U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September 2024 Supplementary Examinations

Programme: B.E. **Semester: II**
Branch: CS Stream and Biotechnology **Duration: 3 hrs.**
Course Code: 23MA2BSMCS / 22MA2BSMCS **Max Marks: 100**
Course: Mathematical Foundation for Computer Science Stream-2

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	PO	Marks
		UNIT - 1			
1	a)	Evaluate $\int_0^1\int_0^1 \frac{dx\,dy}{\sqrt{(1-x^2)(1-y^2)}}$.	1	1	**6**
	b)	Evaluate $\int_0^a\int_y^a \frac{x}{x^2+y^2}\,dx\,dy$ by changing the order of integration.	1	1	**7**
	c)	Derive the relation between the Beta and the Gamma functions.	1	1	**7**
		OR			
2	a)	Express $\int_0^{\pi/2}\left(\sqrt{\tan\theta}+\sqrt{\sec\theta}\right)d\theta$ in terms of the Gamma function.	1	1	**6**
	b)	Find the volume of the tetrahedron bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and the co-ordinate planes by using triple integration.	2	1	**7**
	c)	Evaluate $\iint xy\,dy\,dx$ taken over the region in the first quadrant bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$.	1	1	**7**
		UNIT - 2			
3	a)	If $\vec{r}=x\hat{i}+y\,j+z\,k$ and $r=\lvert\vec{r}\rvert$, then prove that $div\left(grad\, r^n\right)=n(n+1)r^{n-2}$.	1	1	**6**
	b)	If $\vec{F}=(x+y+az)\hat{i}+(bx+2y-z)\,j+(x+cy+2z)\,k$, find a, b, c so that $\text{curl}\,\vec{F}=0$ and then find ϕ such that $\vec{F}=\nabla\phi$.	1	1	**7**
	c)	Derive the expression for the base vectors of the cylindrical polar coordinate system and hence prove that they form an orthogonal curvilinear coordinate system.	1	1	**7**
		UNIT – 3			
4	a)	Determine whether or not the vectors $u=(1,1,2)$, $v=(2,3,1)$ and $w=(4,5,5)$ in R^3 are linearly dependent.	1	1	**6**

<table>
<tr><td></td><td></td><td>b)</td><td>Express M as a linear combination of the matrices A, B, C, where $M=\begin{bmatrix}4 & 7\\ 7 & 9\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and $C=\begin{bmatrix}1 & 1\\ 4 & 5\end{bmatrix}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the basis for the row space, the column space and the null space of the matrix $A=\begin{bmatrix}1 & 1 & 5 & 1\\ 2 & 4 & 14 & 4\\ 2 & 3 & 12 & 3\end{bmatrix}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Verify whether the vector $v=(2,-5,3)$ is in the subspace spanned by $W=\{(1,-3,2),(2,-4,-1),(1,-5,7)\}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Let $G:R^3\to R^3$ be the linear mapping defined by $G(x,y,z)=(x+2y-z,\ y+z,\ x+y-2z)$. Find the basis and the dimension of the (i) image of G (ii) kernel of G.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the condition on a, b, c so that $v=(a,b,c)$ in R^3 belongs to $W=\text{span}(u_1, u_2, u_3)$, where $u_1=(1,2,0)$, $u_2=(-1,1,2)$ and $u_3=(3,0,-4)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT – 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Apply Newton-Raphson method to find the approximate root of the equation $x\log_{10}x=1.2$ correct to 4 decimal places.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the number of students who obtained less than 45 marks from the following table. Also, estimate the number of students scoring marks more than 40 but less than 45.
<table>
<tr><td>Marks</td><td>30 – 40</td><td>40 – 50</td><td>50 – 60</td><td>60 – 70</td><td>70 – 80</td></tr>
<tr><td>No. of Students</td><td>31</td><td>42</td><td>51</td><td>35</td><td>31</td></tr>
</table></td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the approximate value of $\int_0^{\pi/2}\sqrt{\cos\theta}\,d\theta$ by Simpson's 1/3rd rule taking 9 ordinates.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Apply fourth order Runge-Kutta method to compute $y(0.1)$ for the equation $\frac{dy}{dx}=3e^x+2y$ with $y(0)=0$ taking $h=0.1$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply modified Euler's method to find the value of y at $x=0.1$ for the differential equation $\frac{dy}{dx}=\frac{y-x}{y+x}$ with $y(0)=1$ taking h = 0.1. Carry out three modifications.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Milne's predictor-corrector method to find the solution of the equation $\frac{dy}{dx}=\frac{2y}{x}$, $x\neq 0$ at the point $x=2$ for the data
<table>
<tr><td>x</td><td>1.00</td><td>1.25</td><td>1.5</td><td>1.75</td></tr>
<tr><td>y</td><td>2.00</td><td>3.13</td><td>4.5</td><td>6.13</td></tr>
</table>
Use the corrector formula once.</td><td>1</td><td>1</td><td>7</td></tr>
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