U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

December / January 2024 Supplementary Examinations

Programme: B.E. **Semester: II**
Branch: Common to all Branches **Duration: 3 hrs.**
Course Code: 21MA2BSACN **Max Marks: 100**
Course: Advanced Calculus and Numerical Methods

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	PO	Marks
		UNIT - I			
1	a)	Evaluate $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}(2-x)\,dy\,dx$ by changing the order of integration.	CO2	PO1	**6**
	b)	Find the area of the cardioid $r = a\,(1+\cos\theta)$.	CO2	PO1	**7**
	c)	Prove that $\beta(m,n) = \dfrac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$.	CO2	PO1	**7**
		OR			
2	a)	Evaluate $\int_{0}^{a}\int_{0}^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2}\,dx\,dy$ by changing into polar coordinates.	CO2	PO1	**6**
	b)	Find the volume of the sphere $x^2+y^2+z^2=a^2$ using triple integrals.	CO2	PO1	**7**
	c)	Show that $\int_{0}^{\frac{\pi}{2}}\dfrac{d\theta}{\sqrt{\sin\theta}}\times\int_{0}^{\frac{\pi}{2}}\sqrt{\sin\theta}\;d\theta = \pi$.	CO2	PO1	**7**
		UNIT - II			
3	a)	Find the directional derivative of $\Phi = 4xz^3 - 3x^2y^2z$ at $(2,-1,2)$ along $2i-3j+6k$.	CO2	PO1	**6**
	b)	Show that $\vec{F} = (2xy^2+yz)\hat{i} + (2x^2y+xz+2yz^2)\hat{j} + (2y^2z+xy)\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$.	CO2	PO1	7
	c)	Verify Green's theorem for $\oint_C (xy+y^2)\,dx + x^2\,dy$ where C is the closed curve of the region bounded by $y=x$ and $y=x^2$.	CO2	PO1	**7**
		OR			

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<table>
<tr><td></td><td>4</td><td>a)</td><td>Find the angle between the normals to the surface $xy = z^2$ at $(4, 1, 2)$ and $(3, 3, -3)$.</td><td>CO2</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = \left|\vec{r}\right|$ prove that $\nabla \bullet \left(r^n \vec{r} \right) = (n+3) r^n$.</td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Stokes' theorem to evaluate $\oint_C (y^2 + x^2) dx - 2xy\, dy$, where C is the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$.</td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - III</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Form the partial differential equation by eliminating the arbitrary constants from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.</td><td>CO2</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Solve $x(y^2 - z^2)\, p + y(z^2 - x^2)\, q = z(x^2 - y^2)$.</td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Derive the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.</td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - IV</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Apply Newton – Raphson method to find the real root of the equation $xe^x - 2 = 0$ correct to three decimal places.</td><td>CO2</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>The area of a circle (A) corresponding to diameter (D) is given below:
<table>
<tr><td>D</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr>
<tr><td>A</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr>
</table>
Approximate the area when $D = 105$ using appropriate interpolation formula.</td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Simpson's $3/8^{th}$ rule to evaluate $\int_0^1 \frac{1}{1+x} dx$ by taking seven ordinates. Hence deduce the value of $\log_e 2$.</td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - V</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Apply Taylor series method to solve $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$ at $x = 0.1$ and 0.2 up to the third degree.</td><td>CO2</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply modified Euler's method to find the approximate value of y at $x = 1.2$ for the differential equation $\frac{dy}{dx} = 1 + \frac{y}{x}$, $y = 2$ at $x = 1$ by taking step size $h = 0.2$.</td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Runge-Kutta method of fourth order to find $y(0.2)$ for the differential equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.2$.</td><td>CO2</td><td>PO1</td><td>7</td></tr>
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