

	c)	If the length-breadth index of skulls is normally distributed and classified as A and B accordingly as under 75 and over 80. Find approximately the mean and standard deviation of a series of skulls in which A are 58% and B are 4%.	CO1	PO1	7																					
		UNIT - II																								
3	a)	Experimenters injected a growth hormone gene into thousands of carp eggs. Of the 400 carp that grew from these eggs, 20 incorporated the gene into their DNA. Construct a 95% confidence interval for the proportion of carp that would incorporate the gene into their DNA.	CO1	PO1	6																					
	b)	A test is designed to measure the level of anxiety that was administered to a sample of male and female patients just prior to undergoing the same surgical procedure. The sample sizes and the variances computed from the scores were as follows: <table border="1"><tr><td>Male</td><td>Female</td></tr><tr><td>$n_1 = 16$</td><td>$n_2 = 21$</td></tr><tr><td>$s^2_1 = 150$</td><td>$s^2_2 = 275$</td></tr></table> Do these data provide sufficient evidence to indicate that in the represented populations the scores made by females are more variable than those made by males?	Male	Female	$n_1 = 16$	$n_2 = 21$	$s^2_1 = 150$	$s^2_2 = 275$	CO1	PO1	7															
Male	Female																									
$n_1 = 16$	$n_2 = 21$																									
$s^2_1 = 150$	$s^2_2 = 275$																									
	c)	The number of cars passing a given point was observed as follows: <table border="1"><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>f</td><td>419</td><td>352</td><td>154</td><td>56</td><td>19</td></tr></table> Fit a Poisson distribution to the data and test the goodness of fit.	x	0	1	2	3	4	f	419	352	154	56	19	CO1	PO1	7									
x	0	1	2	3	4																					
f	419	352	154	56	19																					
		UNIT - III																								
4	a)	Five pre-school children were given a supplement of multi-purpose food for a period of four months. Their skin-fold thickness (in mm) was measured before the commencement of the programme and also at the end. Apply parametric test to check if there is any change in their skin- fold thickness for the following data. <table border="1"><tr><td>Child</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>At the beginning of the program</td><td>6</td><td>8</td><td>8</td><td>6</td><td>9</td></tr><tr><td>At the end of the training program</td><td>8</td><td>8</td><td>10</td><td>7</td><td>10</td></tr></table>	Child	1	2	3	4	5	At the beginning of the program	6	8	8	6	9	At the end of the training program	8	8	10	7	10	CO1	PO1	6			
Child	1	2	3	4	5																					
At the beginning of the program	6	8	8	6	9																					
At the end of the training program	8	8	10	7	10																					
	b)	In a study of cerebrovascular disease, patients from 3 socio-economic backgrounds were thoroughly investigated. One characteristic measured was the diastolic blood pressure (mm/Hg). Is there any reason to believe that the 3 groups differ with respect to this characteristic? Apply Kruskal - Wallis One-way Analysis of Variance. <table border="1"><tr><td>Group A</td><td>100</td><td>103</td><td>89</td><td>78</td><td>105</td><td></td></tr><tr><td>Group B</td><td>92</td><td>97</td><td>88</td><td>84</td><td>90</td><td>95</td></tr><tr><td>Group C</td><td>81</td><td>102</td><td>86</td><td>83</td><td>99</td><td></td></tr></table>	Group A	100	103	89	78	105		Group B	92	97	88	84	90	95	Group C	81	102	86	83	99		CO1	PO1	7
Group A	100	103	89	78	105																					
Group B	92	97	88	84	90	95																				
Group C	81	102	86	83	99																					

	c)	<p>Consider a Phase II clinical trial designed to investigate the effectiveness of a new drug to reduce symptoms of asthma in children. A total of $n=10$ participants are randomized to receive either the new drug or a placebo. Participants are asked to record the number of episodes of shortness of breath over a 1 week period following receipt of the assigned treatment. The data are shown below.</p> <table border="1"><tr><td>Placebo</td><td>7</td><td>5</td><td>6</td><td>4</td><td>12</td></tr><tr><td>New Drug</td><td>3</td><td>6</td><td>4</td><td>2</td><td>1</td></tr></table> <p>Apply Mann-Whitney test, to check if there is a difference in the number of episodes of shortness of breath over a 1 week period in participants receiving the new drug as compared to those receiving the placebo?</p>	Placebo	7	5	6	4	12	New Drug	3	6	4	2	1	COI	POI	7										
Placebo	7	5	6	4	12																						
New Drug	3	6	4	2	1																						
		OR																									
5	a)	<p>The mean yield of wheat from a district A was 210 lb with $s_1 = 10\text{ lb}$ per acre from a sample of 100 plots. In another district B, the mean yield was 220 lb with $s_2 = 12\text{ lb}$ from a sample of 150 plots. Assuming that the standard deviation of the yield in the entire state was 11 lb, test whether there is any significant difference between the mean yields of crops in the two districts.</p>	COI	POI	6																						
	b)	<p>A study is run to evaluate the effectiveness of an exercise program in reducing systolic blood pressure in patients with pre-hypertension. A total of 10 patients with pre-hypertension enrol in the study, and their systolic blood pressures are measured. Each patient then participates in an exercise training program where they learn proper techniques and execution of a series of exercises. Patients are instructed to do the exercise program 3 times per week for 6 weeks. After 6 weeks, systolic blood pressures are again measured. The data are shown below.</p> <table border="1"><tr><td>Before</td><td>125</td><td>132</td><td>138</td><td>120</td><td>125</td><td>127</td><td>136</td><td>139</td><td>131</td><td>132</td></tr><tr><td>After</td><td>118</td><td>134</td><td>130</td><td>124</td><td>105</td><td>130</td><td>130</td><td>132</td><td>123</td><td>128</td></tr></table> <p>Apply Wilcoxon Signed rank test to determine if there is a difference in systolic blood pressures after participating in the exercise program as compared to before?</p>	Before	125	132	138	120	125	127	136	139	131	132	After	118	134	130	124	105	130	130	132	123	128	COI	POI	7
Before	125	132	138	120	125	127	136	139	131	132																	
After	118	134	130	124	105	130	130	132	123	128																	
	c)	<p>A group of 10 rats fed on a diet A and another group of 8 rats fed on a different diet B, recorded the following increase in weights.</p> <table border="1"><tr><td>Diet A</td><td>5</td><td>6</td><td>8</td><td>1</td><td>12</td><td>4</td><td>3</td><td>9</td><td>6</td><td>10</td></tr><tr><td>Diet B</td><td>2</td><td>3</td><td>6</td><td>8</td><td>10</td><td>1</td><td>2</td><td>8</td><td>-</td><td>-</td></tr></table> <p>Does it show the superiority of diet A over that of diet B when the analysis is performed by considering their average weights?</p>	Diet A	5	6	8	1	12	4	3	9	6	10	Diet B	2	3	6	8	10	1	2	8	-	-	COI	POI	7
Diet A	5	6	8	1	12	4	3	9	6	10																	
Diet B	2	3	6	8	10	1	2	8	-	-																	
		UNIT - IV																									
6	a)	<p>Explain the following types of variables with an example.</p> <p>(i) Intervening variable (ii) Extraneous variable</p> <p>(iii) Composite variable</p>	COI	POI	6																						

