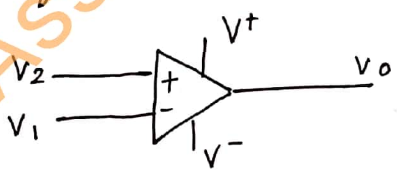
Operational Amplifiers :

An operational amplifier is a direct coupled multi-stage voltage amplifier with extremely high gain. It's behavior can be controlled by adding suitable feedback. It has a very high input impedence and very low output impedence.

Op-amps were basically used for performing mathematical operations such as addition, subtraction, integration and differentiation. Hence the name op-amp. Now they are used for more wider applications such as AC amplification, rectification, filtration, waveform generation etc.

In a practical op-amp a large number of transistors, diodes, passive elements are integrated monolithically in a small chip to make it cost effective and highly reliable.

Circuit symbol of an op-amp :-



It has 5 main terminals

1) non inverting input terminal denoted by (+) symbol
2) inverting input terminal denoted by (-) symbol.
3) output terminal
4) a positive supply voltage terminal V^+
5) a negative supply voltage terminal V^-

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from fig,

$V_1 \rightarrow$ voltage at inverting input

$V_2 \rightarrow$ voltage at non-inverting input

$V_0 \rightarrow$ output voltage

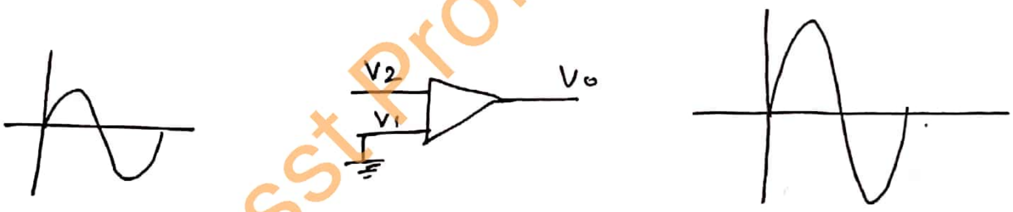
op-amp operates on bipolar or dual power supply i.e., one +ve and one −ve with a common ground terminal.

The range of $V^{\pm} = \pm 9V$ to $\pm 22V$, typically it is set to $\pm 15V$

Voltage applied at non-inverting terminal produces an inphase output.

i.e., when V_2 is applied to non inverting ~~in~~ terminal and $V_1 = 0$ then

$$V_0 = +AV_2.$$

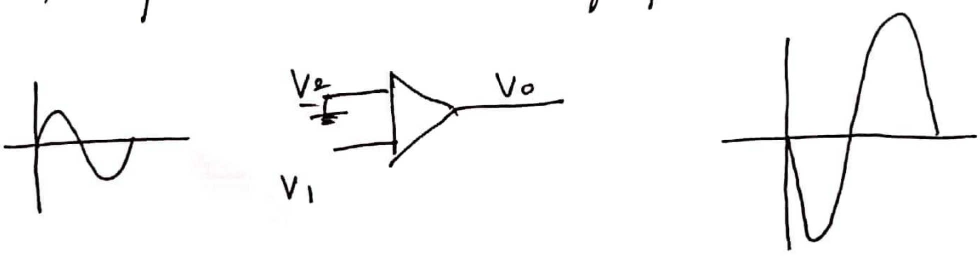


Voltage applied at inverting terminal produces an out of phase output.

i.e., when V_1 is applied to the inverting terminal and $V_2 = 0$ or ground then

$$V_0 = -AV_1$$

i.e., output is 180° out of phase with input.



Hence the output V_0 of the op-amp is proportional to the difference between input voltage i.e.,

$$V_0 = A(V_2 - V_1)$$

where $A \rightarrow$ voltage gain.

$$V_0 = A V_d$$

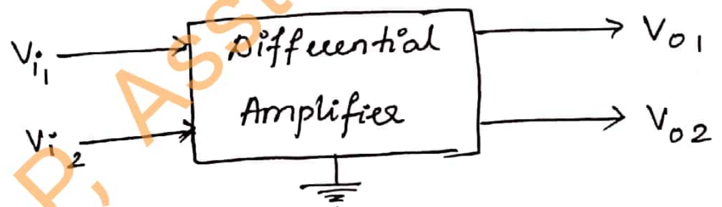
where $V_d \rightarrow$ input difference signal

Hence op-amp is also known as differential or difference amplifier.

Differential or Difference Amplifier :-

An op-amp is basically a differential amplifier which amplifies the difference between its two input voltages.

The block diagram of a differential amplifier is as shown in the figure.



The differential amplifier has two input terminals V_{i1} and V_{i2} and 2 output terminals V_{o1} and V_{o2}.

Different configurations of differential amplifier are.

1) Dual input : two input signals are applied at two input terminals.

2) Single input : Input is applied to any one terminal [other terminal is connected to ground]

3) ~~Balan~~

3) Balanced output: The output voltage is measured between two output terminals.

