U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February / March 2024 Semester End Main Examinations

Programme: B.E. **Semester: II**
Branch: Common to all Branches **Duration: 3 hrs.**
Course Code: 21MA2BSACN **Max Marks: 100**
Course: Advanced Calculus and Numerical Methods

Instructions: Answer any FIVE full questions, choosing one full question from each unit.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	PO	Marks
		UNIT - I			
1	a)	Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dx\,dy$ by changing to polar coordinates.	CO1	PO1	**6**
	b)	Find the volume of the sphere $x^2+y^2+z^2=a^2$ using triple integrals.	CO1	PO1	**7**
	c)	Prove that $\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta \times \int_0^{\pi/2}\frac{1}{\sqrt{\sin\theta}}\,d\theta=\pi$.	CO1	PO1	**7**
		OR			
2	a)	Evaluate: $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration.	CO1	PO1	**6**
	b)	Obtain the area enclosed by the curve $r=a(1+\cos\theta)$ between $\theta=0$ and $\theta=\pi$ using double integration.	CO1	PO1	**7**
	c)	Show that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ with the usual notations.	CO1	PO1	**7**
		UNIT - II			
3	a)	Find the directional derivative of $\phi=x^2yz+4xz^2$ at $(1,-2,-1)$ in the direction of the vector $2\hat{i}-\hat{j}-2\hat{k}$.	CO1	PO1	**6**
	b)	If $\vec{F}=\nabla(xy^3z^2)$, find div $\vec{F}$ and curl $\vec{F}$ at the point $(1,-1,1)$.	CO1	PO1	**7**
	c)	Apply Stokes' theorem to evaluate $\int_C (y^2+x^2)dx-2xydy$, where C is the rectangle bounded by the lines $x=\pm a$, $y=0$ and $y=b$.	CO1	PO1	**7**
		OR			
4	a)	Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at $(2,-1,2)$.	CO1	PO1	**6**
	b)	Show that $\vec{f}=(y+z)\hat{i}+(z+x)\hat{j}+(x+y)\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{f}=\nabla\phi$.	CO1	PO1	**7**
	c)	Apply Green's theorem to evaluate $\int_C (xy+y^2)dx+x^2dy$ where C is the closed curve of the region bounded by $y=x$ and $y=x^2$.	CO1	PO1	**7**

			UNIT - III			
	5	a)	Solve $\frac{\partial^2 z}{\partial x^2} = xy$ by direct integration.	*CO1*	*PO1*	**6**
		b)	Form the partial differential equation by eliminating the arbitrary function from $f(x+y+z, x^2+y^2+z^2) = 0$.	*CO1*	*PO1*	**7**
		c)	Solve: $(y-z)p + (z-x)q = (x-y)$.	*CO1*	*PO1*	**7**
			UNIT - IV			
	6	a)	Evaluate $\int_0^6 \frac{1}{1+x^2}\, dx$ by using Simpson's 1/3rd rule considering seven ordinates.	*CO1*	*PO1*	**6**
		b)	The area A of a circle of diameter d is given in the table below: d: 80, 85, 90, 95, 100 A: 5026, 5674, 6362, 7088, 7854 Calculate the area of the circle of diameter 105 using appropriate interpolation formula.	*CO2*	*PO1*	**7**
		c)	Apply Newton-Raphson method to find a real root of $x \sin x + \cos x = 0$ near $x = \pi$ correct to 4 decimal places.	*CO1*	*PO1*	**7**
			UNIT - V			
	7	a)	Apply Taylor's series method to solve $\frac{dy}{dx} = x^2 + y^2$ given $y(0) = 1$ by considering the terms up to third degree. Hence find $y(0.1)$.	*CO1*	*PO1*	**6**
		b)	Apply Runge-Kutta method of order four to find $y(0.2)$ given $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.2$.	*CO1*	*PO1*	**7**
		c)	Solve the differential equation $\frac{dy}{dx} = \log_{10}(x+y)$ given the initial condition $y(1) = 2$ by using Modified Euler's method at the points $x = 1.2$. Take the step size $h = 0.2$ and carry out two iterations.	*CO1*	*PO1*	**7**

Table in question 6 b):

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

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