

U.S.N.

# B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

## February / March 2025 Semester End Main Examinations

Programme: B.E.

Semester: I

Branch: All Branches (Except IS/CS Clusters)

Duration: 3 hrs.

Course Code: 22MA1BSMES/22MA1BSMCV/22MA1BSMME

Max Marks: 100

Course: Mathematical Foundation for Electrical Stream-1

Mathematical Foundation for Civil Engineering -1

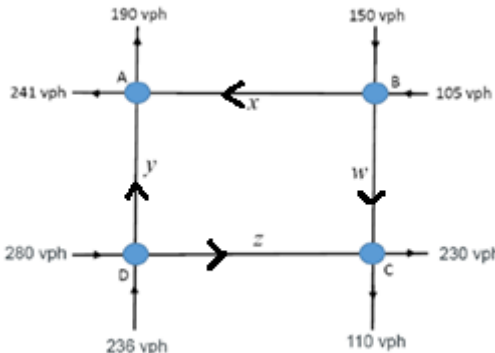
Mathematical Foundation for Mechanical Engineering Stream-1

### Instructions:

1. All units have internal choice, answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.			<b>UNIT - 1</b>	<i>CO</i>	<i>PO</i>	<b>Marks</b>
	1	a)	If $\phi$ be the angle between the radius vector and the tangent at any point on the curve $r = f(\theta)$ , then prove that $\tan \phi = r \frac{dr}{d\theta}$ .	1	1	<b>6</b>
		b)	Find the pedal equation of the curve $\frac{2a}{r} = 1 - \cos \theta$ .	1	1	<b>7</b>
		c)	If $\rho_1$ and $\rho_2$ are the radii of curvature at the extremities of any chord of the cardioid $r = a(1 + \cos \theta)$ which passes through the pole, then show that $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$ .	1	1	<b>7</b>
			<b>OR</b>			
	2	a)	Find the pedal equation of the curve $r = ae^{\theta \cot \alpha}$ where $\alpha$ is a parameter.	1	1	<b>6</b>
		b)	Prove that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally.	1	1	<b>7</b>
		c)	Find the radius of curvature at any point on the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ .	1	1	<b>7</b>
			<b>UNIT - 2</b>			
	3	a)	If $u = F(x - y, y - z, z - x)$ , then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ .	1	1	<b>6</b>
		b)	If $u = \log_e(x^3 + y^3 + z^3 - 3xyz)$ , then prove that (i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x + y + z}$ , (ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x + y + z)^2}$ .	1	1	<b>7</b>

	c)	Apply Taylor's theorem to expand $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x-1)$ and $(y-1)$ up to 2 <sup>nd</sup> degree term.	1	1	7
		<b>OR</b>			
4	a)	If $u = e^{a\theta} \cos(a \log(r))$ , then show that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$ .	1	1	6
	b)	If $x = e^u \cos v$ and $y = e^u \sin v$ , then prove that $JJ' = 1$ .	1	1	7
	c)	A rectangular box open at the top is to have a volume of 32 cubic units. Find the dimensions of the box requiring least material for its construction.	1	1	7
		<b>UNIT - 3</b>			
5	a)	Solve: $xy(1+xy^2) \frac{dy}{dx} = 1$ .	1	1	6
	b)	Find the orthogonal trajectory of the family of conics $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$ , where $\lambda$ is the parameter.	1	1	7
	c)	Solve: $(x^2 + y^2 + x)dx + (xy)dy = 0$ .	1	1	7
		<b>OR</b>			
6	a)	Solve: $y(2xy + e^x)dx - e^x dy = 0$ .	1	1	6
	b)	Solve: $(4xy + 3y^2 - x)dx + x(x+2)dy = 0$ .	1	1	7
	c)	Find the orthogonal trajectories of the family $r^n \cos(n\theta) = a^n$ .	1	1	7
		<b>UNIT - 4</b>			
7	a)	Solve: $(D^2 + 2)y = x^3 + 5^x$ .	1	1	6
	b)	Solve: $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = e^{-2x} - \cos 2x$ .	1	1	7
	c)	Solve: $(x-1)^3 \frac{d^3 y}{dx^3} + 2(x-1)^2 \frac{d^2 y}{dx^2} - 4(x-1) \frac{dy}{dx} + 4y = 4 \log(x-1)$ .	1	1	7
		<b>OR</b>			
8	a)	Solve $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} - 12y = e^{2x} - 3 \sin 2x$ .	1	1	6
	b)	Solve $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = \sin(\log x)$ .	1	1	7
	c)	Apply the method of variation of parameters to solve the differential equation $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = e^{-x} \log x$ .	1	1	7

			<b>UNIT - 5</b>			
9	a)	For what values of $\lambda$ and $\mu$ do the system of equations $x + y + z = 6$ , $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have i) No solution ii) Unique solution iii) Infinitely many solutions.	<i>I</i>	<i>I</i>	<b>6</b>	
	b)	Apply Gauss Seidel method to solve the system of equations $5x - y + z = 12$ , $x + 4y + 2z = 15$ and $x + 2y + 5z = 20$ taking $(1, 0, 3)$ as an initial approximation. Perform 3 iterations.	<i>I</i>	<i>I</i>	<b>7</b>	
	c)	Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ .	<i>I</i>	<i>I</i>	<b>7</b>	
			<b>OR</b>			
10	a)	Solve the system by Gauss elimination method $x + y + z = 4$ , $2x + y - z = 1$ and $x - y + 2z = 2$ .	<i>I</i>	<i>I</i>	<b>6</b>	
	b)	Find the traffic flow in the network of one-way streets with the directions shown below. 	<i>I</i>	<i>I</i>	<b>7</b>	
	c)	Apply Rayleigh Power method to find the dominant eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ by taking $[1 \ 0 \ 0]^T$ as the initial approximation. Perform four iterations.	<i>I</i>	<i>I</i>	<b>7</b>	

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