4) Unbalanced output: output is measured at one terminal with respect to ground.

Based on the above configurations, the different topologies of differential amplifier are

1) Dual input balanced output
2) Dual input unbalanced output
3) Single input, balanced output
4) single input, unbalanced output.

The op-amp uses dual input unbalanced output configuration.

The output of an op-amp is given by,

$$V_o = A_d(V_{i_1} - V_{i_2})$$

$A_d \rightarrow$ differential gain of the amplifier.

The output of an op amp depends on the difference of the two input signals and it also depends on ~~its~~ their average levels.

ie., $V_d = V_{i_1} - V_{i_2}$ $\qquad V_c = \frac{V_{i_1} + V_{i_2}}{2}$

$V_d \rightarrow$ difference mode signal

$V_c \rightarrow$ common mode signal.

$\therefore V_{i_1} = V_c + \frac{V_d}{2}$ $\qquad V_{i_2} = V_c - \frac{V_d}{2}$

applying superposition theorem,

Case 1: when $v_{i_1} \neq 0$ and $v_{i_2} = 0$ $\quad V_o = A_1 V_{i_1}$

Case 2: when $V_{i_1} = 0$ and $V_{i_2} \neq 0$ $\quad V_o = A_2 V_{i_2}$

$$\therefore V_o = A_1 V_{i_1} + A_2 V_{i_2}.$$

$$V_o = A_1\left[V_c + \frac{V_d}{2}\right] + A_2\left[V_c - \frac{V_d}{2}\right]$$

$$V_o = V_c\left[A_1 + A_2\right] + \frac{V_d}{2}\left[A_1 - A_2\right]$$

$\therefore$ $A_c = A_1 + A_2 \rightarrow$ common mode gain

$A_d = \dfrac{A_1 - A_2}{2} \rightarrow$ differential mode gain.

Standard Parameters:

1) Slew Rate:

Slew rate of an op-amp is defined as the maximum time rate of change of output voltage expressed in volts per micro second (V/μsec)

$$SR \text{ or } MSR = \left.\frac{dV_o}{dt}\right|_{max} \text{ V/}\mu\text{sec}.$$

It is a measure of how fast the op-amp output can change in response to changes in input voltage.

$$SR = 2\pi V_m f_{max}$$

or

$$SR = \omega_{max} V_m.$$

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2) Differential Gain (Ad)

When 2 input voltages V_1 and V_2 such that $V_1 \neq V_2$ are applied to the op-amp, then the output voltage is given by,

$$V_0 = AV_d = A_d V_d$$

$$V_0 = A_d(V_2 - V_1)$$

$$A_d = \frac{V_0}{V_2 - V_1} = \frac{V_0}{V_d}.$$

$$\boxed{A_d = \frac{V_0}{V_d}}$$

3) Common mode gain (Ac) :

When two equal input voltages are applied to the op-amp ie., $V_1 = V_2 = V_c$, the op-amp output must be ideally zero. But practically a small non-zero output voltage occurs which is given by,

$$V_0 = A_c V_c \qquad V_c = \frac{V_1 + V_2}{2}.$$

$$A_c = \frac{V_0}{V_c}$$

$A_c \rightarrow$ common mode gain of op-amp.

4) Common-mode rejection ratio [CMRR]

CMRR is a figure of merit or performance measure of an op-amp. It is defined as ratio of differential gain Ad to the common mode gain Ac

$$CMRR = \rho = \frac{|A_d|}{|A_c|}$$

It is expressed in dB

$$\boxed{CMRR = 20\log_{10}\frac{|A_d|}{|A_c|} \text{ dB.}}$$

for an ideal - op-amp CMRR = ∞

But practically Ad is a very large value and Ac is very small, hence CMRR is large.

The CMRR is a measure of the ability of the op-amp to reject signals common to both inputs ie., its ability to produce zero output voltage when the inputs are exactly identical.

