

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2023 Supplementary Examinations

Programme: B.E

Branch: Common to all Branches

Course Code: 18MA2BSEM2

Course: Engineering Mathematics-2

Semester: II

Duration: 3 hrs.

Max Marks: 100

Date: 13.09.2023

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

1 a) Evaluate: i) $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$ ii) $L\left\{\int_0^t \frac{\sin t}{t} dt\right\}$ 6

b) Find the Laplace transform of periodic function $f(t) = \begin{cases} \sin(\omega t) & 0 < t < \frac{\pi}{\omega} \\ 0 & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ 7

$$f\left(t + \frac{2\pi}{\omega}\right) = f(t).$$

c) Express $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ 1 & \pi < t < 2\pi \\ \sin t & t > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. 7

UNIT - II

2 a) Find $L^{-1}\left[\frac{s^2 + 6s + 9}{(s-1)(s-2)(s+4)}\right]$. 6

b) Find $L^{-1}\left[\frac{s}{s^4 + 4a^4}\right]$. 7

c) In an electrical circuit with e.m.f $E(t)$, resistance R and inductance L_1 , the current i builds up at the rate given by $L_1 \frac{di}{dt} + Ri = E(t)$. If the switch is connected at $t = 0$ and disconnected at $t = a$, find the current i at any instant. 7

OR

3 a) Find $L^{-1}\left[\frac{se^{-s/2} + \pi e^{-s}}{s^2 + \pi^2}\right]$. 6

b) Employing Laplace transform method, solve the differential equation $\frac{d^2x}{dt^2} + 9x = \cos(2t)$ if $x\left(\frac{\pi}{2}\right) = -1$ and $x(0) = 1$. 7

c) Apply the method of Laplace transform to solve the simultaneous equation 7

$$\frac{dx}{dt} + y = \sin t \quad \frac{dy}{dt} + x = \cos t \quad \text{being given } x=2, y=0 \text{ when } t=0.$$

UNIT - III

4 a) Form a partial differential equation by eliminating arbitrary function f from 6
the relation $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$.

b) Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$. 7

c) Apply the method of separation of variables to obtain the solution of one 7
dimensional heat equation.

UNIT - IV

5 a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at 6
 $(2, -1, 2)$.

b) Show that $\text{div}(\vec{A} \times \vec{B}) = \vec{B} \cdot \text{curl}(\vec{A}) - \vec{A} \cdot \text{curl}(\vec{B})$ where \vec{A} and \vec{B} are vector point 7
functions of x, y and z .

c) Verify Green's theorem for the integral $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ taken 7
around the closed curve bounded by the curves $x=0, y=0$ and $x+y=1$.

OR

6 a) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ along 6
the normal to the surface $x \log z - y^2 = -4$ at the point $(-1, 2, 1)$.

b) A vector field is given by $\mathbf{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$. Show 7
that the field is irrotational and find its scalar potential.

c) Apply Stokes' theorem to evaluate $\oint_C (2x - y)dx - yz^2dy - y^2zdz$ where C is 7
the projection over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ in the xy -
plane.

UNIT - V

7 a) Show that the spherical polar coordinate system forms an orthogonal 6
curvilinear coordinate system.

b) Express $\vec{F} = 2xi - 3y^2j + zxk$ in cylindrical polar coordinate system. 7

c) Derive an expression for finding $\nabla^2\psi$ in orthogonal curvilinear coordinate 7
system. Write the expression for $\nabla^2\psi$ in cylindrical polar coordinate system
and spherical polar coordinate system.
