

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2023 Supplementary Examinations

Programme: B.E

Branch: Common to all Branches

Course Code: 18MA2BSEM2

Course: Engineering Mathematics-2

Semester: II

Duration: 3 hrs.

Max Marks: 100

Date: 13.09.2023

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

- 1 a) Evaluate: i) $\int_0^{\infty} \frac{e^{-t} - e^{-3t}}{t} dt$ ii) $L \left\{ \int_0^t \frac{\sin t}{t} dt \right\}$ 6
- b) Find the Laplace transform of periodic function $f(t) = \begin{cases} \sin(\omega t) & 0 < t < \frac{\pi}{\omega} \\ 0 & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ 7
- $f\left(t + \frac{2\pi}{\omega}\right) = f(t).$
- c) Express $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ 1 & \pi < t < 2\pi \\ \sin t & t > 2\pi \end{cases}$ in terms of unit step function and hence find 7
- its Laplace transform.

UNIT - II

- 2 a) Find $L^{-1} \left[\frac{s^2 + 6s + 9}{(s-1)(s-2)(s+4)} \right]$. 6
- b) Find $L^{-1} \left[\frac{s}{s^4 + 4a^4} \right]$. 7
- c) In an electrical circuit with e.m.f $E(t)$, resistance R and inductance L , the current i builds up at the rate given by $L \frac{di}{dt} + Ri = E(t)$. If the switch is connected at $t = 0$ and disconnected at $t = a$, find the current i at any instant. 7

OR

- 3 a) Find $L^{-1} \left[\frac{s e^{-s/2} + \pi e^{-s}}{s^2 + \pi^2} \right]$. 6
- b) Employing Laplace transform method, solve the differential equation 7
- $\frac{d^2 x}{dt^2} + 9x = \cos(2t)$ if $x\left(\frac{\pi}{2}\right) = -1$ and $x(0) = 1$.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

- c) Apply the method of Laplace transform to solve the simultaneous equation $\frac{dx}{dt} + y = \sin t$ $\frac{dy}{dt} + x = \cos t$ being given $x = 2, y = 0$ when $t = 0$. 7

UNIT - III

- 4 a) Form a partial differential equation by eliminating arbitrary function f from the relation $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$. 6
- b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$. 7
- c) Apply the method of separation of variables to obtain the solution of one dimensional heat equation. 7

UNIT - IV

- 5 a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$. 6
- b) Show that $\text{div}(\vec{A} \times \vec{B}) = \vec{B} \cdot \text{curl}(\vec{A}) - \vec{A} \cdot \text{curl}(\vec{B})$ where \vec{A} and \vec{B} are vector point functions of x, y and z . 7
- c) Verify Green's theorem for the integral $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ taken around the closed curve bounded by the curves $x = 0, y = 0$ and $x + y = 1$. 7

OR

- 6 a) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ along the normal to the surface $x \log z - y^2 = -4$ at the point $(-1, 2, 1)$. 6
- b) A vector field is given by $F = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$. Show that the field is irrotational and find its scalar potential. 7
- c) Apply Stokes' theorem to evaluate $\oint_C (2x - y)dx - yz^2dy - y^2zdz$ where C is the projection over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ in the xy -plane. 7

UNIT - V

- 7 a) Show that the spherical polar coordinate system forms an orthogonal curvilinear coordinate system. 6
- b) Express $\vec{F} = 2xi - 3y^2j + z^2k$ in cylindrical polar coordinate system. 7
- c) Derive an expression for finding $\nabla^2\psi$ in orthogonal curvilinear coordinate system. Write the expression for $\nabla^2\psi$ in cylindrical polar coordinate system and spherical polar coordinate system. 7