	b)	Seventeen patients admitted to the hospital were diagnosed as having liver problem. The following are the ages of the subjects in the study: 63 72 62 69 71 84 81 78 61 76 84 67 86 69 64 87 76 Calculate (i) five number summary (ii) interquartile range (iii) standard deviation	COI	POI	7																																
	c)	Researchers are conducting a prospective cohort study of the association between being an office worker who uses a computer daily and carpal tunnel syndrome. A total of 300 exposed and 300 unexposed participants are enrolled and followed for 10 years. A total of 25 exposed and 17 unexposed had the outcome of interest over the follow-up period. (i) What is the <i>relative risk</i> for developing carpal tunnel syndrome? (ii) What is the incidence attributable to daily computer use? (iii) If 60% of the population uses a computer daily at work, how much carpal tunnel could we prevent if we implemented a national work-place ergonomics program (and thus eliminated the exposure of daily computer use)?	COI	POI	7																																
		UNIT - V																																			
7	a)	Explain the basic principles of design of experiments.	COI	POI	4																																
	b)	Data recorded on yield of four varieties in an experiment with four replications for which one value is missing. Estimate the missing value and analyse the data. <table border="1"><tr><td>P</td><td>R</td><td>Q</td><td>S</td></tr><tr><td>5.52</td><td>5.57</td><td>5.071</td><td>9.16</td></tr><tr><td>S</td><td>R</td><td>Q</td><td>P</td></tr><tr><td>6.69</td><td>5.14</td><td>-</td><td>6.09</td></tr><tr><td>S</td><td>P</td><td>Q</td><td>R</td></tr><tr><td>2.89</td><td>6.02</td><td>6.53</td><td>2.83</td></tr><tr><td>R</td><td>Q</td><td>S</td><td>P</td></tr><tr><td>9.76</td><td>6.25</td><td>8.9</td><td>9.77</td></tr></table>	P	R	Q	S	5.52	5.57	5.071	9.16	S	R	Q	P	6.69	5.14	-	6.09	S	P	Q	R	2.89	6.02	6.53	2.83	R	Q	S	P	9.76	6.25	8.9	9.77	COI	POI	8
P	R	Q	S																																		
5.52	5.57	5.071	9.16																																		
S	R	Q	P																																		
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S	P	Q	R																																		
2.89	6.02	6.53	2.83																																		
R	Q	S	P																																		
9.76	6.25	8.9	9.77																																		
	c)	An oil company tested 4 different blends - A, B, C and D of gasoline for fuel efficiency according to a Latin Square Design in order to control for the variability of 4 different drivers and 4 different models of cars. Fuel efficiency was measured in miles per gallon after driving the cars over a standard course. The data are presented below. Construct the ANOVA table. <table border="1"><tr><td rowspan="2">Driver</td><td colspan="4">Car Model</td></tr><tr><td>I</td><td>II</td><td>III</td><td>IV</td></tr><tr><td>1</td><td>D-15.5</td><td>B-33.9</td><td>C-13.2</td><td>A-29.1</td></tr><tr><td>2</td><td>B-16.3</td><td>C-26.6</td><td>A-19.4</td><td>D-22.8</td></tr><tr><td>3</td><td>C-10.8</td><td>A-31.8</td><td>D-17.1</td><td>B-30.3</td></tr><tr><td>4</td><td>A-14.7</td><td>D-34</td><td>B-19.7</td><td>C-21.6</td></tr></table>	Driver	Car Model				I	II	III	IV	1	D-15.5	B-33.9	C-13.2	A-29.1	2	B-16.3	C-26.6	A-19.4	D-22.8	3	C-10.8	A-31.8	D-17.1	B-30.3	4	A-14.7	D-34	B-19.7	C-21.6	COI	POI	8			
Driver	Car Model																																				
	I	II	III	IV																																	
1	D-15.5	B-33.9	C-13.2	A-29.1																																	
2	B-16.3	C-26.6	A-19.4	D-22.8																																	
3	C-10.8	A-31.8	D-17.1	B-30.3																																	
4	A-14.7	D-34	B-19.7	C-21.6																																	

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2024 Semester End Main Examinations

Programme: B.E.

