U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

December 2023 Supplementary Examinations

Programme: B.E. **Semester: II**
Branch: CS, IS, ML, BT, DS, IOT, CSB **Duration: 3 hrs.**
Course Code: 22MA2BSMCS **Max Marks: 100**
Course: Mathematical Foundation for Computer Science Stream-2

Instructions:1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - I	***CO***	***PO***	**Marks**
1	a)	Evaluate the integral $\int_0^1 \int_{\sqrt{y}}^{2-y} xy\,dx\,dy$ by changing the order of integration.	CO1	PO1	**6**
	b)	Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ using triple integrals.	CO1	PO1	**7**
	c)	Using the definition $\Gamma(n)=2\int_0^\infty e^{-x^2}x^{2n-1}dx$, prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.	CO1	PO1	**7**
		OR			
2	a)	Evaluate the integral $\int_{-3}^{3}\int_0^{\sqrt{9-x^2}} \sqrt{x^2+y^2}\,dy\,dx$ by changing into polar form.	CO1	PO1	**6**
	b)	Find the area lying inside the cardioid $r=a(1+\cos\theta)$ and outside the circle $r=a$.	CO1	PO1	**7**
	c)	Prove that $\int_0^\infty xe^{-x^8}dx \times \int_0^\infty x^2e^{-x^4}dx=\frac{\pi}{16\sqrt{2}}$.	CO1	PO1	**7**
		UNIT - II			
3	a)	Prove that the cylindrical polar coordinate system is orthogonal curvilinear coordinate system.	CO1	PO1	**6**
	b)	If $\vec{r}=x\hat{i}+yj+zk$ and $r=\lvert\vec{r}\rvert$, then prove that $\nabla^2 f(r)=f''(r)+\frac{2}{r}f'(r)$.	CO1	PO1	**7**
	c)	Show that the vector field $\vec{F}=(y+z)\,\hat{i}+(z+x)\,\hat{j}+(x+y)\,\hat{k}$ is irrotational. Also find its scalar potential.	CO1	PO1	**7**

<table>
<tr><td></td><td></td><td></td><td>UNIT - III</td><td></td><td></td><td></td></tr>
<tr><td>4</td><td>a)</td><td>Express $v=(3,7,-4)$ in $\mathbb{R}^3$ as a linear combination of the vectors $u_1=(1,\ 2,\ 3)$, $u_2=(2,3,7)$, $u_3=(3,5,6)$.</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>Let $\mathbb{R}^+$ be the set of all positive real numbers. Define vector addition as $u+v=uv\ \forall u,v\in\mathbb{R}^+$ and scalar multiplication $k.u=u^k\ \ \forall k\in\mathbb{R}$. Show that $\mathbb{R}^+$ is a vector space over the field of real numbers.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Find the conditions a, b, c so that $v=(a,b,c)$ in $\mathbb{R}^3$ belongs to $W=\text{span}(u_1, u_2, u_3)$ where $u_1=(1,\ 2,\ 0)$, $u_2=(-1,1,2)$, $u_3=(3,0,-4)$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td>5</td><td>a)</td><td>Find the basis and dimension of the subspace W spanned by $u_1=t^3-2t^2+4t+1$, $u_2=2t^3-3t^2+9t-1$, $u_3=t^3+6t-5$ $u_4=2t^3-5t^2+7t+5$.</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>Find the matrix of Linear transformation Let $T:\mathbb{R}^2\to\mathbb{R}^3$ whose matrix is $A=\begin{bmatrix}1 & -1\\ -2 & 3\\ 0 & 1\end{bmatrix}$ relative to the basis $B=\{(1,1),\ (0,2)\}$ for $\mathbb{R}^2$ and $B'=\{(0,1,1),\ (1,0,1),(1,1,0)\}$ for $\mathbb{R}^3$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Let $G:\mathbb{R}^3\to\mathbb{R}^3$ be the linear mapping defined by $G(x,y,z)=(x+2y-z,\ \ y+z,\ \ x+y-2z)$. Find a basis and the dimension of (i) the image of G (ii) the kernel of G.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - IV</td><td></td><td></td><td></td></tr>
<tr><td>6</td><td>a)</td><td>Apply Newton-Raphson method to find the real root of the equation $3x=\cos x+1$ that lies near x = 0.6 correct to 4 decimal places.</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>The area of a circle (A) corresponding to diameter (D) is given below:
<table><tr><td>D</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr><tr><td>A</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr></table>
Find the area corresponding to diameter 105 using an appropriate interpolation formula.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Evaluate $\int_0^1 \frac{xdx}{1+x^2}$ by Weddle’s rule taking seven ordinates and hence find $\log_e 2$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
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			UNIT - V			
	7	a)	Apply Taylor series method to find an approximate solution up to third degree term for the initial value problem $y' = x^2 y - 1$, $y(0) = 1$ at $x = 0.1$ and $x = 0.2$.	CO1	PO1	**6**
		b)	Apply Modified Euler's method to find the solution of the differential equation $y' = x + \left\|\sqrt{y}\right\|$, $y = 1$ when $x = 0$ at the point x = 0.2 by taking h = 0.2. Carry out three modifications.	CO1	PO1	**7**
		c)	Apply Milne's predictor-corrector method to find the solution of the equation $y' = 2e^x - y$ at the point x = 0.4, given that $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.04$ and $y(0.3) = 2.09$.	CO1	PO1	**7**

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