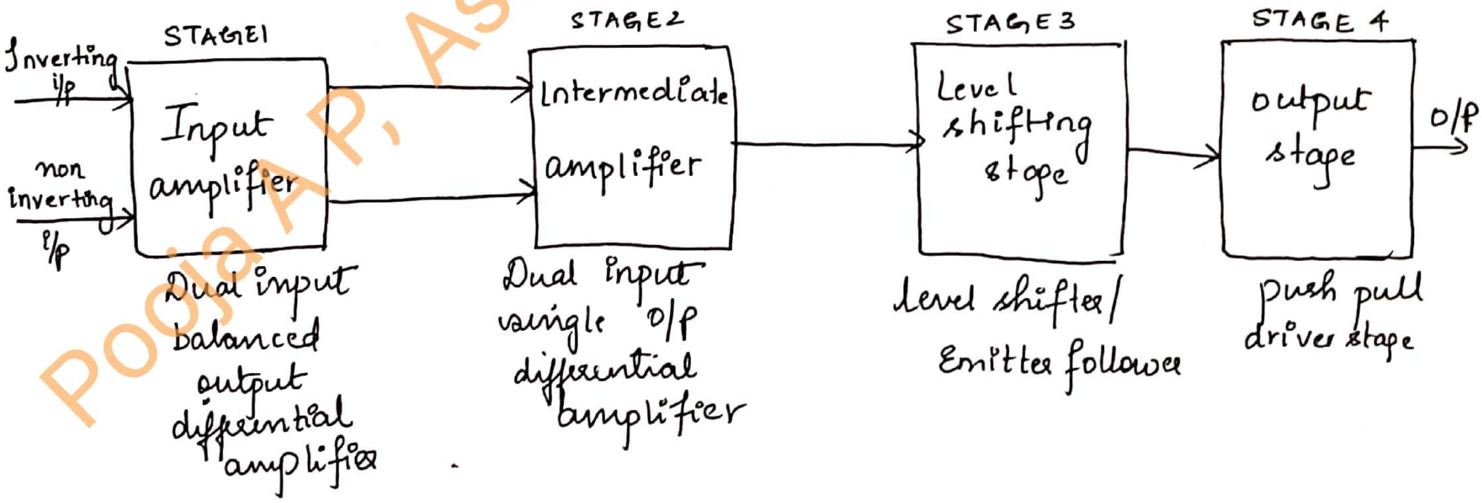
5) Power supply rejection ratio [PSRR].

It is the ability of an amplifier to maintain its output voltage as its DC power supply is varied.

$$\boxed{PSRR = \frac{\text{change in } V_{cc}}{\text{change in } V_{out}}}$$

Block Diagram Representation of an Op-amp :-

A typical op-amp consists of four stage cascaded block structure as shown in the figure below,



Stage 1 : Input Amplifier :

It comprises of a double ended differential amplifier [2 input - 2 output]. It provides very high voltage gain

It rejects the signals common to both the input terminals. It has high input impedance. It has two input terminals i.e., an inverting and a non-inverting input terminal.

STAGE 2 : Intermediate amplifier :-

The output from stage 1 is fed as input to the stage 2. It consists of another differential amplifier and it provides additional gain. It is a dual input single output amplifier.

STAGE 3 : Level Shifting stage :-

Each stage in the amplifier is directly coupled. Because of direct coupling, dc voltage at output of second stage is very large compared to the ground potential and it is undesirable. The high dc drives the transistor to saturation and limits the peak to peak output voltage due to clipping giving rise to erroneous output signal.

Hence stage 3 is a level shifting circuit to shift dc = 0V wrt ground. Emitter follower with voltage divider or with constant current bias is the most simplest level shifting circuit.

STAGE 4 : Output stage :

The output stage of an op-amp requires large output voltage swing capability, low output impedance and low power dissipation. Hence the last stage is a push pull complementary amplifier which satisfies all the above requirements.

Properties of an ideal op-amp :-

1) Infinite open loop gain $A = \infty$

main function of an op-amp is to amplify the input signal, hence more the gain better. Therefor for an ideal opamp gain $= \infty$.

2) Infinite input resistance $R_i = \infty$

The opamp does not draw any current from the voltage sources connected to input terminals.

3) Zero output resistance $R_o = 0$

output resistance of an ideal op-amp is assumed to be zero, so that it can supply as much current as necessary to the load.

4) Infinite bandwidth :

An ideal op-amp has an infinite frequency response and can amplify any frequency signal, so its assumed to have infinite bandwidth

5) Zero output voltage when input voltage is zero.

6) perfect balance ie., zero output voltage when same voltage is applied at both the inputs.

7) Common mode rejection ratio [CMRR] of ideal op-amp $= \infty$

8) Infinite slew rate

ie., output voltage change occurs simultaneously with the change in input voltage.

9) Stable characteristics against temperature variations.

10) PSRR of an ideal opamp $= 0$.

Characteristics of a practical op-amp :-

1) Open loop gain $\neq \infty$; it typically ranges between 10^4 to 10^6 or more.

2) Input resistance $R_i \neq \infty$; but it is very high around $M\Omega$.

3) Output resistance $R_o \neq 0$; but it is less than 100Ω.

4) Bandwidth $\neq \infty$; but it range is 1 to 100MHz.

5) CMRR $\neq \infty$; but it is of the order of 90dB.

6) Negative and positive voltage swings are limited by supply voltages V^+ and V^-

7) Slew rate $\neq \infty$; typically around $0.5V/\mu sec$.

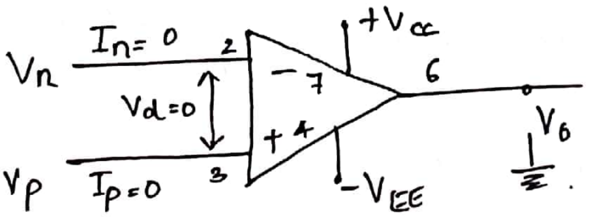
8) produces small output voltage when input voltage $= 0$ [$\approx 2mV$]

9) produces small output voltages when some input is applied to both input terminals.

