U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February / March 2024 Semester End Main Examinations

Programme: B.E. **Semester: II**
Branch: CS, IS, ML, BT, DS, IOT, CSB **Duration: 3 hrs.**
Course Code: 22MA2BSMCS **Max Marks: 100**
Course: Mathematical Foundation for Computer Science Stream-2

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - I	***CO***	***PO***	**Marks**
1	a)	Evaluate $\iint_R xy\ dydx$ by changing the order of integration, where R is the region bounded by x-axis, ordinate $x = 2a$ and the parabola $x^2 = 4ay$.	CO1	PO1	**6**
	b)	Find the area lying inside the cardioid $r = a(1+\cos\theta)$ and outside the circle $r = a$.	CO2	PO1	**7**
	c)	Using the definition $\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2n-1} dx$, prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.	CO1	PO1	**7**
		OR			
2	a)	Evaluate $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dx\, dy$ by changing into polar coordinates.	CO1	PO1	**6**
	b)	Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	CO2	PO1	**7**
	c)	Prove that $\int_0^\infty x^2 e^{-x^4} dx \times \int_0^\infty e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$.	CO1	PO1	**7**
		UNIT - II			
3	a)	Prove that the cylindrical polar coordinate system is orthogonal curvilinear coordinate system.	CO1	PO1	**6**
	b)	Find the constants a and b so that the surfaces $\phi = ax^2y + z^3$ and $5x^2 - byz = 9x$ intersect orthogonally at the point (1, -1, 2).	CO1	PO1	**7**
	c)	Show that the fluid motion is given by $\vec{F}$ is irrotational, where $\vec{F} = (6xy+z^3)\hat{i} + (3x^2-z)\hat{j} + (3xz^2-y)\hat{k}$ and find its scalar potential.	CO1	PO1	**7**
		UNIT - III			
4	a)	Determine whether the set $V = \{(x,y)/x, y \in R\}$ is a vector space over the field of reals when the vector addition is the standard vector addition and the scalar multiplication is defined as $k.(x,y) = (0,ky)$.	CO1	PO1	**6**

<table>
<tr><td></td><td></td><td>b)</td><td>Find the basis and dimension of the row space, column space and null space of the matrix $\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & 1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the matrix of linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(a,\, b) = (a+b,\, a-b,\, 2b)$ with respect to the bases $B_1 = \{(1,\,0),(1,\,1)\}$ for $\mathbb{R}^2$ and $B_2 = \{(1,\,0,\,0),(0,\,1,\,0),(0,\,0,\,1)\}$ for $\mathbb{R}^3$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Determine the vectors $V_1 = (1,\,2,\,-3,\,1)$, $V_2 = (3,\,7,\,1,\,-2)$ and $V_3 = (1,\,3,\,7,\,-4)$ in $\mathbb{R}^4$ are linearly dependent or independent.</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Determine whether (1,1,1,1), (1,2,3,2), (2,5,6,4) and (2,6,8,5) form a basis of $\mathbb{R}^4$. If not find the dimension of the subspace they span.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the bases of the range space, null space and hence verify the rank-nullity theorem for the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x,y,z) = (x+2y-z,\, y+z,\, x+y-2z)$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - IV</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Apply Newton-Raphson method to find the root of the equation $x\log_{10} x = 1.2$ in (2,3) correct to four decimal places.</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>The following table gives the viscosity of an oil as a function of temperature. Find the viscosity of oil at a temperature of 140^0c using appropriate interpolation formula.
<table>
<tr><td>Temp</td><td>110</td><td>130</td><td>160</td><td>190</td></tr>
<tr><td>Viscosity</td><td>10.8</td><td>8.1</td><td>5.5</td><td>4.8</td></tr>
</table></td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's 1/3rd rule, find the velocity of the rocket at $t = 80$ sec.
<table>
<tr><td>t (sec)</td><td>0</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td></tr>
<tr><td>f (cm/sec^2)</td><td>30</td><td>31.63</td><td>33.34</td><td>35.47</td><td>37.75</td><td>40.33</td><td>43.25</td><td>46.69</td><td>50.67</td></tr>
</table></td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - V</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Solve the initial value problem $y' = 2y + 3e^x$, $y(0) = 0$ at $x = 0.1$ and $x = 0.2$ using Taylor's series method by considering terms up to fourth degree.</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply modified Euler's formula to solve $y' = xy + y^2$, $y(0) = 1$ at $x = 0.2$ by taking $h = 0.2$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Using Runge-Kutta method of fourth-order, solve the differential equation $y' = \dfrac{2xy + e^x}{x^2 + xe^x}$, with $y(1) = 0$ at $x = 1.1$ by taking $h = 0.1$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
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