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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February / March 2025 Semester End Main Examinations

Programme: B.E. | **Semester: I**
Branch: Common to all Branches | **Duration: 3 hrs.**
Course Code: 21MA1BSCDE | **Max Marks: 100**
Course: Calculus and Differential Equations

Instructions:
1. All units have internal choice, answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	PO	Marks
		UNIT - 1			
1	a)	Derive an expression for the angle between the radius vector and the tangent to the polar curve $r = f(\theta)$.	1	1	**6**
	b)	Obtain the pedal equation of the curve $r^m \cos(m\theta) = a^m$.	1	1	**7**
	c)	Determine the radius of curvature of $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.	1	1	**7**
		OR			
2	a)	Show that the pair of curves $r = 4\sec^2\left(\frac{\theta}{2}\right)$ and $r = 9\,\mathrm{cosec}^2\left(\frac{\theta}{2}\right)$ intersect each other orthogonally.	1	1	**6**
	b)	Find the pedal equation of the polar curve $\frac{2a}{r} = 1 - \cos(\theta)$.	1	1	**7**
	c)	If ρ_1 and ρ_2 are the radii of curvature at the extremities of a chord through the pole for the polar curve $r = a(1+\cos\theta)$, prove that $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$.	1	1	**7**
		UNIT - 2			
3	a)	Obtain the Taylor's series expansion of $f(x) = \cos(x)$ in powers of $\left(x - \frac{\pi}{3}\right)$ up to third degree term.	1	1	**6**
	b)	If $w = r^m$, show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = m(m+1)r^{m-2}$ where $r^2 = x^2 + y^2 + z^2$.	1	1	**7**
	c)	Determine the Jacobian $J\left(\frac{x, y, z}{u, v, w}\right)$ when $x = \sqrt{vw}$, $y = \sqrt{uw}$ and $z = \sqrt{uv}$.	1	1	**7**

			OR			
	4	a)	Obtain the Maclaurin's series expansion of $f(x)=\tan^{-1}(x)$ up to the second degree term.	*1*	*1*	**6**
		b)	Find the value of n for which the function $\theta=t^n e^{-r^2/4t}$ satisfies $\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial\theta}{\partial r}\right]=\frac{\partial\theta}{\partial t}$.	*1*	*1*	**7**
		c)	Find the extreme values of the function $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$.	*1*	*1*	**7**
			UNIT - 3			
	5	a)	Determine the values of a and b for which the system of equations $x+2y+3z=6$, $x+3y+5z=9$ and $2x+5y+az=b$ have i) No solution ii) Unique solution iii) Infinitely many solutions.	*1*	*1*	**6**
		b)	Apply Gauss-Seidel iteration method to obtain an approximate solution of the system of equations $10x+y+z=12$; $2x+10y+z=13$ and $2x+2y+10z=14$. Perform 3 iterations.	*1*	*1*	**7**
		c)	Apply Rayleigh's power method to determine the dominant eigenvalue and the corresponding eigenvector of the matrix $A=\begin{bmatrix}25&1&2\\1&3&0\\2&0&-4\end{bmatrix}$ choosing $\begin{bmatrix}1\\0\\0\end{bmatrix}$ as the initial approximation. Perform 4 iterations.	*1*	*1*	**7**
			OR			
	6	a)	For what values of k the system of equations $\begin{matrix}x+y+z=1\\2x+y+4z=k\\4x+y+10z=k^2\end{matrix}$ has a solution.	*1*	*1*	**6**
		b)	Apply Gauss- Seidel iteration method to obtain an approximate solution of $5x+2y+z=12$; $x+4y+2z=15$ and $x+2y+5z=20$ by taking the initial approximation as (1, 0, 3). Carry out 3 iterations.	*1*	*1*	**7**
		c)	Obtain the eigenvalues and the corresponding eigenvectors of the matrix $\begin{bmatrix}2&0&1\\0&2&0\\1&0&2\end{bmatrix}$.	*1*	*1*	**7**
			UNIT - 4			
	7	a)	Solve $\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4}dy=0$.	*1*	*1*	**6**
		b)	Show that the family of curves $y^2=4a(x+a)$ is self-orthogonal.	*1*	*1*	**7**
		c)	Solve the differential equation $p^2+2py\cot(x)=y^2$ by solving for p.	*1*	*1*	**7**

			OR			
	8	a)	Solve $y' + \frac{y}{2x} = \frac{x}{y^3}$ subject to $y(1) = 2$.	*1*	*1*	**6**
		b)	Solve $\left(3x^2y^4 + 2xy\right)dx + \left(2x^3y^3 - x^2\right)dy = 0$.	*1*	*1*	**7**
		c)	Obtain the orthogonal trajectories of the family of curves $r = a\left(1 + \cos(\theta)\right)$.	*1*	*1*	**7**
			UNIT – 5			
	9	a)	Solve $\left(D^2 + 1\right)y = x^3 + \cos 3x$.	*1*	*1*	**6**
		b)	Solve $y'' + y = \sec x$ by the method of variation of parameters.	*1*	*1*	**7**
		c)	Solve $x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 4x - 6$.	*1*	*1*	**7**
			OR			
	10	a)	Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 5y = e^{-3x} + 5$.	*1*	*1*	**6**
		b)	Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec}(x)$.	*1*	*1*	**7**
		c)	Solve $(2x+5)^2\frac{d^2y}{dx^2} - 6(2x+5)\frac{dy}{dx} + 8y = 6x$.	*1*	*1*	**7**

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