Virtual Ground Concept :-

Op-amps circuits can be simplified by making two assumptions based on the characteristics of an ideal op-amp [ie., $A = \infty$, $R_i = \infty$ & $R_o = 0$].

Consider an ideal op-amp with input voltage and current levels as shown in figure below,



where $V_d \rightarrow$ difference input voltage
$V_o \rightarrow$ output voltage
$I_n \rightarrow$ i/p current at inverting terminal
$I_p \rightarrow$ i/p current at non-inverting terminal.

for an ideal op-amp $R_i = \infty$ i.e., none of the input terminals draw any current $[I_n = 0$ and $I_p = 0]$.
practically $I_n = I_p \approx \mu A$ or nA, hence they are very small and they can be approximated to be equal to zero.
[Assumption 1]

for an ideal op-amp $A_d = \infty$

$$\text{i.e.,} \quad A_d = \frac{V_o}{V_d} = \frac{V_o}{V_2 - V_1} = \infty$$

i.e., for a finite o/p voltage the input difference voltage $V_d = 0$

Eg: If $V_o = 10V$ and $A_v = 2 \times 10^5$ the the i/p required is

$$V_d = \frac{V_o}{A_v} = \frac{10}{2 \times 10^5} = 50\mu V.$$

$$V_d = 50\mu V$$

$50\mu V$ is a very small value, hence practically it is assumed to be equal to zero.

$$\text{i.e.,} \quad V_d = \frac{V_o}{A_v} = 0$$

$$(V_2 - V_1) = \frac{V_o}{A_v} = 0$$

$$\therefore \quad A_v = \frac{V_o}{V_2 - V_1}$$

for $A_v = \infty$ $\boxed{V_2 = V_1}$

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hence the equivalent ckt of an ideal op-amp can be modeled as shown in figure below

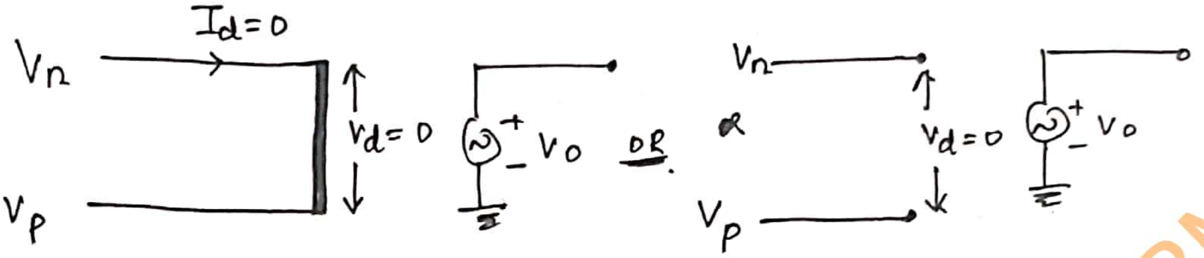
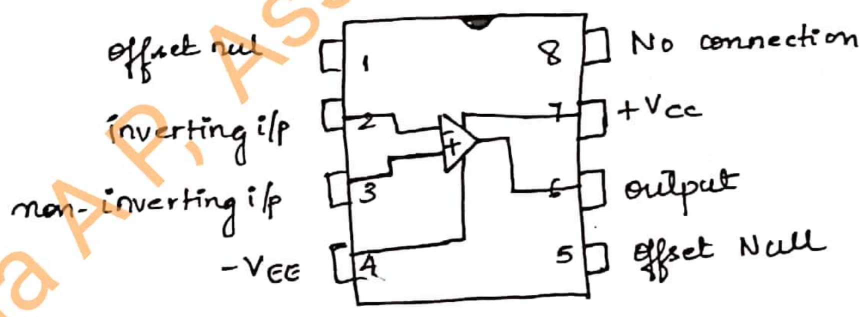


fig:- equivalent ckt.

from fig $A = \infty$ when $V_d = 0$ and $I_i = 0$ $\therefore R_i = \infty$;

thus inspite of a short ckt at its input port, the op-amp does not draw any current at i/p. So we assume that at the i/p of the amplifier there exists a virtual ground or virtual short circuit. This is known as virtual ground concept.

Pin diagram of IC µA741 op-amp :-



The most widely used op-amp IC is µA741. The pin diagram is as shown in the figure above. It is an 8 pin IC. The pin details is as below,

pin 2 :- inverting input

pin 3 :- non inverting input.

pin 4 : -ve power supply i.e., $-V_{EE}$

pin 7 : +ve power supply i.e., $+V_{CC}$

pin 6 : op-amp output.

pin 8 : no connection, it is a dummy pin.

