U.S.N. | | | | | | | | | | |

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

October 2024 Supplementary Examinations

Programme: B.E. | **Semester: II**
Branch: CS Stream and Biotechnology | **Duration: 3 hrs.**
Course Code: 23MA2BSMCS / 22MA2BSMCS | **Max Marks: 100**
Course: Mathematical Foundation for Computer Science Stream-2

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	***CO***	***PO***	**Marks**
1	a)	Derive the relation between Beta and the Gamma functions.	*1*	*1*	**6**
	b)	Evaluate $\iint_R x^2 y\,dx\,dy$ where R is the region bounded by the lines $y=x$, $y=0$ and $x+y=2$.	*1*	*1*	**7**
	c)	Find the area lying inside the circle $r=a\sin(\theta)$ and outside the cardioid $r=a\left(1-\cos(\theta)\right)$.	*1*	*1*	**7**
		OR			
2	a)	Evaluate $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4-a^2x^2}}\,dy\,dx$ by changing the order of integration.	*1*	*1*	**6**
	b)	Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ using triple integrals.	*2*	*1*	**7**
	c)	Express $\int_0^1 x^m\left(1-x^n\right)^p dx$ in terms of the Beta function and hence evaluate $\int_0^1 x^5\left(1-x^3\right)^{10} dx$.	*1*	*1*	**7**
		UNIT – 2			
3	a)	Derive the expression for the base vectors of the cylindrical polar coordinate system and hence prove that they form an orthogonal curvilinear coordinate system.	*1*	*1*	**6**
	b)	Find the constants a and b so that the surfaces $ax^2-byz=(a+2)x$ is orthogonal to the surface $4x^2y+z^3=4$ at the point $(1,-1,2)$.	*1*	*1*	**7**
	c)	If $\vec{r}=x\hat{i}+y\,j+zk$ and $r=\left\|\vec{r}\right\|$ then show that $div\left(r^n\,\vec{r}\right)=(n+3)r^n$ and hence deduce that is $\frac{\vec{r}}{r^3}$ is solenoidal.	*1*	*1*	**7**

<table>
<tr><td></td><td></td><td></td><td>UNIT – 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Determine whether or not the vectors $u=(1,1,2)$, $v=(2,3,1)$ and $w=(4,5,5)$ in R^3 are linearly dependent.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the matrix of the linear transformation $T:R^2\to R^2$ defined by $T(x,y)=(2x+3y,4x-5y)$ with respect to the basis $S=\{(1,2),(2,5)\}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the basis and dimension of the row space and null space of the matrix $A=\begin{bmatrix}1 & 3 & -5 & 1 & 5\\ -2 & -5 & 8 & 0 & -17\\ 3 & 11 & -19 & 7 & 1\\ 1 & 7 & -13 & 5 & -3\end{bmatrix}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Verify whether $v=(1,5,4)$ in $\mathbb{R}^3$ belongs to the Span(S) or not where $S=\{(1,3,-2),(2,7,-1),(1,6,7)\}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the linear transformation $T:\mathbb{R}^3\to\mathbb{R}^3$ such that $T(1,2,3)=(-1,-2,-3)$, $T(1,1,1)=(1,1,1)$ and $T(0,0,1)=(1,0,1)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Verify the rank nullity theorem for the linear mapping $A:\mathbb{R}^4\to\mathbb{R}^3$ where $A=\begin{bmatrix}1 & 2 & 3 & 1\\ 1 & 3 & 5 & -2\\ 3 & 8 & 13 & -3\end{bmatrix}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Apply Newton–Raphson method to obtain an approximate root of the equation $x\log_{10}(x)=1.4$ correct to 4 decimal places.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>The velocity *v* (km/min) of a car which starts from rest, is given at fixed intervals of time *t* (min) as follows:
<table>
<tr><td>t</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>14</td><td>16</td><td>18</td><td>20</td></tr>
<tr><td>v</td><td>10</td><td>18</td><td>25</td><td>29</td><td>32</td><td>20</td><td>11</td><td>5</td><td>2</td><td>0</td></tr>
</table>
Estimate approximately the distance covered in 20 minutes by numerical integration.</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Estimate the number of students who have scored less than 85 marks from the following data using an appropriate interpolation formula.
<table>
<tr><td>Marks</td><td>0-20</td><td>20-40</td><td>40-60</td><td>60-80</td><td>80-100</td></tr>
<tr><td>Number of Students</td><td>41</td><td>62</td><td>65</td><td>50</td><td>17</td></tr>
</table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Apply Modified Euler's method to find the solution of the differential equation $y'=x+\sin y$ at $x=0.2$ with $y(0)=1$ taking $h=0.2$. Carry out three modifications.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Runge-Kutta method of order four to solve the differential equation $\frac{dy}{dx}=\frac{y-x}{y+x}$ at $x=0.1$ with $y(0)=1$ taking $h=0.1$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Milne's predictor-corrector method to compute $y(0.4)$ for the differential equation $y'=x(x^2+y^2)e^{-x}$ with $y(0)=1$, $y(0.1)=1.005$, $y(0.2)=1.018$ and $y(0.3)=1.04$.</td><td>1</td><td>1</td><td>7</td></tr>
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