U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

May 2023 Semester End Main Examinations

Programme: B.E. **Semester: I**
Branch: Common to all Branches **Duration: 3 hrs.**
Course Code: 18MA1BSEM1 **Max Marks: 100**
Course: Engineering Mathematics-1 **Date: 24.05.223**

Instructions to Candidates: Answer FIVE FULL questions, choosing one from each unit.

UNIT - I

1	a)	Find the angle of intersection between the curves $r = \sin(\theta) + \cos(\theta)$ and $r = 2\sin(\theta)$.	**06**
	b)	Obtain the radius of curvature of the parabola $y^2 = 4ax$ at any point on it.	**07**
	c)	Obtain the Maclaurin's series expansion of the function $\log(\sec(x))$ up to third degree terms.	**07**

UNIT - II

2	a)	If $u = \log(tanx + tany + tanz)$, then prove that $\sin(2x)\frac{\partial u}{\partial x} + \sin(2y)\frac{\partial u}{\partial y} + \sin(2z)\frac{\partial u}{\partial z} = 2$.	**06**
	b)	Determine $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$ given $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$ and $w = x + y + z$.	**07**
	c)	Obtain the extreme values of $f(x,y) = x^3\, y^2(1 - x - y)$.	**07**

OR

3	a)	Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$, for $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$.	**06**
	b)	If $u = f(x,y)$ where $x = r\cos(\theta)$ and $y = r\sin(\theta)$, then show that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$.	**07**
	c)	Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ using Taylor's theorem up to second degree terms.	**07**

UNIT - III

4	a)	Find the area enclosed by the curve $r = a(1 + cos\theta)$ between $\theta = 0$ and $\theta = \pi$.	**06**
	b)	Change the order of integration and hence evaluate $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2 + y^2)\, dydx$.	**07**

c) Evaluate $\int_0^2 (8-x^3)^{-1/3}\,dx$ using Beta-Gamma functions. **07**

OR

5 a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (xyz)\,dzdydx$. **06**

b) Show that the area of the region outside the circle $r = a$ and inside the cardioid $r = a(1+\cos(\theta))$ is $\frac{a^2}{4}(\pi + 8)$. **07**

c) Apply Beta-Gamma functions to evaluate $\int_0^3 \frac{x^{3/2}}{\sqrt{3-x}}\,dx$. **07**

UNIT - IV

6 a) Find the orthogonal trajectories for the family of semi-cubical parabola $ay^2 = x^3$. **06**

b) Solve: $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$. **07**

c) Solve: $(y\log(x) - 2)ydx = xdy$ given $y(1) = 1$. **07**

UNIT - V

7 a) Solve: $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \cos(2x)$. **06**

b) Apply the method of variation of parameters to solve the differential equation $\frac{d^2y}{dx^2} - y = \frac{2}{(1+e^x)}$. **07**

c) Solve: $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = \log(x)$. **07**

******