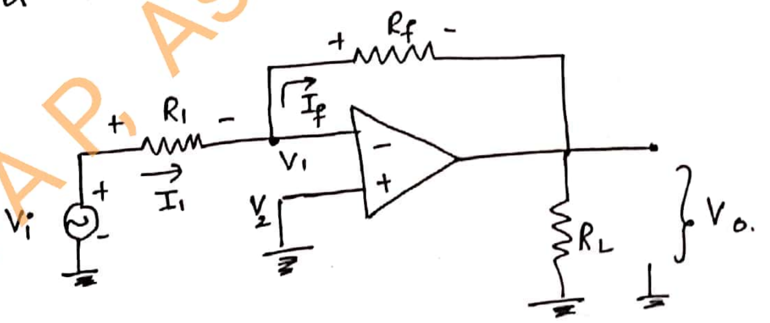
pin 1, 5 : offset null

When both inputs are zero, ideally the output of the op-amp must be zero. But practically there exists a small output voltage known as output offset voltage. The pins 1 and 5 are used to nullify the effects of offset voltages.

Applications of Op-amps :-

1) Inverting Amplifier :-

The circuit diagram of an inverting amplifier is as shown in the figure below,



The input is applied to the inverting terminal of the op-amp through R_1.

The non-inverting ~~ter~~ i/p terminal of the op-amp is connected to ground.

Hence the output of the inverting amplifier is 180° out of phase wrt the input signal.

feedback from the output is provided through feedback resistor connected between output and inverting ~~inverting~~ input terminal. This connection provides negative feedback to the system. This configuration represents a voltage shunt feedback amplifier.

from fig $V_2 = 0$ or ground.

due to virtual short, the inverting and non-inverting terminals are at same ~~potentiy~~ potential.

$$\therefore \quad V_1 = V_2 = 0$$

wkt $R_i = \infty$ $\therefore$ current flowing through $V_1 = 0$

hence same current flows through R_f and R_1

$$\text{ie.,} \quad I_1 = I_f \quad \text{--- (1)}$$

from fig $I_1 = \frac{V_i - V_1}{R_1} = \frac{V_i}{R_1}$

$$I_1 = \frac{V_i}{R_1} \quad \text{--- (2)}$$

$$I_f = \frac{V_1 - V_0}{R_f} = \frac{-V_0}{R_f}$$

$$I_f = -\frac{V_0}{R_f} \quad \text{--- (3)}$$

using (3) and (2) in (1).

$$\frac{V_i}{R_1} = \frac{-V_0}{R_f}$$

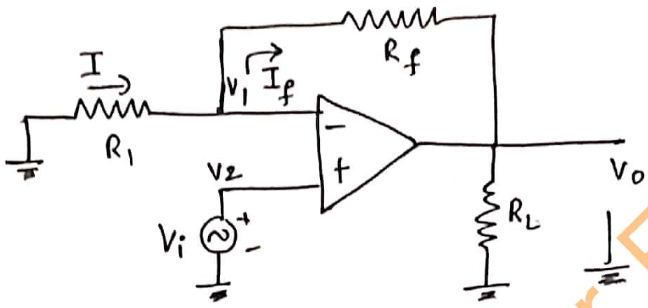
$$\therefore \quad \frac{V_0}{V_i} = \frac{-R_f}{R_1} = A_f$$

$$\therefore \quad \boxed{A_f = \frac{-R_f}{R_1}}$$

The negative sign indicates that the polarity of output is opposite to that of input. Hence it is called an inverting amplifier.

2) <u>Non Inverting Amplifier</u> :-

The circuit diagram of a non-inverting amplifier is as shown in the figure below.



The input (V_i) is applied to the non-inverting ~~ter~~ input terminal of the op-amp.

The inverting input terminal is connected to ground.

Hence the output of the non-inverting amplifier is in phase wrt the input signal. ie., there is no phase change between input and output.

→ The feedback is connected between the output and the -ve input terminal. The feedback is -ve feedback.

from figure $V_2 = V_i$ —— (1).

due to virtual short, the inverting and non inverting terminals are at the same potential.

$$\therefore \quad V_1 = V_2 = V_i \quad —— (2).$$

wkt $R_i = \infty$, hence current flowing into inverting input = 0. Hence same current flows through

R_1 and R_f

i.e., $I_s = I_f$ — (3)

from fig. $I = \frac{-V_1}{R_1} = \frac{-V_i}{R_1}$ — (4)

$$I_f = \frac{V_1 - V_o}{R_f} = \frac{V_i - V_o}{R_f} \quad — (5)$$

using (4) and (5) in (3)

