U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

May 2023 Semester End Main Examinations

Programme: B.E. **Semester: I**
Branch: Computer Science Stream **Duration: 3 hrs.**
Course Code: 22MA1BSMCS **Max Marks: 100**
Course: Mathematical Foundation for Computer Science Stream – 1 **Date: 12.05.2023**

Instructions: Answer any FIVE full questions, choosing one full question from each unit.

UNIT - I

1 a) Obtain the pedal equation of the polar curve $r^m \cos(m\theta) = a^m$ where 'a' is constant. **6**

b) Find the angle of intersection of the polar curves $r(1+\cos\theta) = a$ and $r(1-\cos\theta) = b$. **7**

c) If ρ be the radius of curvature at any point P on the parabola $y^2 = 4ax$ and S be its focus, then show that ρ^2 varies as $(SP)^3$. **7**

UNIT - II

2 a) Let $F=F(x, y, z)$ and $x = u + v + w,\ y = vw + wu + uv$ and $z = uvw$, then show that $u\frac{\partial F}{\partial u} + v\frac{\partial F}{\partial v} + w\frac{\partial F}{\partial w} = x\frac{\partial F}{\partial x} + 2y\frac{\partial F}{\partial y} + 3z\frac{\partial F}{\partial z}$. **6**

b) A flat circular plate is heated so that the temperature at any point (x, y) is $u(x,y) = x^2 + 2y^2 - x$. Find the coldest point on the plate. **7**

c) Expand $f(x,y) = e^x \log(1+y)$ in powers of x and y up to third degree terms. **7**

OR

3 a) If $v = \log_e(x^2+y^2+z^2)$ then prove that $(x^2+y^2+z^2)\left[v_{xx} + v_{yy} + v_{zz}\right] = 2$. **6**

b) If $u = x + \frac{y^2}{x}$, $v = \frac{y^2}{x}$ then verify that $JJ' = 1$ where $J = \frac{\partial(u,v)}{\partial(x,y)}$ and $J' = \frac{\partial(x,y)}{\partial(u,v)}$. **7**

c) Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3y - 12x + 20$. **7**

UNIT - III

4 a) Solve: $\frac{dy}{dx} = \frac{y}{x + \sqrt{xy}}$. **6**

b) Solve $xdx + (x^2y + 4y)dy = 0, y(4) = 0.$ **7**

c) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, where λ is the parameter. **7**

UNIT - IV

5 a) Find the remainder when 4^{1000} is divided by 7. **6**

b) A general counts the number of surviving soldiers of a battle by aligning them successively in rows of certain sizes. Each time, he counts the number of remaining soldiers who failed to fill a row. The general initially had 1200 soldiers before the battle; after the battle **7**

- aligning them in rows of 7 soldiers leaves 4 remaining soldiers;
- aligning them in rows of 11 soldiers leaves 6 remaining soldiers;
- aligning them in rows of 13 soldiers leaves 9 remaining soldiers;

How many soldiers survived the battle?

c) Apply RSA algorithm to find the public and private key using $p = 3$ and $q = 11$ and hence encrypt a certain message with plain text numeral "29". **7**

OR

6 a) Show that $63! \equiv -1 \pmod{71}$. **6**

b) A small clothing manufacturer produces two styles of sweaters: cardigan and pullover. She sells cardigans for Rs.31 each and pullovers for Rs.28 each. If her total revenue from a day's production is Rs.146, how many of each type might she manufacture in a day? **7**

c) Solve: $x^3 + 2x + 2 \equiv 0 \pmod{49}$. **7**

UNIT - V

7 a) Determine the values of a and b for which the system $x+2y+3z=6$; $x+3y+5z=9$ and $2x+5y+az=b$ have (i) no solution (ii) unique solution (iii) infinite number of solutions. **6**

b) Apply Gauss-Seidel method to solve the system of equations $2x+y+6z=9$; $8x+3y+2z=13$ and $x+5y+z=7$ by taking the initial approximation as (0,0,0). Perform three iterations. **7**

c) Apply Rayleigh Power method to find the numerically largest eigenvalue and the corresponding eigenvector of the matrix $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking initial approximation to the eigenvector as $\begin{bmatrix} 1 & 0.8 & -0.8 \end{bmatrix}^T$. Perform three iterations. **7**

****