Branch: Common to AS/ME /EEE/ECE/ET/MD/CIVIL/EIE

Course Code: 22MA4BSCPS

Course: Complex Analysis, Probability and Statistical Methods

Semester: IV

Duration: 3 hrs.

Max Marks: 100

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.
3. Statistical tables are permitted

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.			UNIT - 1	CO	PO	Marks
	1	a)	Construct the analytic function $f(z) = u + iv$ if $u - v = (x - y)(x^2 + 4xy + y^2)$.	CO1	PO1	6
		b)	Discuss the transformation $w = z + \frac{k^2}{z}$, $z \neq 0$.	CO1	PO1	7
		c)	Verify Cauchy's theorem for $\oint_C \frac{1}{z} dz$ where C is the boundary of the triangle having vertices (1, 2), (1, 4) & (3, 2).	CO1	PO1	7
			OR			
	2	a)	Find the orthogonal trajectories of the family of curves $e^x \cos y - xy = c$.	CO1	PO1	6
		b)	If $f(z)$ is analytic function, then prove that $\left\{ \frac{\partial}{\partial x} f(z) \right\}^2 + \left\{ \frac{\partial}{\partial y} f(z) \right\}^2 = f'(z) ^2$.	CO1	PO1	7
		c)	Apply Cauchy's integral formula to evaluate $\oint_C \frac{\sin^2 z}{(z - \pi/6)^3} dz$ where C is the circle $ z = 1$.	CO1	PO1	7
			UNIT - 2			
	3	a)	Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ and $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.	CO1	PO1	6
		b)	Obtain series solution of Legendre's differential equation $(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + n(n+1)y = 0$.	CO1	PO1	7
		c)	Express the function $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre polynomial.	CO1	PO1	7

		UNIT - 3																									
4	a)	The following table gives the marks obtained by 10 students in the subjects English and Mathematics. Find the rank correlation between the subjects. <table><tr><td>English</td><td>56</td><td>75</td><td>45</td><td>71</td><td>62</td><td>64</td><td>58</td><td>80</td><td>76</td><td>61</td></tr><tr><td>Mathematics</td><td>66</td><td>70</td><td>40</td><td>60</td><td>65</td><td>56</td><td>59</td><td>77</td><td>67</td><td>63</td></tr></table>	English	56	75	45	71	62	64	58	80	76	61	Mathematics	66	70	40	60	65	56	59	77	67	63	CO1	PO1	6
English	56	75	45	71	62	64	58	80	76	61																	
Mathematics	66	70	40	60	65	56	59	77	67	63																	
	b)	Obtain the lines of regression and hence find the coefficient of correlation for the following data: <table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>y</td><td>9</td><td>8</td><td>10</td><td>12</td><td>11</td><td>13</td><td>14</td></tr></table>	x	1	2	3	4	5	6	7	y	9	8	10	12	11	13	14	CO1	PO1	7						
x	1	2	3	4	5	6	7																				
y	9	8	10	12	11	13	14																				
	c)	Fit a power function $y = ax^b$ to the following data pertaining to demand for a product and its price charged at five different cities. Predict the demand when price of the product is Rs. 12 <table><tr><td>Price (Rs.): x</td><td>20</td><td>16</td><td>10</td><td>11</td><td>14</td></tr><tr><td>Demand: y (1000 units)</td><td>22</td><td>41</td><td>120</td><td>89</td><td>56</td></tr></table>	Price (Rs.): x	20	16	10	11	14	Demand: y (1000 units)	22	41	120	89	56	CO1	PO1	7										
Price (Rs.): x	20	16	10	11	14																						
Demand: y (1000 units)	22	41	120	89	56																						
		UNIT - 4																									
5	a)	Derive an expression for mean and variance of Poisson distribution.	CO1	PO1	6																						
	b)	A manufacturer of air-mail envelopes knows from experience that the weight of the envelopes is normally distributed with mean 1.95 gm and standard deviation 0.05 gm. About how many envelopes weighting (i) 2 gm or more (ii) 2.05 gm or less (iii) more than 2.05 but less than 1.9 can be expected in a given packet of 100 envelopes. [Given $P(0 < z < 1) = 0.3413$, $P(0 < z < 2) = 0.4772$].	CO2	PO1	7																						
	c)	The joint probability distribution of two random variables X and Y is given in the table below. Find the marginal distributions of X and Y and find the co-variance of X and Y. <table><tr><td>X/Y</td><td>-2</td><td>-1</td><td>4</td><td>5</td></tr><tr><td>1</td><td>0.1</td><td>0.2</td><td>0</td><td>0.3</td></tr><tr><td>2</td><td>0.2</td><td>0.1</td><td>0.1</td><td>0</td></tr></table>	X/Y	-2	-1	4	5	1	0.1	0.2	0	0.3	2	0.2	0.1	0.1	0	CO1	PO1	7							
X/Y	-2	-1	4	5																							
1	0.1	0.2	0	0.3																							
2	0.2	0.1	0.1	0																							
		UNIT - 5																									
6	a)	In a random sample of 10 bolts produced by a machine the mean length of bolt is 0.53 mm and standard deviation 0.03 mm. Can we claim from this that the machine is in proper working order if in the past it produced bolts of length 0.50 mm? Use level of significance as 5%.	CO2	PO1	6																						
	b)	A random sample of 40 ‘geysers’ produced by company A have a mean lifetime (mlt) of 647 hours of continuous use with a standard deviation of 27 hours, while a sample 40 produced by another company B have mlt of 638 hours with standard deviation 31 hours. Does this substantiate the claim of company A that their ‘geysers’ are superior to those produced by company B at 0.01 level of significance?	CO2	PO1	7																						

	c)	According to a theory in genetics, the proportion of beans of four types A, B, C and D in a generation should be 9:3:3:1. In an experiment, among 1600 beans, the frequency of beans of each of the above four types were 882, 313, 287, and 118 respectively. Does the result support the theory at 5% level of significance?	CO2	PO1	7																		
		OR																					
7	a)	<p>A corporate training institution claimed that its training program can greatly enhance the efficiency of call center employees. A big call center sent some of its employees for the training program. The efficiency was measured by the number of deals closed by each employee in a one-month period. Data was collected for a one-month period before sending the employees for the training program. After the training program, data was again collected on the same employees for a one-month period. Test the validity of the claim made by the training institution that its training program improves the efficiency of call center employees for the following data at $\alpha = 5\%$.</p> <table><tr><td>Employee</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Before the training program</td><td>41</td><td>46</td><td>35</td><td>42</td><td>40</td></tr><tr><td>After the training program</td><td>44</td><td>39</td><td>36</td><td>40</td><td>48</td></tr></table>	Employee	1	2	3	4	5	Before the training program	41	46	35	42	40	After the training program	44	39	36	40	48	CO2	PO1	6
Employee	1	2	3	4	5																		
Before the training program	41	46	35	42	40																		
After the training program	44	39	36	40	48																		
	b)	<p>Out of random sample of 9 mice, suffering with a disease, 5 mice were treated with new serum while the remaining were not treated. From the time commencement of experiment, the following are the survival times:</p> <table><tr><td>Treatment</td><td>2.1</td><td>5.3</td><td>1.4</td><td>4.6</td><td>0.9</td></tr><tr><td>Not Treatment</td><td>1.9</td><td>0.5</td><td>2.8</td><td>3.1</td><td>—</td></tr></table> <p>Test whether the serum treatment is effective in curing the disease at 5% level of significance, assuming that the two distributions are normally distributed with equal variance.</p>	Treatment	2.1	5.3	1.4	4.6	0.9	Not Treatment	1.9	0.5	2.8	3.1	—	CO2	PO1	7						
Treatment	2.1	5.3	1.4	4.6	0.9																		
Not Treatment	1.9	0.5	2.8	3.1	—																		
	c)	<p>The number of computer malfunctions per day is recorded for 260 days with the following results.</p> <table><tr><td>Number of malfunctions</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Number of days</td><td>77</td><td>87</td><td>55</td><td>30</td><td>5</td><td>6</td></tr></table> <p>Fit a Poisson distribution and test for the goodness of fit at $\alpha = 5\%$.</p>	Number of malfunctions	0	1	2	3	4	5	Number of days	77	87	55	30	5	6	CO2	PO1	7				
Number of malfunctions	0	1	2	3	4	5																	
Number of days	77	87	55	30	5	6																	

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2024 Semester End Main Examinations

Programme: B.E.

Branch: CS, IS and AI & ML

Course Code: 22MA4BSLIA

Course: Linear Algebra

Semester: IV

Duration: 3 hrs.

Max Marks: 100

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.			UNIT - I	CO	PO	Marks
	1	a)	Determine whether the set $V = \{(x, y) \mid x, y \in \mathbb{R}\}$ is a vector space over the field of reals when the vector addition is the standard vector addition and the scalar multiplication is defined as $k \cdot (x, y) = (0, ky)$.	CO1	PO1	6
		b)	Find the basis of row space, Column space, null space for the matrix $A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & 1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & 4 \end{bmatrix}$.	CO1	PO1	7
		c)	Find a homogeneous system whose solution set W is spanned by $\{u_1, u_2, u_3\} = \{(1, -2, 0, 3), (1, -1, -1, 4), (1, 0, -2, 5)\}$.	CO1	PO1	7
			UNIT - II			
	2	a)	Verify whether the linear transformation $T: P_2(t) \rightarrow M_{2 \times 2}$ defined by $T(at^2 + bt + c) = \begin{bmatrix} a & 2b \\ 0 & a \end{bmatrix}$ is one-one and onto.	CO1	PO1	6
		b)	Verify Rank-Nullity theorem for the linear transformation $G: R^3 \rightarrow R^3$ defined by $G(x, y, z, t) = (x - y + z + t, 2x - 2y + 3z + 4t, 3x - 3y + 4z + 5t)$.	CO1	PO1	7
		c)	Let T be a linear operator defined on R^3 through $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Is T invertible? If so, find a formula for T^{-1} and T^{-2} .	CO1	PO1	7
			OR			