$$-\frac{V_i}{R_1} = \frac{V_i - V_o}{R_f}$$

$$\frac{V_o}{R_f} = V_i\left[\frac{1}{R_f} + \frac{1}{R_1}\right]$$

$$V_o = V_i\left[1 + \frac{R_f}{R_1}\right]$$

$$\therefore \frac{V_o}{V_i} = A_f = 1 + \frac{R_f}{R_i}$$

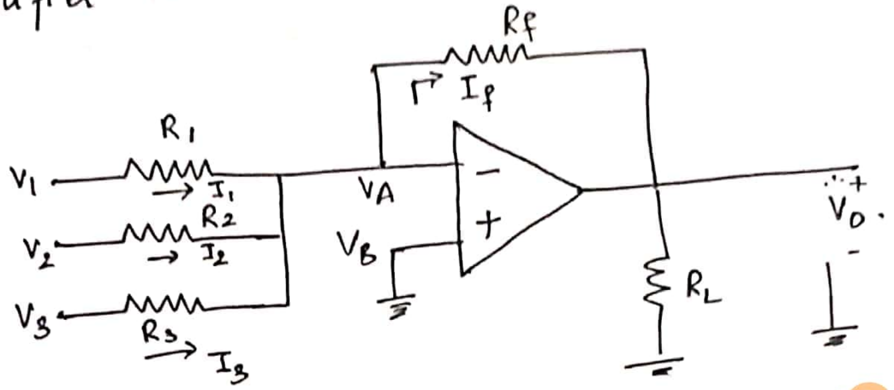
$$\boxed{A_f = 1 + \frac{R_f}{R_i}}$$

The gain of the non-inverting amplifier is positive which means input and output voltages are in phase.

The gain of this amplifier is stable and its minimum value is equal to 1.

3) Summing Amplifier :-

An op-amp can be used to add more than one signal. The circuit diagram of a summing amplifier is as shown in the figure below,



The inverting amplifier configuration is used for performing summing action. Here the non-inverting input terminal V_B is connected to ground.

The input signals V_1, V_2 and V_3 ~~and~~ are connected to the inverting terminal of the op-amp through resistances R_1, R_2 and R_3 respectively.

As we are using inverting amplifier configuration, the output is 180° out of phase wrt the input signal.

from fig, $V_B = 0$ or ground,

due to virtual short, the inverting and non inverting terminal are at the same potential.

i.e., $V_A = V_B = 0$.

~~$R_i = \infty$~~ ~~∴ current flowing into~~

from fig,

$$I_1 = \frac{V_1 - V_A}{R_1} = \frac{V_1}{R_1} \quad \text{—(1)}$$

$$I_2 = \frac{V_2 - V_A}{R_2} = \frac{V_2}{R_2} \quad \text{—(2)}$$

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$$I_3 = \frac{V_3 - V_A}{R_3} = \frac{V_3}{R_3} \quad \text{—(3)}$$

$$I_f = \frac{V_A - V_o}{R_f} = -\frac{V_o}{R_f} \quad \text{—(4)}$$

wkt $R_i = \infty$, hence zero current flows into the input terminal,

applying KCL at node A

$$I_f = I_1 + I_2 + I_3 \quad \text{—(5)}$$

using 1, 2, 3, 4 in (5).

$$-\frac{V_o}{R_f} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$V_o = -\left[\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right]$$

if $R_f = R_1 = R_2 = R_3$ then,

$$\boxed{V_o = -[V_1 + V_2 + V_3]}$$

Hence the output voltage is equal to negative sum of all the inputs, hence the circuit is known as summing amplifier.

If different values of R_1, R_2 and R_3 are used, then the circuit is known as weighted or scaling amplifier.

→ The summing circuit can also be used as averaging circuit.

Here output voltage is equal to average of all the input voltages,

ie., if $R_1 = R_2 = R_3 = R$ and $\frac{R_f}{R} = \frac{1}{n}$

$n \rightarrow$ no. of i/p.

∴ from fig $\frac{R_f}{R} = \frac{1}{3}$

$$V_o = -\frac{[V_1 + V_2 + V_3]}{3} \Rightarrow \text{average of 3 i/p signals.}$$

4) Voltage follower :

Voltage follower is also known as buffer amplifier.
A good voltage follower should have the following properties,

1) High input impedence so that it practically does not load the circuit.

2) Extremely low output impedence so that another circuit does not load it.

3) unit voltage gain so that the input voltage appears at the output without change in phase.

A buffer circuit when placed between two networks, it removes the loading on first network.

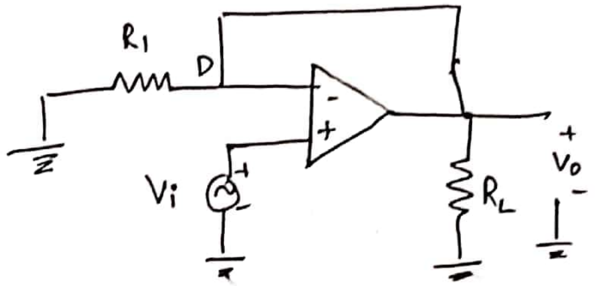
An opamp having unity gain, very high input resistance and large bandwidth is known as a voltage follower.

In an voltage follower the output is equal to and is in phase with the input i.e., output follows input.

If in a non-inverting amplifier, the feedback resistance R_f is replaced by a short circuit i.e., $R_f = 0$ then the amplifier behaves as a voltage follower.

In this circuit configuration all the output voltage is fed back into the inverting terminal of the op-amp.

Hence the gain of ckt = 1



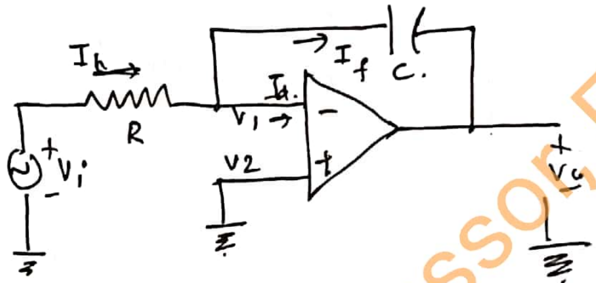
the circuit is used for isolating high impedence source from low impedence load.

5) Integrator :

An integrator is a modified inverting amplifier circuit in which the output voltage waveform is the integral of the input voltage waveform.

If the feedback resistor R_f in an inverting amplifier is replaced by a capacitor C then the circuit functions as an integrator.

The circuit diagram of an integrator is as shown in the figure below,



from fig, $V_2 = 0$

due to virtual short V_1 and V_2 are assumed to be at same potential, $V_1 = V_2 = 0$ —①.

$R_i = \infty$ ∴ I flowing into op-amp is 0. hence.

$$I = I_f \quad \text{—(2)}$$

from figure

$$I = \frac{V_i - V_1}{R_1} = \frac{V_i}{R_1} \quad \text{—(3)}$$

$$I_f = \frac{V_1 - V_o}{X_{C_f}}$$

$$I_f = C_f \frac{d}{dt}(V_A - V_o)$$

$$I_f = -C_f \frac{dV_o}{dt} \quad \text{—— (4)}$$

using (3) & (4) in (2)

$$\frac{V_{in}}{R_1} = -C_f\frac{dV_o}{dt}$$

integrating both sides.

$$\int_0^t \frac{V_i}{R_1}\,dt = -C_f\int_0^t \frac{dV_o}{dt}.dt$$

$$\int_0^t \frac{V_i}{R_1}\,dt = -C_f\, dV_o.$$

$$\therefore\ V_o = -\frac{1}{C_f R_1}\int_0^t V_i dt + V_o(0).$$

where $V_o(0)$ is constant of integration.

Hence from the above Eqn, the output is $-\frac{1}{R_1C_f}$ times the ~~sep~~ integral of input.

$R_1C_f \rightarrow$ time constant of integral.

-ve sign indicates that the output is 180° out of phase wrt input signal.

Applications

→ used in analog computer.

→ used to solve differential equations.

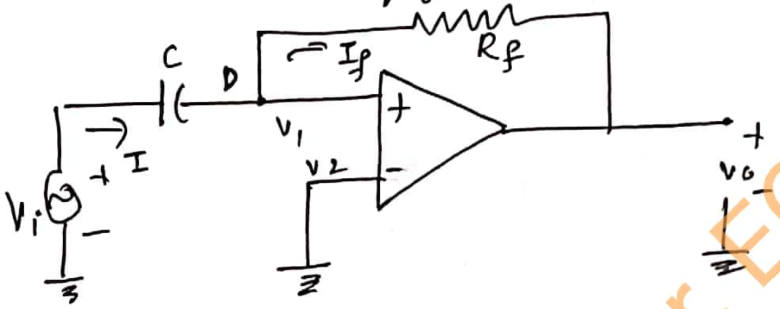
→ used in ADC

→ used in ramp generator.

2) Differentiator :

A differentiator circuit perform the mathematical operation of differentiation i.e., the output waveform is a derivative of the input waveform.

In an inverting amplifier when the input resistance R_1 is replaced by a capacitor C the circuit behaves as a differentiator. The circuit diagram of a differentiator is as shown in the figure below,



here the o/p voltage is proportional to the time rate of change of the input voltage.

from fig, $V_2 = 0$.

due to virtual short, V_1 and V_2 are at the same potential i.e., $V_1 = V_2 = 0$ —— ①.

$R_i = \infty$, hence current flowing into op-amp is zero

∴ from fig $I = I_f$ —(2)

$$I = \frac{C\, d(V_i - V_1)}{dt} \quad —(3)$$

$$I_f = \frac{V_1 - V_o}{R_f} = \frac{-V_o}{R_f} \quad —(4)$$

using ③ & ④ in ②.

$$\frac{C\, d(V_i - V_1)}{dt} = -\frac{V_o}{R_f}$$

$$\therefore \boxed{V_o = -R_f C \frac{dV_i}{dt}}$$

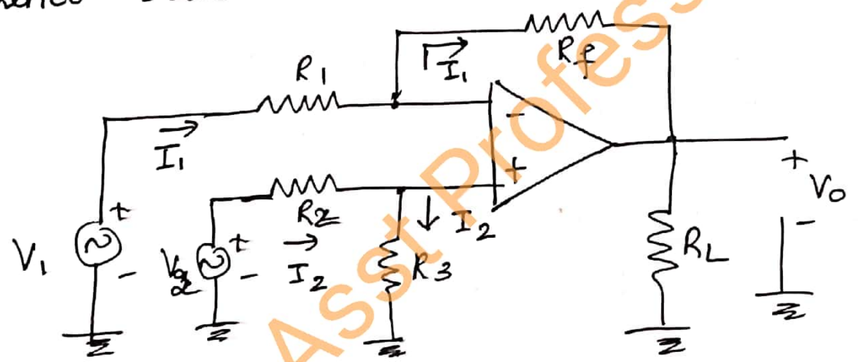
the above equation indicates that the output voltage is equal to $R_f C$ times the -ve instantaneous rate of change of i/p voltage V_i with time.

