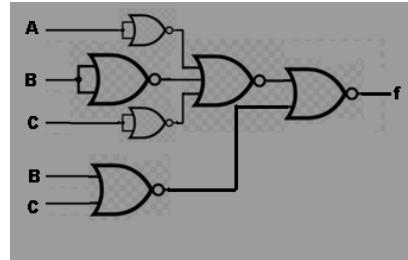
Ans:

$$(i) f(A, B, C) = A\bar{B}C + \bar{C} + BCA + BC$$

$$f(A, B, C) = A + \bar{C} + B$$



$$(ii) f(A, B, C) = \bar{A}\bar{B} + \bar{B} + \bar{C}A + \bar{B}\bar{C}$$



$$f(A, B, C) = \bar{B}\bar{C} + BCA$$

6. If the variables A, B and C can take only the values 0 and 1, prove the following identities of Boolean algebra (10 marks)

$$i) \bar{X}\bar{Y}Z + \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} = \bar{X}(\bar{Y} + \bar{Z})$$

$$ii) \bar{A}B\bar{C} + B\bar{C}\bar{D} + AD = AD + B\bar{C} + B\bar{D}$$

Ans:

$$i) \bar{X}\bar{Y}Z + \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} = \bar{X}(\bar{Y} + \bar{Z})$$

$$\begin{aligned} LHS &= \bar{X}\bar{Y}Z + \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} \\ &= \bar{X}\bar{Y} + \bar{X}Y\bar{Z} \\ &= \bar{X}(\bar{Y} + Y\bar{Z}) \end{aligned}$$

$$= \bar{X}(\bar{Y} + \bar{Z})$$

= RHS

$$ii) \bar{A}B\bar{C} + B\bar{C}\bar{D} + AD = AD + B\bar{C} + B\bar{D}$$

$$\begin{aligned} LHS &= \bar{A}B\bar{C} + B\bar{C}\bar{D} + AD \\ &= \bar{A}B\bar{C} + B(\bar{C} + \bar{D}) + AD \\ &= B\bar{C}(\bar{A} + 1) + B\bar{D} + AD \end{aligned}$$

$$= B\bar{C} + B\bar{D} + AD$$

= RHS

7. Simplify the following functions by K map method

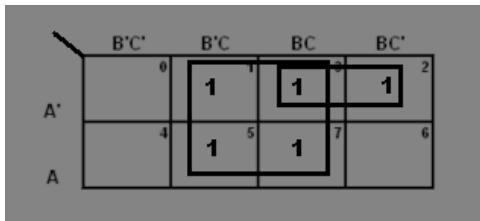
(10 marks)

$$i) F = \bar{A}B + \bar{A}\bar{C} + BC + A\bar{B}\bar{C}$$

$$ii) F = \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C$$

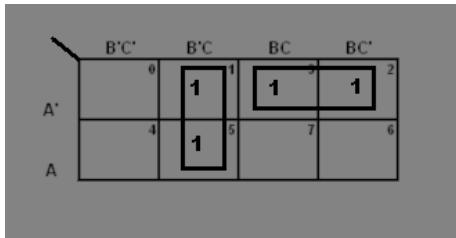
Ans:

i) $F = \bar{A}B + \bar{A}C + BC + A\bar{B}C$



$$F = \bar{A}B + C$$

ii) $F = \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C$

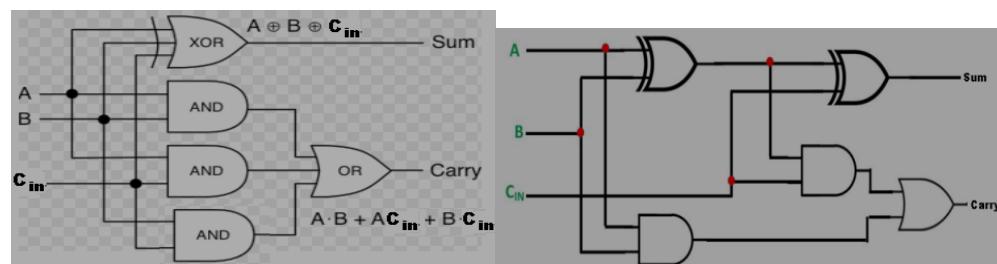
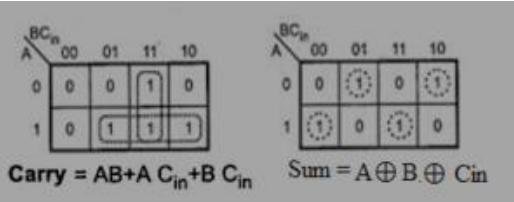


$$F = \bar{A}B + \bar{B}C$$

14. Write the truth table and gate level logic diagram for the full adder, realize the same using Half adders (10 marks)

Ans:

Input			Output	
A	B	Cin	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



8. Simplify the following functions by K map method

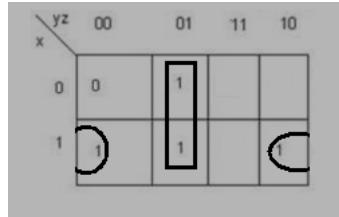
(10 marks)

i) $F = xy' + x'y'z + xyz'$

ii) $X = a'b'c' + a'bc' + a'bc + ab'c + abc'$

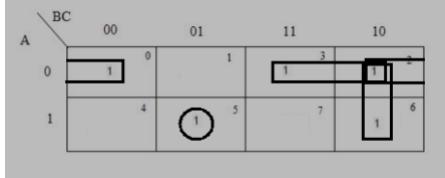
Ans:

i) $F = xy' + x'y'z + xyz'$



$$F = y'z' + xz'$$

ii) $X = a'b'c' + a'bc' + a'bc + ab'c + abc'$



$$X = a'c' + a'b + ab'c + bc'$$

9. If the variables A, B and C can take only the values 0 and 1, prove the following identities of Boolean algebra (10 marks)

i) $\bar{A}BC + A\bar{B}\bar{C} + ABC + A\bar{B}\bar{C} = BC + A\bar{C}$

ii) $AB + C(\bar{A} \odot B) = AB + BC + CA$

Ans:

i). $\bar{A}BC + A\bar{B}\bar{C} + ABC + A\bar{B}\bar{C} = BC + A\bar{C}$

$$\begin{aligned} RHS &= BC(\bar{A} + A) + (\bar{B} + B)A\bar{C} \\ &= BC + A\bar{C} = LHS \end{aligned}$$

ii). $AB + C(\bar{A} \odot B) = AB + BC + CA$

$$\begin{aligned} RHS &= AB + BC + AC \\ &= AB + BC(A + \bar{A}) + AC(B + \bar{B}) \end{aligned}$$

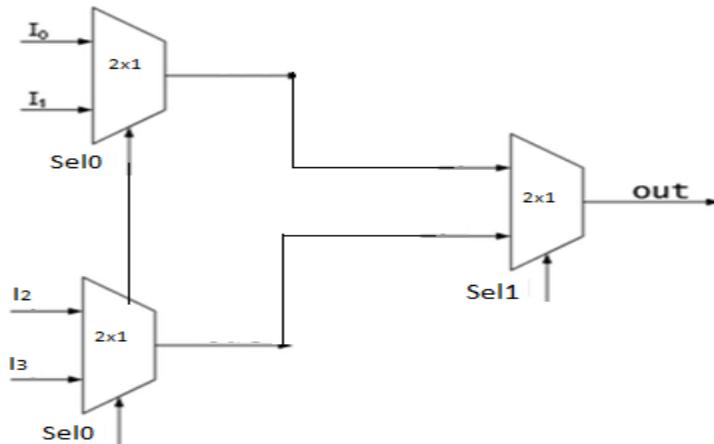
$$= AB(1 + C + C) + C(\bar{A}B + A\bar{B})$$

$$= AB + C(\bar{A}B + A\bar{B})$$

$$= AB + C(\bar{A}B + A\bar{B}) = AB + C(\bar{A} \odot B)$$

$$= LHS$$

10. Implement a 4x1 mux using 2x1 mux. explain the concept.



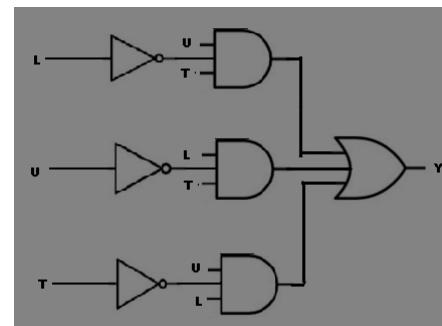
Part III (Analyse, Design, Questions from GATE)

1. The rocket motor of an air-launched missile with three inputs (Launch, Unsafe- height and target-lock), will operate if and only if any two inputs are high. Design a suitable logic circuit with minimum logic gates (5 marks)

Ans:

Launch(L)	Unsafe Height(U)	Target Lock(T)	Output(Y)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$$Y = \bar{L}U + L\bar{U} + LU$$



