

Hashing $\rightarrow$ is useful for Searching purpose. ①

Linear Search - $O(n)$

	0	1	2	3	4	5	6	7	8	9
A	8	5	12	6	15	9	4	3	7	10

Binary Search - $O(\log n)$

A	3	4	5	6	7	8	9	10	12	15
	0	1	2	3	4	5	6	7	8	9

elements must be sorted.

We want algorithm which takes $O(1)$

Hashing.

Keys - 8, 3, 13, 6, 4, 10

A				3	4		6		8		10			13	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

if a list of ele are given and array is given.

The value is also considered as indexed.

EX: 8 is stored in 8

3 is stored in 3.

Now search is directly go to index and search.

if I have 50

array must be 50

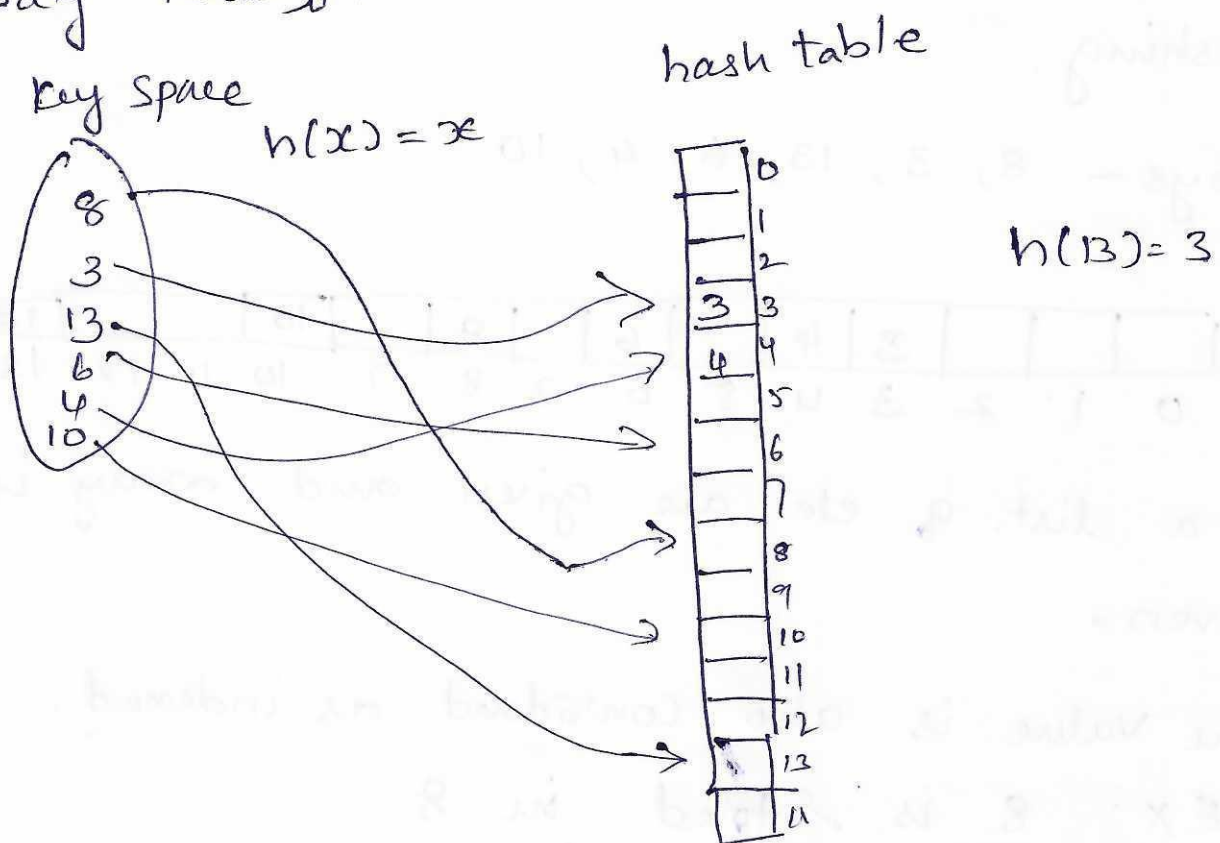
lot of space is wasted.