3	a)	Discuss the following maps on R^2 and represent them graphically: i) Horizontal contraction ii) Vertical contraction iii) Horizontal shear.	COI	POI	6
	b)	Let $T:P_1(t) \rightarrow P_2(t)$ be the linear transformation defined by $T[f(t)] = t f(t)$. Find the matrix of linear transformation with respect to the standard basis of $P_1(t)$ and $B = \{1+t, t-1, t^2\}$ for $P_2(t)$.	COI	POI	7
	c)	Consider the mapping $F:R^2 \rightarrow R^2$ defined by $F(x,y) = (3y, 2x)$. Let 'S' be the unit circle in R^2 , that is the solution of $x^2 + y^2 = 1$. i) Identify $F(S)$, ii) Find $F^{-1}(S)$.	COI	POI	7
		UNIT - III			
4	a)	Apply Cayley –Hamilton theorem to find A^4 if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$.	COI	POI	6
	b)	Obtain the Eigen space for the linear transformation $T:R^3 \rightarrow R^3$ defined by $T(x,y,z) = (2x+y, y-z, 2y+4z)$.	COI	POI	7
	c)	Find all possible Jordan canonical form of linear transformation T, whose minimal polynomial is $(t-8)^2(t+8)^2$ and algebraic multiplicity of the eigen values -8 and 8 are 5 and 4 respectively.	COI	POI	7
		OR			
5	a)	Apply Cayley –Hamilton theorem to find A^{-1} and A^{-2} if $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$.	COI	POI	6
	b)	Obtain the Eigen space for the linear transformation $T:P_2 \rightarrow P_2$ defined by $T(at^2 + bt + c) = (2a-c)t^2 + (2a+b-2c)t + (-a+2c)$.	COI	POI	7
	c)	Find the characteristic and minimal polynomials of the matrix $A = \begin{bmatrix} 3 & 2 & 0 & 0 & 0 \\ 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$.	COI	POI	7

		UNIT - IV																	
6	a)	Consider the following polynomial in $P_2(t)$ with the inner product $\langle f \mid g \rangle = \int_{-1}^1 f(t)g(t)dt \quad f(t)=t+2, \quad g(t)=t^2-3t+4.$ Find the matrix of $\langle f \mid g \rangle$ with respect to the basis $\{1, t, t^2\}$.	COI	POI	6														
	b)	Find QR decomposition of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.	COI	POI	7														
	c)	A sales organization obtains the following data relating the number of salespersons to annual sale. Let x and y denotes the number of salespersons and annual sales respectively. Find the least square line of the form $y = a + bx$ and estimate the annual sales when there are 14 salespersons. <table border="1"><tr><td>x</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>y</td><td>2.3</td><td>3.2</td><td>4.1</td><td>5.0</td><td>6.1</td><td>7.2</td></tr></table>	x	5	6	7	8	9	10	y	2.3	3.2	4.1	5.0	6.1	7.2	COI	POI	7
x	5	6	7	8	9	10													
y	2.3	3.2	4.1	5.0	6.1	7.2													
		UNIT-V																	
7	a)	Compute the Hessian matrix at the point $(1, 1, 1)$ of the function $f(x, y, z) = x^2y^2 + z^3 + 2xy + 3xz + x^2 + 3.$	COI	POI	4														
	b)	Determine the modal matrix that reduces the quadratic form $3x^2 + 3y^2 + 3z^2 - 2yz + 2zx + 2xy$ to its canonical form and hence find the nature of the quadratic form.	COI	POI	8														
	c)	Given the data in table, reduce the dimension from 2 to 1 using principal component analysis. <table border="1"><tr><td>X</td><td>4</td><td>8</td><td>13</td><td>7</td></tr><tr><td>Y</td><td>11</td><td>4</td><td>5</td><td>14</td></tr></table>	X	4	8	13	7	Y	11	4	5	14	COI	POI	8				
X	4	8	13	7															
Y	11	4	5	14															

	b)	Number of sales of household appliances, X , Whirlpool representative Darlene makes in a day is given by the following probability distribution. <table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$P(X)$</td><td>k</td><td>$3k$</td><td>$5k$</td><td>$7k$</td><td>$9k$</td><td>$11k$</td><td>$13k$</td></tr></table> Find (i) the value of k (ii) mean (iii) variance.	X	0	1	2	3	4	5	6	$P(X)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$	CO1	PO1	7
X	0	1	2	3	4	5	6														
$P(X)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$														
	c)	In a certain examination, the percentage of candidates passing and getting distinctions were 45 and 9 respectively. Assuming Normal distribution, estimate the average marks and standard deviation obtained by the candidates, the minimum pass and distinction marks being 40 and 75 respectively.	CO1	PO1	7																
		UNIT - II																			
3	a)	If X and Y are independent random variables, X takes values 2, 5, 7 with probability $1/2$, $1/4$, $1/4$ respectively and Y takes values 3, 4, 5 with probability $1/3$, $1/3$, $1/3$ respectively. Find (i) Joint probability distribution of X and Y , (ii) Covariance of X and Y .	CO1	PO1	6																
	b)	In a certain city, the weather on a day is reported as sunny, cloudy or rainy. If a day is sunny, the probability that the next day is sunny is 70%, cloudy is 20% and rainy is 10%. If a day is cloudy, the probability that the next day is sunny is 30%, cloudy is 20% and rainy is 50%. If a day is rainy, the probability that the next day is sunny is 30%, cloudy is 30% and rainy is 40%. (i) If the Sunday is sunny, find the probability that the Wednesday is rainy (ii) Find the steady state distribution.	CO1	PO1	7																
	c)	Two fruits are selected at random from a bag containing 3 Apples, 2 Oranges and 4 Mangoes. If X and Y are respectively, the number of Apples and the number of Oranges included among the two fruits drawn from the bag, find the probability associated with all the possible pair of values (x, y) . Also find $Cov(X, Y)$.	CO1	PO1	7																
		UNIT - III																			
4	a)	Random samples drawn from two countries gave the following data relating to the heights of adult males: <table><tr><td></td><td>Country A</td><td>Country B</td></tr><tr><td>Mean height (in inches)</td><td>67.42</td><td>67.25</td></tr><tr><td>Standard deviation (in inches)</td><td>2.58</td><td>2.5</td></tr><tr><td>Number of samples</td><td>1000</td><td>1200</td></tr></table> Is the difference between the standard deviations significant at 5% level of significance??		Country A	Country B	Mean height (in inches)	67.42	67.25	Standard deviation (in inches)	2.58	2.5	Number of samples	1000	1200	CO1,2	PO1	6				
	Country A	Country B																			
Mean height (in inches)	67.42	67.25																			
Standard deviation (in inches)	2.58	2.5																			
Number of samples	1000	1200																			
	b)	Researchers suspect that myopia, or nearsightedness, is becoming more common over time. A study from the year 2000 showed 132 cases of myopia in 400 randomly selected people. A separate study from 2015 showed 228 cases in 600 randomly selected people. Is the researchers suspect true at 1% level of significance?	CO1,2	PO1	7																

	c)	To test whether a new low-fat diet actually helps obese people to lose weight, 100 randomly assigned obese people are assigned to group 1 and put on low-fat diet. Another 100 randomly assigned obese people are assigned to group 2 and put on a diet of approximately the same amount of food, but not as low in fat. After 4 months, the mean weight loss was 9.31 <i>lbs.</i> for group1 with standard deviation 4.67 <i>lbs.</i> While the mean weight loss was 7.4 <i>lbs.</i> for group 2 with standard deviation 4.04 <i>lbs.</i> Check whether the scientists claim is true at 1% level of significance.	CO1,2	PO1	7																						
		UNIT – IV																									
5	a)	The mean diameter of rivets produced by two firms A and B are practically the same but their standard deviations are different. For 16 rivets manufactured by firm A, the standard deviation is 3.8 mm while for 22 rivets mean manufactured by firm B is 2.9 mm. Using F-test, check whether products of firm B are of better quantity than those of firm A at 5% level of significance?	CO1,2	PO1	6																						
	b)	Consider a random sample of 500 U.S. adults who are questioned regarding their political affiliation and opinion on a tax reform bill. Test if the political affiliation and their opinion on a tax reform bill are dependent at a 5% level of significance. The observed contingency table is given below. <table border="1"><tr><td></td><td>favor</td><td>indifferent</td><td>opposed</td></tr><tr><td>democrat</td><td>138</td><td>83</td><td>64</td></tr><tr><td>republican</td><td>64</td><td>67</td><td>84</td></tr></table>		favor	indifferent	opposed	democrat	138	83	64	republican	64	67	84	CO1,2	PO1	7										
	favor	indifferent	opposed																								
democrat	138	83	64																								
republican	64	67	84																								
	c)	A group of 10 rats fed on a diet A and another group of 8 rats fed on a different diet B, recorded the following increase in weights. <table border="1"><tr><td>Diet A</td><td>5</td><td>6</td><td>8</td><td>1</td><td>12</td><td>4</td><td>3</td><td>9</td><td>6</td><td>10</td></tr><tr><td>Diet B</td><td>2</td><td>3</td><td>6</td><td>8</td><td>10</td><td>1</td><td>2</td><td>8</td><td>-</td><td>-</td></tr></table> Does it show the superiority of diet A over that of B. Choose $\alpha = 5\%$.	Diet A	5	6	8	1	12	4	3	9	6	10	Diet B	2	3	6	8	10	1	2	8	-	-	CO1,2	PO1	7
Diet A	5	6	8	1	12	4	3	9	6	10																	
Diet B	2	3	6	8	10	1	2	8	-	-																	
		OR																									
6	a)	Memory capacity of students was tested before and after giving the nourishing food (Chavanprash). Check whether Chavanprash was effective or not from the following data: <table border="1"><tr><td>Before</td><td>12</td><td>14</td><td>11</td><td>8</td><td>7</td><td>10</td><td>3</td></tr><tr><td>After</td><td>15</td><td>16</td><td>10</td><td>7</td><td>5</td><td>12</td><td>10</td></tr></table>	Before	12	14	11	8	7	10	3	After	15	16	10	7	5	12	10	CO1,2	PO1	6						
Before	12	14	11	8	7	10	3																				
After	15	16	10	7	5	12	10																				
	b)	The number of cars passing a given point was observed as follows: <table border="1"><tr><td><i>x</i></td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td><i>f</i></td><td>419</td><td>352</td><td>154</td><td>56</td><td>19</td></tr></table> Fit a Poisson distribution to the data and test the goodness of fit.	<i>x</i>	0	1	2	3	4	<i>f</i>	419	352	154	56	19	CO1,2	PO1	7										
<i>x</i>	0	1	2	3	4																						
<i>f</i>	419	352	154	56	19																						

	c)	<p>The chloride content of water was measured at three different stations of a lake. The seasonal averages of chloride in mg/l are given below. Perform two factor analysis of variance for the variations in season and stations.</p> <table><tr><th rowspan="2">Season</th><th colspan="3">Stations</th></tr><tr><th>S₁</th><th>S₂</th><th>S₃</th></tr><tr><td>Summer</td><td>187.4</td><td>341.8</td><td>240.6</td></tr><tr><td>Monsoon</td><td>172.9</td><td>198.2</td><td>150.2</td></tr><tr><td>Winter</td><td>154.3</td><td>157.4</td><td>137.1</td></tr></table>	Season	Stations			S ₁	S ₂	S ₃	Summer	187.4	341.8	240.6	Monsoon	172.9	198.2	150.2	Winter	154.3	157.4	137.1	CO1,2	PO1	7										
Season	Stations																																	
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Winter	154.3	157.4	137.1																															
		UNIT - V																																
7	a)	Define (a) Treatment (b) Replication (c) Randomization (d) Local Control.	CO1,2	PO1	4																													
	b)	<p>A pediatrician speculated that frequency of visits to his office may be influenced by type of medical insurance coverage. As an exploratory study, she randomly chose 15 patients: 5 whose parents belong to a health maintenance organization (A), 5 whose parents had traditional medical insurance (B), and 5 whose parents were uninsured(C). Using the frequency of visits per year given below, identify the design and test the hypothesis that type of insurance coverage has no effect on frequency of visits.</p> <p>A-12, C-3, B-6,C-1, B-1, B-5, B-7, A-6, B-5, A-8, C-2, C-5, A-7, A-6, C-3</p>	CO1,2	PO1	6																													
	c)	<p>An oil company tested 4 different blends - A, B, C and D of gasoline for fuel efficiency according to a Latin Square Design in order to control for the variability of 4 different drivers and 4 different models of cars. Fuel efficiency was measured in miles per gallon after driving the cars over a standard course. The data are presented below. Analyse the data.</p> <table><tr><th rowspan="2">Driver</th><th colspan="4">Car Model</th></tr><tr><th>I</th><th>II</th><th>III</th><th>IV</th></tr><tr><td>1</td><td>D-15.5</td><td>B-33.9</td><td>C-13.2</td><td>A-29.1</td></tr><tr><td>2</td><td>B-16.3</td><td>C-26.6</td><td>A-19.4</td><td>D-22.8</td></tr><tr><td>3</td><td>C-10.8</td><td>A-31.8</td><td>D-17.1</td><td>B-30.3</td></tr><tr><td>4</td><td>A-14.7</td><td>D-34</td><td>B-19.7</td><td>C-21.6</td></tr></table>	Driver	Car Model				I	II	III	IV	1	D-15.5	B-33.9	C-13.2	A-29.1	2	B-16.3	C-26.6	A-19.4	D-22.8	3	C-10.8	A-31.8	D-17.1	B-30.3	4	A-14.7	D-34	B-19.7	C-21.6	CO1,2	PO1	10
Driver	Car Model																																	
	I	II	III	IV																														
1	D-15.5	B-33.9	C-13.2	A-29.1																														
2	B-16.3	C-26.6	A-19.4	D-22.8																														
3	C-10.8	A-31.8	D-17.1	B-30.3																														
4	A-14.7	D-34	B-19.7	C-21.6																														

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2024 Semester End Main Examinations

Programme: B.E.

Branch: Industrial Engineering and Management

Course Code: 22IM4BSSFE

Course: Statistics For Engineers

Semester: IV

Duration: 3 hrs.

Max Marks: 100

- Instructions:** 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

- 1 a) Mention the effective method of collecting data for understanding the population or process that is being considered in engineering field? Explain with an example. **09**
- b) The compressive strengths (in kgf) of 45 specimens are given below. **11**

190	218	208	181	228	176	165	157	201
193	154	245	163	142	146	178	131	229
110	97	76	149	183	156	168	105	207
186	169	237	200	196	194	151	87	180
134	123	221	121	199	160	141	174	143

- i) Construct frequency distribution using the lowest limit of lower class as 75kgf.
- ii) Find the mean, median, mode and standard deviation of the above specimens.
- iii) Construct a histogram

UNIT - II

- 2 a) The data from 200 endothermic reactions involving sodium bicarbonate are summarized as follows: **06**

Final Temperature conditions	Number of Reactions
266K	48
271K	60
274K	92

- i. Calculate the probability mass function of final temperature.
- ii. Determine the cumulative distribution function for the random variable
- iii. Calculate the mean and variance for the random variable

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

- b) Heart failure is due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances or foreign objects. Natural occurrences are caused by arterial blockage, disease, and infection. Suppose that 20 patients will visit an emergency room with heart failure. Assume that causes of heart failure for the individuals are independent. 06
- i. What is the probability that three individuals have conditions caused by outside factors?
 - ii. What is the probability that three or more individuals have conditions caused by outside factors?
 - iii. What are the mean and standard deviation of the number of individuals with conditions caused by outside factors?
- c) The number of flaws in bolts of cloth in textile manufacturing is assumed to be Poisson distributed with a mean of 0.1 flaws per square meter. 08
- i. What is the probability that there are two flaws in one square meter of cloth?
 - ii. What is the probability that there is one flaw in 10 square meters of cloth?
 - iii. What is the probability that there are no flaws in 20 square meters of cloth?
 - iv. What is the probability that there are at least two flaws in 10 square meters of cloth?

OR

- 3 a) The waiting time for service at a hospital emergency department (in hours) follows a distribution with probability density function $f(x) = 0.5\exp(-0.5x)$ for $0 < x$. Determine the following: 08
- i. $P(X < 0.5)$
 - ii. $P(X > 2)$
 - iii. Value x (in hours) exceeded with probability 0.05.
 - iv. Determine the cumulative distribution function for the random variable. Use the cumulative distribution function to determine the probability that $40 < X \leq 60$.
- b) The weight of a sophisticated running shoe is normally distributed with a mean of 12 ounces and a standard deviation of 0.5 ounce. 06
- i. What is the probability that a shoe weighs more than 13 ounces?
 - ii. What must the standard deviation of weight be in order for the company to state that 99.9% of its shoes weights less than 13 ounces?
 - iii. If the standard deviation remains at 0.5 ounce, what must the mean weight be for the company to state that 99.9% of its shoes weighs less than 13 ounces?

