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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

May 2023 Semester End Main Examinations

Programme: B.E. **Semester: I**
Branch: Common to all Branches **Duration: 3 hrs.**
Course Code: 21MA1BSCDE **Max Marks: 100**
Course: Calculus and Differential Equations **Date: 24.05.2023**

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

UNIT - I

1 a) Show that $\tan^{-1}\left(\frac{2e}{1-e^2}\right)$ is the angle of intersection for the pair of curves $r=\frac{a}{\log\theta}$ and $r=a\log\theta$. **6**

b) Find the pedal equation of the polar curve $r(1-\cos\theta)=2a$ and hence prove that ρ^2 varies as r^3, if ρ is the radius of curvature. **7**

c) Obtain the radius of curvature for the curve Folium $x^3+y^3=3axy$ at the point $\left(3a/2, 3a/2\right)$. **7**

UNIT - II

2 a) If $\theta=t^n e^{-r^2/4t}$, what value of n will make $\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)\right]=\frac{\partial\theta}{\partial t}$. **6**

b) Verify that $JJ'=1$ if $u=x+\frac{y^2}{x}$, $v=\frac{y^2}{x}$. **7**

c) The altitude of a right circular cone is $15cm$ and is increasing at $0.2\,cm/s$. The radius of the base is $10cm$ and is decreasing at $0.3\,cm/s$. How fast is the volume changing? **7**

OR

3 a) Expand the function $f(x)=2x^3+7x^2+x-6$ in powers of $(x-2)$. **6**

b) If $x^x y^y z^z=c$, show that $z_{xy}=-\left[x\log_e(ex)\right]^{-1}$ when $x=y=z$. **7**

c) In a plane triangle find the maximum value of $\cos A\cos B\cos C$ where A, B and C are the angles of the triangle. **7**

UNIT - III

4 a) For what values of 'k' the system of equations $x+y+z=1; x+2y+4z=k$ and $x+4y+10z=k^2$ have a solution and solve them completely in each case. **6**

b) Apply Gauss-Seidel iteration method to solve the system of equations $20x+y-2z=17$; $2x-3y+20z=25$ and $3x+20y-z=-18$ by carrying out three iterations. **7**

c) Apply Rayleigh power method to determine the dominant Eigen value and the corresponding Eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by performing three iterations using the initial approximation $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$. **7**

UNIT - IV

5 a) Solve the differential equation $\left(x^3y^2+xy\right)dx=dy.$ **6**

b) Solve the differential equation $\left(xy^2-e^{1/x^3}\right)dx-x^2ydy=0.$ **7**

c) Find the general and singular solutions of the non-linear differential equation $\sin(px)\cos y=\cos(px)\sin y+p.$ **7**

OR

6 a) Solve the differential equation $\left(\frac{y}{(x+y)^2}-1\right)dx+\left(1-\frac{x}{(x+y)^2}\right)dy=0.$ **6**

b) Solve the differential equation $y^2y'-y^3\tan x-\sin x\cos^2 x=0.$ **7**

c) Show that the family of curve $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$, 'λ' being the parameter is self-orthogonal. **7**

UNIT - V

7 a) Solve the differential equation $\left(D^2+4D+4\right)y=x^2+2x$ given $y(0)=0$ and $y'(0)=0$. **6**

b) Apply the method of variation of parameters to solve the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^x\sin x.$ **7**

c) Solve the differential equation $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=\sin[2\log(1+x)].$ **7**

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