2. Analyze the logic circuit shown in fig. Determine the Boolean function for y and state its truth table. (5 marks)

Ans:

$$Y = (\bar{A} + \bar{B}) \cdot \bar{B} + \bar{A}C$$

$$Y = \bar{A} + \bar{C}$$

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

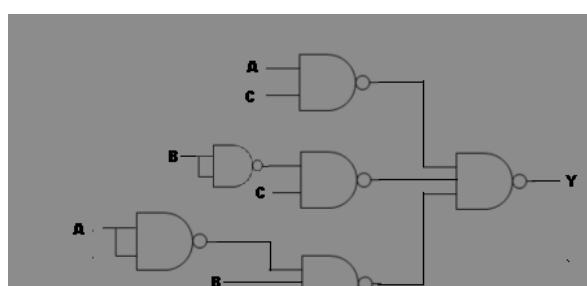
3. A logic circuit has 3 inputs A, B, C and one output Y. Y=B XOR C when A=0, and Y=C when A=1. Simplify the output expression and realize the same using only NAND gates.

(5 marks)

Ans:

$$Y = \bar{B}C + AC + \bar{A}BC$$

A	B	C	Y
0	0	0	0



0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

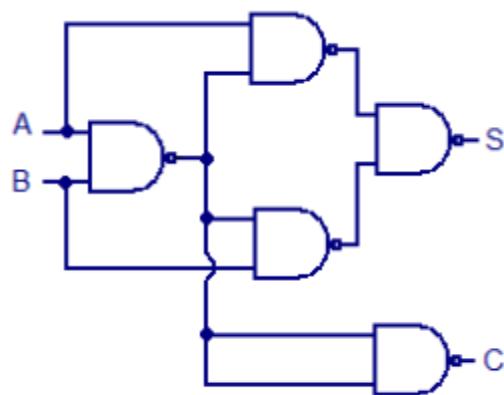
4. Design a two input adder using only NAND gates

(5 marks)

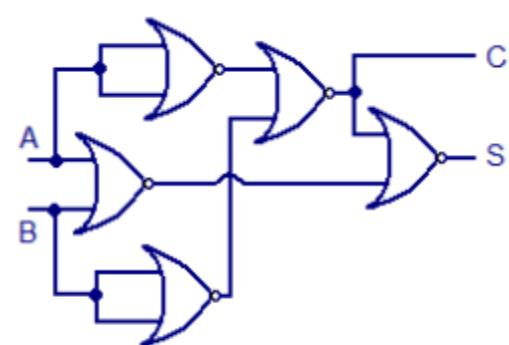
Ans:

Inputs		Outputs	
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$S = A \oplus B \quad C = AB$$



Half adder using NAND logic



Half adder using NOR logic

5. Design a logic circuit using minimum number of NOR gates to implement the following expression.

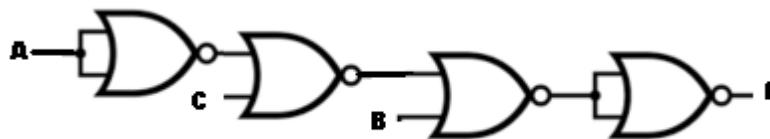
(5 marks)

Ans:

$$f(A, B, C) = \bar{A}B + B\bar{C} + BC + A\bar{B}\bar{C}$$

$$f(A, B, C) = B(\bar{A} + \bar{C} + C) + A\bar{B}\bar{C}$$

$$f(A, B, C) = B + A\bar{C}$$



6. Design a combinational circuit to realize a full adder using only NAND gates. (5 marks)

Ans:

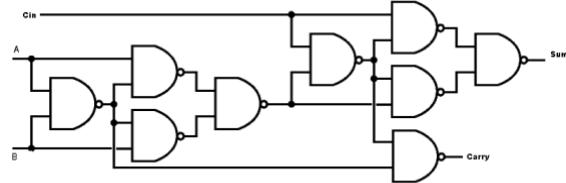
Input			Output	
A	B	Cin	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

BC _{in}		00	01	11	10
A	Cin	0	0	0	1
0	0	0	0	1	0
1	0	1	1	1	1

BC _{in}		00	01	11	10
A	Cin	0	0	1	0
0	1	0	1	0	1
1	1	1	0	0	0

$$\text{Carry} = AB + A C_{in} + B C_{in}$$

$$\text{Sum} = A \oplus B \oplus C_{in}$$



Full-Adder circuit using NAND gates

7. Design a combinational circuit to implement full adder using logic gates. (5marks)

Ans:

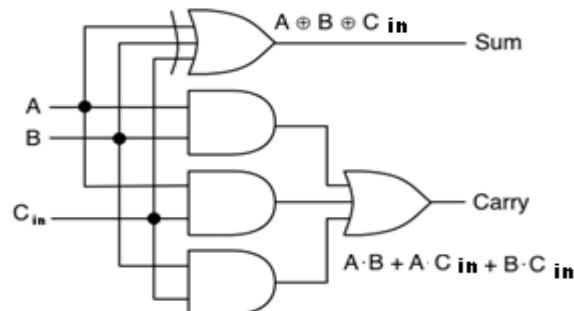
Input			Output	
A	B	Cin	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

BC_{in}	00	01	11	10
0	0	0	1	0
1	0	1	1	1

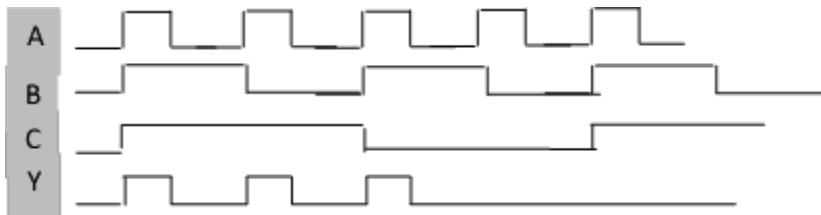
$$\text{Carry} = AB + A C_{in} + B C_{in}$$

BC_{in}	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$\text{Sum} = A \oplus B \oplus C_{in}$$



8. Analyze input and output waveforms shown in Figure 2 and write the truth table.(5 marks)



Inputs			Output
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

9. Design a circuit that has three bit binary input and a single output(Z) specified as follows:

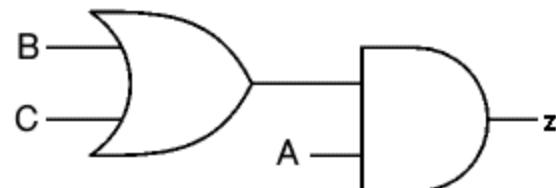
- $Z = 0$; when the input is less than $(5)_{10}$
- $Z = 1$; otherwise

Ans:

Inputs			Output
A	B	C	Z
0	0	0	0
0	0	1	0

$$Z = AC + AB$$

$$\text{or } Z = A(C + B)$$



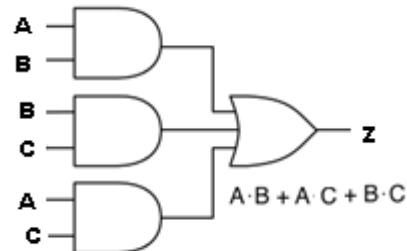
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

10. A bank locker consists of three keys; the locker will get open if any 2 keys are correctly inserted. Design a digital circuit for this scenario. (5 marks)

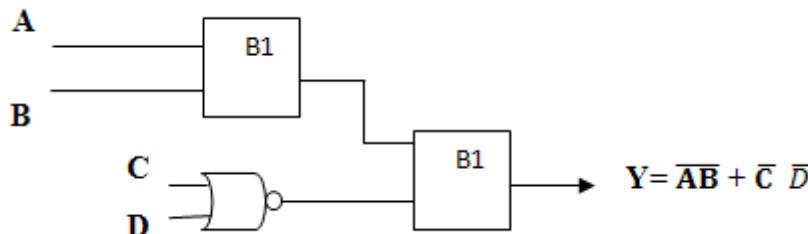
Ans:

Inputs			Output
A	B	C	Z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

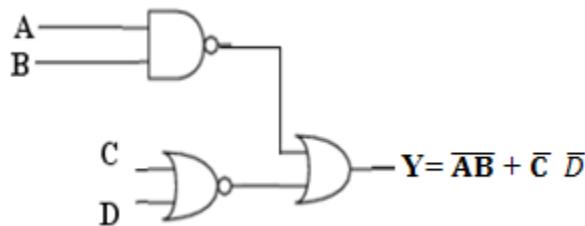
$$Z = AB + BC + AC$$



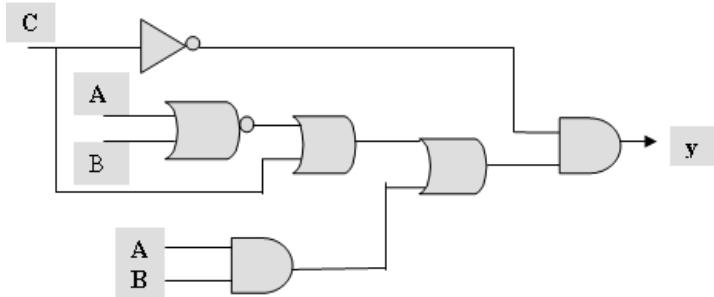
11. Analyze the given circuit to identify the logic used in the blocks B1 and B2 to obtain output Y. (5 marks)