The differentiator circuits are used in FM Demodul-ators, ~~waveshaping~~. waveshaping circuits to detect

They are used in waveshaping circuits to detect high frequency component in an input signal.

6) The Difference Amplifier (subtractor)

The circuit diagram of a difference amplifier is as shown in the figure below; Here the amplified difference between the two signals V_1 and V_2 are



The circuit subtracts the input signals applied to inverting and non inverting i/p terminals when the gain of the amplifier is adjusted to 1.

Linear superposition theorem is used to calculate the output voltage V_o.

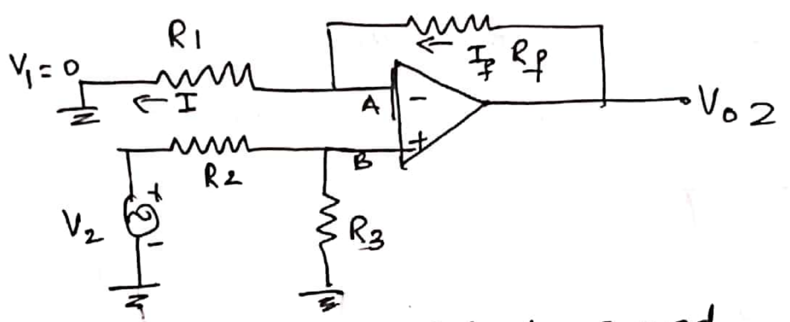
let V_{o1} be output when V_1 is i/p and $V_2 = 0$

let V_{o2} be output when V_2 is i/p and $V_1 = 0$

when $V_2 = 0$, $\rightarrow$ inverting amplifier.

$$V_{01} = -\frac{R_f}{R_1} V_1 \quad \text{—— ①}$$

when $V_1 = 0$ ~~⟹~~ ~~non~~ the equivalent ckt is



from fig due to virtual ground

$$V_A = V_B \quad \text{—— (1)}$$

applying voltage divider rule,

$$V_B = \frac{V_2 R_3}{R_2 + R_3} \quad \text{—— ②}$$

$$I = \frac{V_A}{R_1} = \frac{V_B}{R_1} \quad \text{—— ③}$$

$$I_f = \frac{V_{o2} - V_A}{R_f} = \frac{V_{o2} - V_B}{R_f} \quad \text{—— ④}$$

$R_i = \infty$ $\therefore$ $I = I_f$

$$\therefore \frac{V_B}{R_1} = \frac{V_{o2} - V_B}{R_f}$$

$$V_{o2} = \cancel{R_f} V_B \left[\frac{R_f + R_1}{R_1 \cancel{R_f}} \right]$$

$$V_{o2} = V_B \left[\frac{R_f + R_1}{R_1} \right] \quad \text{—— ⑤}$$

using ② in ⑤

$$V_{o2} = V_2\left[\frac{R_3}{R_2+R_3}\right]\left[1+\frac{R_f}{R_1}\right] \quad \text{--- (6)}$$

using superposition theorem,

$$V_o = V_{o1} + V_{o2}$$

$$V_o = -\frac{R_f}{R_1}V_1 + \left[1+\frac{R_f}{R_1}\right]\left[\frac{R_3}{R_2+R_3}\right]V_2$$

$$V_o = \left[1+\frac{R_f}{R_1}\right]\left[\frac{R_3}{R_2+R_3}\right]V_2 - \frac{R_f}{R_1}V_1$$

$$V_o = \frac{R_f}{R_1}\left[\left[\frac{R_1}{R_f}+1\right]\left[\frac{R_3}{R_2+R_3}\right]V_2 - V_1\right]$$

$$V_o = \frac{R_f}{R_1}\left[\left[\frac{R_1}{R_f}+1\right]\left[\frac{R_2}{R_3}+1\right]^{-1}V_2 - V_1\right] \quad \text{--- (7)}$$

selecting resistors such that $\frac{R_2}{R_3} = \frac{R_1}{R_f}$, the above equation reduces as,

$$\boxed{V_o = \frac{R_f}{R_1}\left[V_2 - V_1\right]}$$

The above Eqn represents the output voltage is the difference of $(V_2 - V_1)$ multiplied by a factor $\frac{R_f}{R_1}$

if $R_f = R_1$ then

$$\boxed{V_o = \left[V_2 - V_1\right]}$$

the above Equation represents the difference amplifier o/p with inverting gain = 1