To improve we use mathematical model.
for hashing based on functions.

so let us see next how it is model

The list of elements what we have we
call them as key space and the place
we have to store them is called hash table.

and we say that there is some function
Say ~~$H(x) = x$~~ $H(x) = x$ which mapping



elements from key space to hash table.

$H(x) = x$ is hash function which is
• mapping key elements to hash table
and this ideal hash function.

if I have key = 50 index must be 50 2
hash table must be lengthy waste of
space.

$H(x) = x$ — one to one function.

many-one — any other.

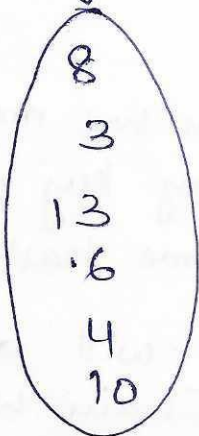
$h(x)$ is responsible for mapping to table.
modify the function to efficiently use space.

we will use.

bmc

$H(x) = x \% \underline{10}$ Size 8 hash table

Key space.



hash table Size 10 (ex)

0	
1	
2	
3	3
4	
5	
6	
7	
8	8
9	

$$h(x) = x \% \text{Size}$$
$$x \% 10$$

$$8 = 8 \% 10 = 8$$

$$3 = 3 \% 10 = 3$$

$$13 = 13 \% 10 = 3 - \text{collision.}$$

Two or more
ele are
mapped to
same index

We need to resolve collisions

Collision Resolution Methods

1. Open Hashing
Chaining

2. Closed hashing

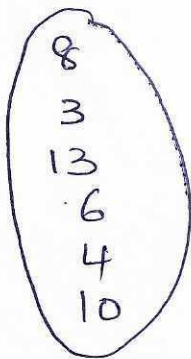
Open addressing

1. Linear Probing

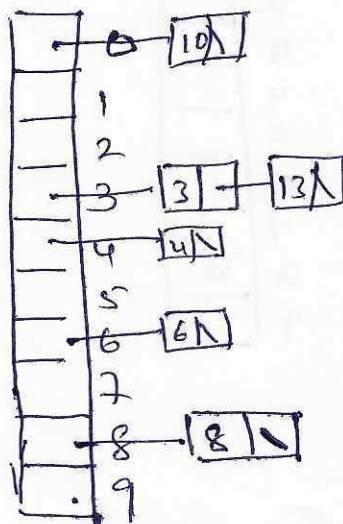
2. Quadratic Probing

1. Chaining.

Key Space



hash table



When we map
any key using
same hash function

we will add a
chain which is
linked list.

$$h(x) = x \% \text{Size}$$

$$8 \% 10 = 8$$

When there is a collision we insert a key element
at the same index in a chain

3

When we want to search an element let us say 13 then we should use hash function H

$$H(13) = 13 \cdot \frac{1}{10} = 3$$

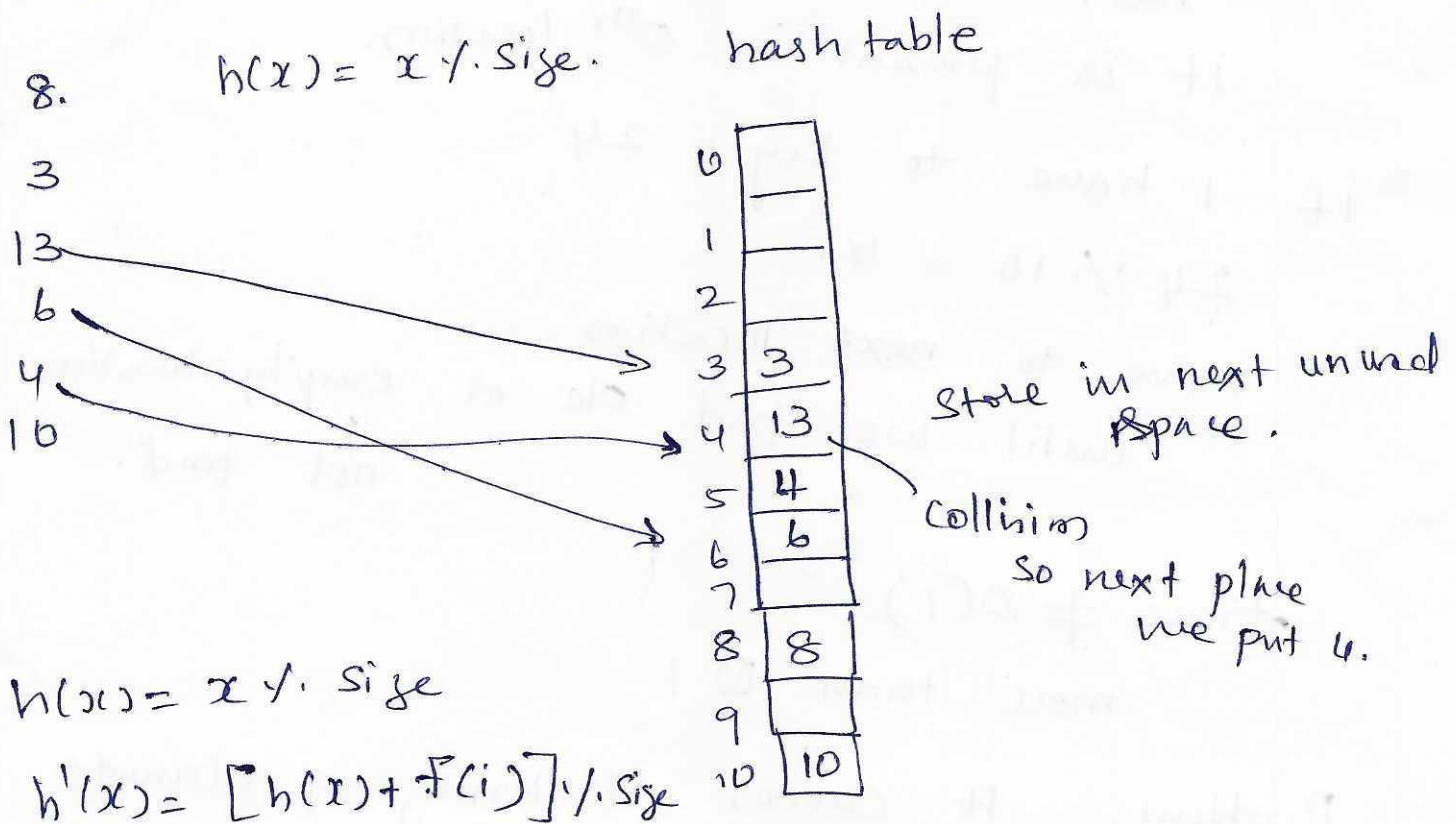
So go to index 3 and search for key in the chain.

There may be more than one element. It should search entire chain

time complexity will be more than 1.

Closed Hashing.

Linear probing



$f(i) = i$ where i takes values
 $i = 0, 1, 2, \dots$

when we use 13 $h'(13) = [h(13) + f(0)] \cdot \frac{1}{10}$
3 - collision

$$h'(13) = [h(13) + f(1)] \% 10$$

$$= \underline{\underline{4}}$$

so 13 is stored in 4.

if there is a collision again we take next value of i

When we want to search for element

we use $h(x) = x \% \text{size}$

4 not present

next look at next location

it is present in 5th location.

if I have to key = 24

$$24 \% 10 = 4.$$

move to next location.

until we find ele or empty location
not found.

time $\neq O(1)$ -

more than $\otimes 1$

Problem: It causes clustering of elements at consecutive locations

— lot of time

To avoid clustering of elements

Linear probing.
12, 44, 67, 22, 58, 64, 52, 89

(4)

$$h(x) = (h(x) + i) \bmod 10$$

$$i = 0, 9$$

0	
1	
2	12
3	22
4	44
5	64
6	52
7	67
8	58
9	89

$$h(12) = (h(12) + 0) \cdot 10$$

$$2 + 0 \cdot 10$$

$$2$$

$$h(44) = 4 + 0 \cdot 10$$

$$4$$

$$h(67) = 7 + 0 \cdot 10$$

$$= 7$$

$$52 = 2 + 0 \cdot 10$$

$$2$$

$$2 + 1 \cdot 10$$

$$3 \cdot 10$$

$$= 3$$

$$h(22) = 2 + 0 \cdot 10$$

$$= \underline{2} \text{ collision.}$$

$$(2 + 1) \cdot 10$$

$$3 \cdot 10$$

$$= \underline{3}$$

$$2 + 2$$

$$4$$

$$2 + 3 = \underline{5}$$

$$2 + 4 = 6 \cdot 10$$

$$= 6$$

$$h(58) = (8 + 0) \cdot 10$$

$$\underline{8}$$

$$89 = (9 + 0) \cdot 10$$

$$10$$

$$\underline{9}$$

$$64 = (4 + 0) \cdot 10$$

$$= 4$$

$$4 + 1 \cdot 10$$

$$5 \cdot 10 = 5$$

quadratic probing

$$h(x) = x \bmod 10$$

$$\text{keys} = \{52, 24, 72, 80, 44, 64\}$$

$$h(x) = (h(x) + i^2) \bmod 10$$

$$i = 0 \text{ to } 9$$

0	80
1	
2	52
3	72
4	24
5	44
6	
7	
8	64
9	

$$h(52) = (2 + 0) \bmod 10$$

$$2$$

$$h(24) = (4 + 0) \bmod 10$$

$$4$$

$$h(72) = (2 + 0) \bmod 10$$

$$= 2 - \text{collision}$$

$$(2 + 1^2) \bmod 10$$

$$3 \bmod 10$$

$$= \underline{3}$$

$$80 = (8 + 0) \bmod 10$$

$$8$$

$$44 = (4 + 0) \bmod 10$$

$$4 - \text{collision}$$

$$(4 + 1) \bmod 10$$

$$\underline{5}$$

$$64 = (\cancel{5} + 0) \bmod 10$$

$$4 - \text{collision}$$

$$(4 + 1^2) \bmod 10$$

$$5 - \text{collision}$$

$$(4 + 2^2) \bmod 10$$

$$4 + 4 \bmod 10$$

$$8 \bmod 10$$

$$= 8 - \text{collision}$$

we use quadratic probing.

(5)

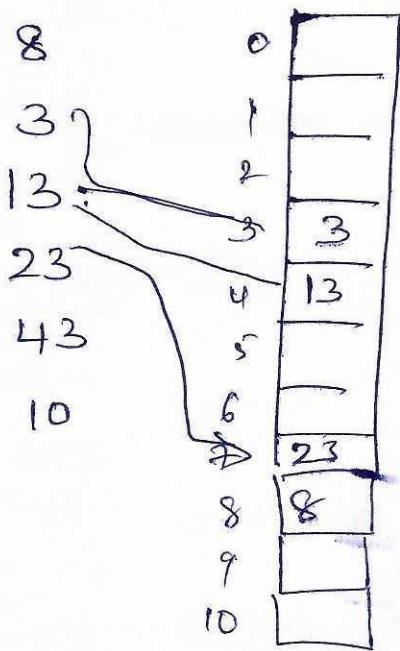
it is same as linear probe

$$h(x) = x \% \text{ size}$$

$$h'(x) = [h(x) + f(i)] \% \text{ size}$$

$$f(i) = \underline{i^2}$$

$$i = 0, 1, 2, 3, \dots$$



$$h'(23) = [h(23) + f(1)] \% 10$$

3 collision.

$$[h(23) + f(1)] \% 10$$

$$3 + 1 \% 10$$

$$4$$

Collision.

$$[h(23) + f(2^2)] \% 10$$

$$\cancel{4} + 4$$

$$3 + 4$$

$$7 \% 10 = 7$$

So place 23 in 7.

it is not close, it place away.

This will avoid clustering.

③

if ϵ is small enough

$$h(x) = x^2$$

$$h'(x) = [h(x) + \epsilon(x)]' = (x^2 + \epsilon)'$$

$$h'(x) = 2x$$

$$h'(x) = 2x + \epsilon'$$

$$h(x) = [h(x) + \epsilon(x)] - \epsilon(x)$$

$$h'(x) = 2x - \epsilon'$$

$$h'(x) = 2x + \epsilon'$$

$$h'(x) = 2x + \epsilon'$$

$$h'(x) = 2x$$

$$h'(x) = 2x$$

$$h'(x) = 2x + \epsilon'$$

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it is not clear if there is any

relationship between ϵ and ϵ'