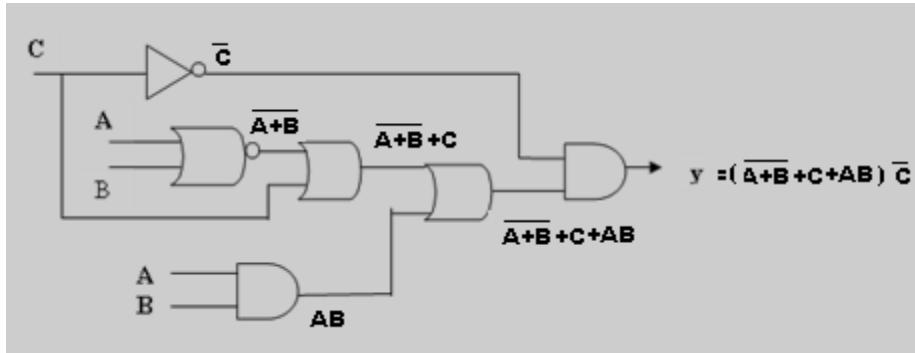
Ans:



12. Analyze the below given logic circuit for an output Y when input C=0. Write expression at each stage (5 marks)



Ans:



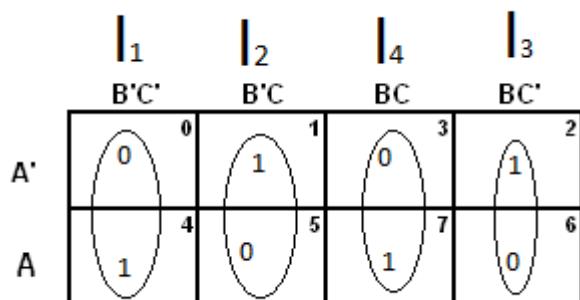
Given $C=0$, $y = \bar{A} \cdot \bar{B} + AB$

13. Design a Full adder using 4x1 mux.

For sum output : input B and C acts as select lines, as per the truth table of 4x1 mux output follows: I_1 when $BC=00$, I_2 when $BC=01$, I_3 when $BC=10$ and I_4 when $BC=11$.

Truth table :

Input			Output	
A	B	Cin	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



From the K-map expression for 4 inputs of the MUX can be written as

$$I_1 = A \quad I_2 = A' \quad I_3 = A' \quad I_4 = A$$

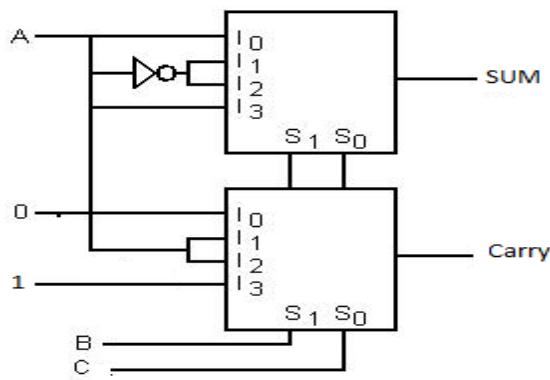
For Carry output:

From the K-map expression for 4 inputs of the MUX can be written as

$$I_1 = 0 \quad I_2 = A \quad I_3 = 1 \quad I_4 = A$$

I_1	I_2	I_3	I_4
$B'C'$	$B'C$	BC	BC'
0	1	3	2
4	5	7	6

Logic diagram for the full adder using 2 4X1 mux:



14. Design a full adder using 3 to 8 decoder.

Truth table :

Input			Output	
A	B	Cin	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Expressions:

Equation for sum

$$S = ab'c' + a'b'c + a'bc' + abc \\ = \Sigma(1,2,4,7)$$

Equation for carry

$$C_{out} = ab + ac + bc \\ = abc + a'bc + ab'c + abc' \\ = \Sigma(3, 5, 6, 7)$$

FULL ADDER

So we can implement it from decoder using OR gates as follow:

