U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February / March 2025 Semester End Main Examinations

Programme: B.E. | **Semester: II**
Branch: Common to all Branches | **Duration: 3 hrs.**
Course Code: 21MA2BSACN | **Max Marks: 100**
Course: Advanced Calculus and Numerical Methods

Instructions:
1. All units have internal choice, answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	*PO*	Marks
		UNIT - 1			
1	a)	Evaluate $\iint_R x\,y\,dx\,dy$, where R is the region bounded by $x-$axis, the line $x=2a$ and the curve $x^2=4ay$.	1	1	**6**
	b)	Apply double integral to find the area of the cardioid $r=a(1+\cos\theta)$.	1	1	**7**
	c)	With usual notations, derive the relation between beta and gamma functions in the form $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	1	1	**7**
		OR			
2	a)	Evaluate $\int_0^a\int_y^a \dfrac{x\,dx\,dy}{\sqrt{x^2+y^2}}$ by changing the order of integration,	1	1	**6**
	b)	Evaluate $\int_0^a\int_0^{\sqrt{a^2-z^2}}\int_0^{\sqrt{a^2-y^2-z^2}} x\,dxdydz$.	1	1	**7**
	c)	Prove that $\int_0^\infty xe^{-x^8}dx\times\int_0^\infty x^2e^{-x^4}dx=\dfrac{\pi}{16\sqrt{2}}$.	1	1	**7**
		UNIT - 2			
3	a)	If $\vec{F}=grad\left(x^3+y^3+z^3-3xyz\right)$ then find $div\vec{F}$ and $curl\vec{F}$.	1	1	**6**
	b)	Find the constants a and b so that the surface $ax^2-byz=(a+2)x$ is orthogonal to the surface $4x^2y+z^3=4$ at the point $(1,-1,2)$.	1	1	**7**
	c)	Apply Green's theorem to evaluate $\int_C\left(3x-8y^2\right)dx+\left(4y-6xy\right)dy$, where C is bounded by $x=0$, $y=0$ and $x+y=1$.	1	1	**7**

			OR			
	4	a)	Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.	1	1	**6**
		b)	A fluid motion is given by $\vec{V} = (y+z)\hat{i} + (z+x)j + (x+y)k$. Is this motion irrotational? If so, find the velocity potential.	2	1	**7**
		c)	Apply Stokes' theorem to evaluate $\int_C (y^2 + x^2)dx - 2xydy$, where C is the rectangle bounded by the lines $x = \pm a$, $y = 0$ and $y = b$.	1	1	**7**
			UNIT - III			
	5	a)	Form the partial differential equation by eliminating the arbitrary constants from $z = a\log\left\{\frac{b(y-1)}{1-x}\right\}$.	1	1	**6**
		b)	Solve the partial differential equation $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.	1	1	**7**
		c)	Derive the one-dimensional heat equation in the form $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$.	1	1	**7**
			OR			
	6	a)	Form the partial differential equation by eliminating the arbitrary function from $z = x^n f\left(\frac{y}{x}\right)$.	1	1	**6**
		b)	Solve $\frac{\partial^2 z}{\partial x^2} = \cos x$ by using direct integration method.	1	1	**7**
		c)	Derive the one-dimensional wave equation in the form $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$.	1	1	**7**
			UNIT - 4			
	7	a)	Apply Newton-Raphson method to approximate the root of $x\tan x + 1 = 0$ near $x = 2.5$. Perform 3 iterations.	1	1	**6**
		b)	The area of a circle (A) corresponding to diameter (D) is given below: Find the area corresponding to diameter 105 using an appropriate interpolation formula.	1	1	**7**

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

		c)	A rocket is launched from the ground. Its acceleration f, generated during the first 80 seconds is tabulated below. Apply Simpson's $\left(\frac{1}{3}\right)^{\text{rd}}$ rule to find the velocity of the rocket at $t = 80$.	1	1	**7**

t(sec)	0	10	20	30	40	50	60	70	80
$f\,(cms^{-2})$	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

			OR			

<table>
<tr><td></td><td>8</td><td>a)</td><td>Apply Lagrange's interpolation formula to find y at $x = 10$ given
<table>
<tr><td>x</td><td>5</td><td>6</td><td>9</td><td>11</td></tr>
<tr><td>y</td><td>12</td><td>13</td><td>14</td><td>16</td></tr>
</table></td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>The following table gives the temperature θ of a cooling body at different instant of time t (in seconds)
<table>
<tr><td>t</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td></tr>
<tr><td>θ</td><td>85.3</td><td>74.5</td><td>67</td><td>60.5</td><td>54.3</td></tr>
</table>
Calculate θ at $t = 2$ using Newton's forward interpolation formula.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos(x)}\, dx$ by dividing the interval into six equal parts using Simpson's 3/8th rule.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>9</td><td>a)</td><td>Apply Taylor's series method to approximate the value of y at $x = 0.1$ for $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$ considering up to third degree.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Runge-Kutta method to solve the differential equation $\frac{dy}{dx} = \frac{y - x}{y + x}$ at $x = 0.2$ taking $y(0) = 1$ and $h = 0.2$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Compute y at $x = 0.8$ by applying Milne's predictor-corrector method to approximate the differential equation $\frac{dy}{dx} = x - y^2$ with $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$ and $y(0.6) = 0.1762$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>10</td><td>a)</td><td>Apply Milne's predictor-corrector method to find $y(0.4)$ given $\frac{dy}{dx} = 2e^x - y$ with $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.04$ and $y(0.3) = 2.09$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Runge-Kutta method of fourth order to find approximate value of y at $x = 0.2$ given $\frac{dy}{dx} = \frac{(y^2 - x^2)}{(y^2 + x^2)}$ with $y(0) = 1$, taking $h = 0.2$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Modified Euler's method to find the approximate solution of $y' = y + e^x$, $y(0) = 0$ at $x = 0.2$ taking $h = 0.2$. Perform two iterations.</td><td>1</td><td>1</td><td>7</td></tr>
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