- c) A test instrument needs to be calibrated periodically to prevent measurement errors. After some time of use without calibration, it is known that the probability density function of the measurement error is $f(x) = 1 - 0.5x$ for $0 < x < 2$ millimeters. **06**
- If the measurement error within 0.5 millimeters is acceptable, what is the probability that the error is not acceptable before calibration?
 - What is the value of measurement error exceeded with probability 0.2 before calibration?
 - What is the probability that the measurement error is exactly 0.22 millimeters before calibration?

UNIT - III

- 4 a) A civil engineer is analyzing the compressive strength of concrete. Compressive strength is normally distributed with $\sigma^2 = 1000(\text{psi})^2$. A random sample of 12 specimens has a mean compressive strength of $\bar{x} = 3250$ psi. **06**
- Construct a 95% two-sided confidence interval on mean compressive strength.
Construct a 99% two-sided confidence interval on mean compressive strength. Compare the width of this confidence interval with the width of the one found in part (I).
- b) Differentiate between point estimate and interval estimate with an example **06**
- c) An article in the Australian Journal of Agricultural Research [“Non-Starch Polysaccharides and Broiler Performance on Diets Containing Soyabean Meal as the Sole Protein Concentrate” (1993, Vol. 44(8), pp. 1483–1499)] determined that the essential amino acid (Lysine) composition level of soybean meals is as shown here (g/kg): **08**
- | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|
| 22.2 | 24.7 | 20.9 | 27.0 | 26.0 | 23.9 | 25.6 | 23.8 | 24.8 | 26.5 |
|------|------|------|------|------|------|------|------|------|------|
- Construct a 99% two-sided confidence interval for σ^2 .
 - Calculate a 99% lower confidence bound for σ^2 .
 - Calculate a 90% lower confidence bound for σ .
- Compare the intervals that you have computed.

UNIT - IV

- 5 a) An article in Growth: A Journal Devoted to Problems of Normal and Abnormal Growth [“Comparison of Measured and Estimated Fat-Free Weight, Fat, Potassium and Nitrogen of Growing Guinea Pigs” (1982, Vol. 46(4), pp. 306–321)] reported the results of a study that measured the body weight (in grams) for guinea pigs at birth. 12

421.0	452.6	456.1	494.6	373.8
110.7	241.0	290.9	88.8	258.5
90.5	296.0	687.6	296.0	296.0
96.4	317.0	705.7	273.0	279.3
81.7	256.5	879.0	268.0	227.5
102.4	447.8			

- i. Test the hypothesis that mean body weight is 300 grams. Use $\alpha = 0.05$.
 - ii. What is the smallest level of significance at which you would be willing to reject the null hypothesis?
 - iii. Explain how you could answer the question in part (I) with a two-sided confidence interval on mean body weight.
- b) In a random sample of 85 automobile engine crankshaft bearings, 10 have a surface finish roughness that exceeds the specifications. Do these data present strong evidence that the proportion of crankshaft bearings exhibiting excess surface roughness exceeds 0.10? 08
- i. State and test the appropriate hypotheses using $\alpha = 0.05$.
 - ii. If $p = 0.15$, how large would the sample size have to be for us to have a probability of correctly rejecting the null hypothesis of 0.9?

OR

- 6 a) Let X denotes the number of flaws observed on a large coil of galvanized steel. Of 75 coils inspected, the following data were observed for the values of X : 12

No. of Flaws	1	2	3	4	5	6	7	8
Observed data	1	11	8	13	11	12	10	9

- i. Does the assumption of the Poisson distribution seem appropriate as a probability model for these data? Use $\alpha = 0.01$.
 - ii. Calculate the P-value for this test.
- b) Extracts of St. John's Wort are widely used to treat depression. An article in the April 18, 2001, issue of the Journal of the American Medical Association (“Effectiveness of St. John's Wort on Major Depression: A Randomized Controlled Trial”) compared the efficacy of a standard extract of St. John's Wort with a placebo in 200 outpatients diagnosed with major depression. Patients were randomly assigned to two groups; one group received the St. John's Wort, and the other received the placebo. After eight weeks, 19 of the placebo-treated patients showed improvement, and 27 of those treated with St. John's Wort improved. Is there any reason to believe that St. John's Wort is effective in treating major depression? Use $\alpha = 0.05$. 08

UNIT - V

- 7 a) An article in the Journal of Sound and Vibration (Vol. 15 1, 199 1, pp. 383-394) described a study investigating the relationship between noise exposure (x) and hypertension (Y). The following data (sorted by sound pressure level) are representative of those reported in the article: 16

No	Sound Pressure level (x;unit: dB)	Blood Pressure Rise (Y;unit:m mHg)	No	Sound Pressure level (x;unit: dB)	Blood Pressure Rise (Y;unit:m mHg)
1	60	1	11	85	5
2	63	0	12	89	4
3	65	1	13	90	6
4	70	2	14	90	8
5	70	5	15	90	4
6	70	1	16	90	5
7	80	4	17	94	7
8	80	6	18	100	9
9	80	2	19	100	7
10	80	3	20	100	6

Assume that x and Y are linearly related.

- i. Calculate the least squares estimates of the intercept and slope of the linear model for x and Y.
 - ii. Calculate the residual of $y = 5$ mmHg at $x = 85$ dB.
 - iii. Estimate the error variance σ^2 .
- b) Discuss briefly the Multi linear Regression. With its application 04
