

# B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

## September / October 2023 Supplementary Examinations

Programme: B.E.

Branch: Common to all Branches

Course Code: 18MA1BSEM1

Course: Engineering Mathematics - 1

Semester: I

Duration: 3 hrs.

Max Marks: 100

**Instructions to Candidates: Answer FIVE FULL questions, choosing one from each unit.**

### UNIT - I

- 1 a) Derive an expression for the angle between radius vector and tangent in the form of  $\tan(\phi) = r \frac{d\theta}{dr}$ . 6
- b) Show that  $\rho$  for the curve  $r^n = a^n \cos n\theta$  varies inversely as  $r^{n-1}$ . 7
- c) Obtain the Maclaurin's series expansion of the function  $\log \left( \sqrt{\frac{1+x}{1-x}} \right)$  up to third degree term. 7

### UNIT - II

- 2 a) If  $u = e^x \sin(yz)$  where  $x = t^3$ ,  $y = t - 1$  and  $z = \frac{1}{t}$ , then find  $\frac{du}{dt}$  at  $t = 1$ . 6
- b) If  $z = f(x, y)$  where  $x = r \cos(\theta)$  and  $y = r \sin(\theta)$ , then show that  $\left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 = \left( \frac{\partial z}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial z}{\partial \theta} \right)^2$ . 7
- c) Obtain the Maclaurin's series expansion of the function  $e^x \log(1 + y)$  up to the third degree terms. 7

### OR

- 3 a) If  $x = r \sin(\theta) \cos(\phi)$ ,  $y = r \sin(\theta) \sin(\phi)$  and  $z = r \cos(\theta)$ , then find  $J = \frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ . 6
- b) Obtain the extreme values of the function  $f(x, y) = x^3 + 3xy^2 - 15y^2 - 15x^2 + 72x$ . 7
- c) Expand  $xy^2 + \cos(xy)$  about the point  $\left(1, \frac{\pi}{2}\right)$  using the Taylor's theorem up to the second degree terms. 7

### UNIT - III

- 4 a) Evaluate  $\int_1^2 \int_1^{x^2} (x^2 + y^2) dy dx$  by changing the order of integration. 6
- b) Evaluate  $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$ . 7

**Important Note:** Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

- c) Prove that  $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ . 7

**OR**

- 5 a) Evaluate  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$  by changing into polar coordinates. 6
- b) Find the area enclosed by the curve  $r = a(1 + \cos\theta)$  between  $\theta = 0$  and  $\theta = \pi$ . 7
- c) Show that  $\int_0^{\pi/2} \sqrt{\sin\theta} d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$ . 7

#### UNIT - IV

- 6 a) Solve:  $x^3 \frac{dy}{dx} - x^2 y = -y^4 \cos(x)$ . 6
- b) Solve:  $y(2x - y + 1)dx + x(3x - 4y + 3)dy = 0$ . 7
- c) Find the orthogonal trajectories of the family of curves  $r = 4a \sec(\theta) \tan(\theta)$ . 7

#### UNIT - V

- 7 a) Solve:  $(D^2 - 4D + 4)y = 8(e^{2x} + \sin(2x) + x^2)$ . 6
- b) Apply the method of variation of parameters to solve  $(D^2 + 2D + 2)y = e^{-x} \sec^3 x$ . 7
- c) Solve:  $(3x + 2)^2 y'' + 3(3x + 2)y' - 36y = 3x^2 + 4x + 1$ . 7

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