U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February / March 2025 Semester End Main Examinations

Programme: B.E. | **Semester: II**
Branch: CS, CS-BS, CS-DS, CS-IOT, AI-ML, AI-DS, IS and BT | **Duration: 3 hrs.**
Course Code: 23MA2BSMCS / 22MA2BSMCS | **Max Marks: 100**
Course: Mathematical Foundation for Computer Science Stream -2

Instructions:
1. All units have internal choice, answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	*PO*	**Marks**
		UNIT - 1			
1	a)	With usual notations, prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.	1	1	**6**
	b)	Evaluate $\iint_R x\,y\,dx\,dy$ over the region R in the first quadrant bounded by $y = x^2, y = 0$ and $y = 4$.	1	1	**7**
	c)	Find the volume of the sphere $x^2 + y^2 + z^2 = 9$ using triple integration.	1	1	**7**
		OR			
2	a)	Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\,d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}}\,d\theta = \pi$ using Beta and Gamma functions.	1	1	**6**
	b)	Evaluate $\int_0^2 \int_x^{\sqrt{8-x^2}} \left(\frac{1}{5+x^2+y^2}\right) dy dx$ by changing into polar coordinates.	1	1	**7**
	c)	Change the order of integration and hence evaluate $\int_0^{2a} \int_0^{x^2/2a} xy\,dy\,dx$.	1	1	**7**
		UNIT - 2			
3	a)	If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = \lvert\vec{r}\rvert$ then show that $\frac{\vec{r}}{r^3}$ is solenoidal.	1	1	**6**
	b)	Obtain an angle between the surfaces $4x^2y + z^3 = 4$ and $5x^2 - 2yz = 9x$ at the point $(1, -1, 2)$.	1	1	**7**
	c)	Express the vector $\vec{f} = y\hat{\imath} - z\hat{\jmath} + x\hat{k}$ in spherical polar coordinates.	1	1	**7**
		OR			
4	a)	If $\vec{F} = (x + y + 1)\hat{i} + j - (x + y)k$ show that $\vec{F} \cdot curl\,\vec{F} = 0$.	1	1	**6**

<table>
<tr><td></td><td></td><td>b)</td><td>If the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(-1, 1, 2)$ has maximum magnitude of 32 units in the direction parallel to y-axis find a, b and c.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Prove that the cylindrical polar coordinate system is orthogonal curvilinear coordinate system.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Express the vector $(3, -7, 6)$ as a linear combination of the vectors $\{(1, -3, 2), (2, 4, 1), (1, 1, 1)\}$ in $\mathbb{R}^3$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Show that the set $B = \{(1,1), (1, -1)\}$ is a basis of the vector space $\mathbb{R}^2$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ such that $T(2,1) = (3,1,4)$ and $T(3,2) = (4,1,6)$. Hence find $T(1,1)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Show that the subset $W = \{(x_1, x_2, x_3) | x_1 + x_2 + x_3 = 0\}$ of the vector space $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Determine the basis and dimension of the subspace spanned by $(1, 2, 3, 2)$, $(3, 1, 0, 4)$, $(-2, 1, 3, 4)$ and $(2, 4, 6, 10)$ in $\mathbb{R}^4$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Let $T : V \to W$ be a linear transformation defined by $T(x, y, z) = (x + y, x - y, 2x + z)$. Find the range space, null space and hence verify the rank-nullity theorem.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Apply Lagrange's interpolation formula to find y at $x = 10$ given
<table>
<tr><td>x</td><td>5</td><td>6</td><td>9</td><td>11</td></tr>
<tr><td>y</td><td>12</td><td>13</td><td>14</td><td>16</td></tr>
</table></td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>The following table gives the temperature θ of a cooling body at different instant of time t (in seconds)
<table>
<tr><td>t</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td></tr>
<tr><td>θ</td><td>85.3</td><td>74.5</td><td>67</td><td>60.5</td><td>54.3</td></tr>
</table>
Calculate θ at $t = 2$ using Newton's forward interpolation formula.</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos(x)}\, dx$ by dividing the interval into six equal parts using Simpson's 3/8th rule.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>8</td><td>a)</td><td>Apply Newton-Raphson method to find an approximate root of the equation $3x = \cos x + 1$ near $x = 0.5$. Perform three iterations.</td><td>1</td><td>1</td><td>6</td></tr>
</table>

<table>
<tr><td></td><td></td><td>b)</td><td>From the following data estimate the number of students who have scored less than 70 marks using backward interpolation formula.
<table><tr><td>Marks</td><td>0 – 20</td><td>20 – 40</td><td>40 – 60</td><td>60 – 80</td><td>80 – 100</td></tr><tr><td>No. of Students</td><td>41</td><td>62</td><td>65</td><td>50</td><td>17</td></tr></table></td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Simpson's $(1/3)^{rd}$ rule to compute the area bounded by the curve $y = f(x)$, x-axis and the extreme ordinates from the following table.
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>0</td><td>2</td><td>2.5</td><td>2.3</td><td>2</td><td>1.7</td><td>1.5</td></tr></table></td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>9</td><td>a)</td><td>Employ Taylor's series method to obtain the approximate value of y at $x = 0.1$ for the differential equation $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ considering terms up to third degree.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Runge-Kutta method of fourth order with $h = 0.1$ to find an approximate value for the solution of the initial value problem $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1.5$ at $x = 1.1$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Milne's predictor – corrector method to find the solution of the differential equation $\frac{dy}{dx} = x^2 - y$ at $x = 0.4$ given $y(0) = 1$, $y(0.1) = 0.9051, y(0.2) = 0.8212$ and $y(0.3) = 0.7491$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>10</td><td>a)</td><td>Apply Taylor's series method to approximate the value of y at $x = 0.1$ for $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$ considering up to third degree.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Runge-Kutta method to solve the differential equation $\frac{dy}{dx} = \frac{y - x}{y + x}$ at $x = 0.2$ taking $y(0) = 1$ and $h = 0.2$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Intensity of radiation is directly proportional to the amount of remaining radioactive substance. The governing differential equation is $\frac{dy}{dt} = -ky$, where $k = 0.01, t_0 = 0, y_0 = 100g$. Determine how much substance will remain at the moment $t = 50$ sec by Modified Euler's method with $h = 25$. Perform two iterations.</td><td>2</td><td>1</td><td>7</td></tr>
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