U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2023 Semester End Main Examinations

Programme: B.E. **Semester: II**
Branch: CS, IS, ML, BT, DS, IOT, CSB **Duration: 3 hrs.**
Course Code: 22MA2BSMCS **Max Marks: 100**
Course: Mathematical Foundation for Computer Science Stream-2 **Date: 27.09.2023**

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

				CO	PO	Marks
			UNIT - I			
1	a)		Change the order of integration and evaluate $\int_0^1 \int_{\sqrt{y}}^{2-y} xy\, dxdy$.	CO1	PO1	**06**
	b)		Find the volume of the tetrahedron bounded by the planes $x = 0,\ y = 0,\ z = 0$, $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.	CO1	PO1	**07**
	c)		Prove that $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$.	CO1	PO1	**07**
			OR			
2	a)		Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2)\, dz\, dy\, dx$.	CO1	PO1	**06**
	b)		Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dxdy$ by changing into polar coordinates.	CO1	PO1	**07**
	c)		Show that $\int_0^\infty x\, e^{-x^8} dx \times \int_0^\infty x^2\, e^{-x^4} dx = \frac{\pi}{16\sqrt{2}}$.	CO1	PO1	**07**
			UNIT - II			
3	a)		Show that $\vec{F} = \frac{x\,\hat{\imath} + y\,\hat{\jmath}}{x^2+y^2}$ is solenoidal and also irrotational.	CO1	PO1	**06**
	b)		If $\vec{A} = x\,z^3\hat{\imath} - 2\,x^2yz\,\hat{\jmath} + 2y\,z^4\hat{k}$ find $\nabla \cdot \vec{A}$, $\nabla \times \vec{A}$ and $\nabla \cdot (\nabla \times \vec{A})$.	CO1	PO1	**07**
	c)		Represent $\vec{F} = y\,\hat{\imath} - z\,\hat{\jmath} + x\,\hat{k}$ in spherical polar coordinates.	CO1	PO1	**07**
			UNIT - III			
4	a)		Show that $(1,1,1),\ (1,-1,1), (0,1,1)$ are the basis of $V_3(R)$.	CO1	PO1	**06**
	b)		Let $\mathbb{R}^+$ be the set of all positive real numbers. Define vector addition as $u + v = uv\ \forall u, v \in \mathbb{R}^+$ and scalar multiplication $k.u = u^k \quad \forall k \in \mathbb{R}$. Show that $\mathbb{R}^+$ is a vector space over the field of real numbers.	CO1	PO1	**07**
	c)		Let $T: R^3 \to R^3$ be a linear transformation defined by $T(x,y,z) = (x + 2y - z,\ y + z,\ x + y - 2z)$. Find the bases of the range space and null space of T. Hence verify the Rank-nullity theorem.	CO1	PO1	**07**

			OR			
	5	a)	Express $p = 3t^2 + 7t - 4$ in $P_2(t)$ as a linear combination of the vectors $p_1 = t^2 + 2t + 3$, $p_2 = 2t^2 + 3t + 7$, $p_3 = 3t^2 + 5t + 6$.	*CO1*	*PO1*	**06**
		b)	Find the basis and dimension of the row space, column space and null space of the matrix $\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & 1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$.	*CO1*	*PO1*	**07**
		c)	Find the matrix of the linear transformation T on $V_3(R)$ defined as $T(a,b,c) = (2b + c,\ a - 4b,\ 3a)$ with respect to the basis $\{(1,1,1)\ (1,1,0), (1,0,0)\}$.	*CO1*	*PO1*	**07**
			UNIT - IV			
	6	a)	Apply Newton-Raphson method to find the root of the equation $x \log_{10} x = 1.2$ in (2,3) correct to four decimal places.	*CO1*	*PO1*	**06**
		b)	From the following table estimate the number of students who obtained marks between 40 and 45 Marks: 30-40 | 40-50 | 50-60 | 60-70 | 70-80 No. of students: 31 | 42 | 51 | 35 | 31	*CO2*	*PO1*	**07**
		c)	Evaluate $\int_0^{\pi/2} (1 - 0.612\ \sin^2 x)\, dx$ numerically using suitable integration rule taking 9 equal parts.	*CO1*	*PO1*	**07**
			UNIT - V			
	7	a)	Employ Taylor series method to find y at $x = 0.1$ correct to four decimal places of the linear equation $\frac{dy}{dx} = 3e^x + 2y$ whose solution passes through the origin.	*CO1*	*PO1*	**06**
		b)	Apply Runge-Kutta method of 4th order to find $y(0.2)$ given $(y^2 - x^2)dx = (y^2 + x^2)dy$, $y(0) = 1$ with $h = 0.2$.	*CO1*	*PO1*	**07**
		c)	Apply Milne's predictor - corrector method to compute $y(1.4)$ correct to four decimal places if $\frac{dy}{dx} = x^2 + \frac{y}{2}$, given $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.2469$ and $y(1.3) = 2.7514$.	*CO1*	*PO1*	**07**

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