

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February / March 2025 Semester End Main Examinations

Programme: B.E.

Semester: II

Duration: 3 hrs.

Max Marks: 100

Course:

Mathematical Foundation for Civil and Mechanical Engineering Stream-2

Mathematical Foundation for Mechanical Engineering Stream – 2

Mathematical Foundation for Civil Engineering-2

Instructions: 1. All units have internal choice, answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - 1			CO	PO	Marks
1	a)	Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dy dx dz$.	1	1	6
	b)	Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy dy dx$ by changing the order of integration.	1	1	7
	c)	Derive the relation between beta and gamma function in the form $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	1	1	7
OR					
2	a)	Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates.	1	1	6
	b)	Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$.	1	1	7
	c)	Prove that $\int_0^{\infty} x e^{-x^8} dx \times \int_0^{\infty} x^2 e^{-x^4} dx = \frac{\pi}{16\sqrt{2}}$ using Beta and Gamma functions.	1	1	7
UNIT - 2					
3	a)	Find the angle between surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$.	1	1	6
	b)	Evaluate div \vec{F} and curl \vec{F} at the point $(1, 2, 3)$ given that $\vec{F} = x^2 y z \hat{i} + x y^2 z \hat{j} + x y z^2 \hat{k}$.	1	1	7
	c)	Find the work done in moving a particle in the force field $\vec{F} = 3x^2 \hat{i} + (2xz - y) \hat{j} + z \hat{k}$ along the straight line from $(0, 0, 0)$ to $(2, 1, 3)$.	1	1	7
OR					

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

4	a)	Find the directional derivative of $f(x, y, z) = 4e^{2x+y+z}$ at the point $P = (1, 1, -1)$ in the direction of \vec{PQ} where $Q = (-3, 5, 6)$.	1	1	6												
	b)	Find the value of a if the vector \vec{F} has zero divergence, where $\vec{F} = (ax^2y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{k}$. Find the curl of the above vector which has zero divergence.	1	1	7												
	c)	Apply Green's theorem to evaluate $\int_C (3x - 8y^2)dx + (4y - 6xy)dy$, where C is bounded by $x = 0, y = 0$ and $x + y = 1$.	1	1	7												
		UNIT - 3															
5	a)	Form a partial differential equation by eliminating arbitrary function ϕ from $\phi(x + y + z, x^2 + y^2 + z^2) = 0$.	1	1	6												
	b)	Solve: $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.	1	1	7												
	c)	Solve $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} = 0$ by using the method of separation of variables.	1	1	7												
		OR															
6	a)	Solve $\frac{\partial^2 z}{\partial x \partial y} = x^2$ by direct integration method.	1	1	6												
	b)	Form a partial differential equation by eliminating the arbitrary constants from $(x - a)^2 + (y - b)^2 = z$.	1	1	7												
	c)	Derive the one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.	1	1	7												
		UNIT - 4															
7	a)	Apply Newton-Raphson method to find a real root of $x^3 - 2x - 5 = 0$ near $x = 2.5$. Carryout 3 iterations.	1	1	6												
	b)	Apply Lagrange's interpolation formula to compute $f(3)$ from the following data:	1	1	7												
		<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td><td>0</td><td>1</td><td>2</td><td>5</td></tr> <tr> <td>$f(x)$</td><td>2</td><td>3</td><td>12</td><td>147</td></tr> </table>	x	0	1	2	5	$f(x)$	2	3	12	147					
x	0	1	2	5													
$f(x)$	2	3	12	147													
	c)	Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by Simpson's $\frac{3}{8}$ th rule taking 7 ordinates.	1	1	7												
		OR															
8	a)	Apply Newton Raphson method to find an approximate root of the equation $x \log_{10} x = 1.2$ that is near 2.5. Carryout three iterations.	1	1	6												
	b)	Find the number of students who have obtained marks between 40 and 45 from the given data:	1	1	7												
		<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>Marks</td><td>0-40</td><td>40-50</td><td>50-60</td><td>60-70</td><td>70-80</td></tr> <tr> <td>No. of students</td><td>31</td><td>42</td><td>51</td><td>35</td><td>31</td></tr> </table>	Marks	0-40	40-50	50-60	60-70	70-80	No. of students	31	42	51	35	31			
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No. of students	31	42	51	35	31												
	c)	Apply Simpson's 3/8 th rule with $h = 0.2$, to find the approximate area under the curve $y = \frac{x^2 - 1}{x^2 + 1}$ between $x = 1$ and $x = 2.8$, by taking 6 equal subintervals.	1	1	7												

UNIT - 5					
9	a)	Apply Taylor's series method to find y at $x = 0.1$ considering terms up to third degree, given that $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$.	1	1	6
	b)	Apply fourth order Runge-Kutta method to find y at $x = 0.1$ given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$ taking $h = 0.1$.	1	1	7
	c)	Apply Milne's predictor-corrector method to find $y(0.4)$ correct to four decimal places given $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1.0000$, $y(0.1) = 1.1113$, $y(0.2) = 1.2507$ and $y(0.3) = 1.426$.	1	1	7
OR					
10	a)	Find $y(0.1)$ using Taylor's series method, given that $\frac{dy}{dx} = e^x - y^2$, $y(0) = 1$, considering terms up to 3 rd degree.	1	1	6
	b)	Solve $y' = \log_e(x + y)$, $y(0) = 2$ at $x = 0.2$ using modified Euler's method. (Take $h = 0.2$). Perform two iterations.	1	1	7
	c)	Find an approximate solution of $\frac{dy}{dx} = 2e^x - y$ at $x = 0.4$ using Milne's predictor-corrector method given $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.04$ and $y(0.3) = 2.09$.	1	1	7
