U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February / March 2025 Semester End Main Examinations

Programme: B.E. **Semester: II**
Branch: EEE, ETE, ECE, MD, EIE **Duration: 3 hrs.**
Course Code: 23MA2BSMES / 22MA2BSMES **Max Marks: 100**
Course: Mathematical Foundation for Electrical Stream - 2

Instructions:
1. All units have internal choice, answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.

			CO	PO	Marks
		UNIT - 1			
1	a)	Evaluate $\iint_R x\,y\,dx\,dy$, where R is the region bounded by $x-$ axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$.	1	1	**6**
	b)	Find the area lying inside the cardioid $r = a(1+\cos\theta)$ and outside the circle $r = a$.	1	1	**7**
	c)	Prove that $\int_0^\infty xe^{-x^8}dx \times \int_0^\infty x^2e^{-x^4}dx = \frac{\pi}{16\sqrt{2}}$.	1	1	**7**
		OR			
2	a)	Derive the relation between beta and gamma function in the form $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	1	1	**6**
	b)	Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}dx\,dy$ by transforming into polar coordinates.	1	1	**7**
	c)	Calculate the volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$.	1	1	**7**
		UNIT - 2			
3	a)	Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.	1	1	**6**
	b)	Apply Green's theorem to evaluate $\int_C (3x-8y^2)dx + (4y-6xy)dy$ where C is bounded by $x=0, y=0$ and $x+y=1$.	1	1	**7**
	c)	If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = \lvert\vec{r}\rvert$, show that $r^n\vec{r}$ is solenoidal for $n = -3$.	1	1	**7**
		OR			
4	a)	If $\vec{r} = x\hat{i} + yj + zk$ and $r = \lvert\vec{r}\rvert$, then prove that $div(grad(r^n)) = n(n+1)r^{n-2}$.	1	1	**6**

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

<table>
<tr><td></td><td></td><td>b)</td><td>A vector field is given by $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2yx + y)\hat{j}$. Show that the field is irrotational and find its scalar potential.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Stokes' theorem to evaluate $\oint_C (2x - y)\,dx - yz^2 dy - y^2 z dz$ where C is the projection over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ in the xy - plane.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Obtain the matrix of linear transformation $T : R^2 \rightarrow R^3$ defined by $T(a,b) = (a+b, a-b, 2b)$ with respect to the basis $B = \{(1,0), (1,1)\}$ for R^2 and C a standard basis for R^3.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Let T: $V \rightarrow W$ be a linear transformation defined by $T(x, y, z) = (x + y, x - y, 2x + z)$. Verify the rank-nullity theorem.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the basis and dimension of row space, column space and null space of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Find the linear transformation $T : R^2 \rightarrow R^2$ such that $T(1,2) = (3,0)$ and $T(2,1) = (1,2)$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the basis and dimension of the subspace spanned by the vectors $\{(1,-2,3), (1,-3,4), (-1,1,-2)\}$ in the vector space $V_3(R)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Verify whether $V = (1,-2,5)$ in R^3 is a linear combination of the vectors $u_1 = (1,1,1), u_2 = (1,2,3), u_3 = (2,-1,1)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Approximate the root of the equation $3x = \cos x + 1$ near $x = 0.5$ by Newton-Raphson method. Perform three iterations.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Lagrange's interpolation formula inversely to obtain a root of the equation $f(x) = 0$ given that $f(30) = -30$, $f(34) = -13$ and $f(38) = 3$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>A solid of revolution is formed by rotating about the x –axis, the lines $x = 0$, $x = 1$ and curve through the points with the following coordinates:<table><tr><td>x</td><td>0</td><td>0.25</td><td>0.5</td><td>0.75</td><td>1</td></tr><tr><td>y</td><td>1</td><td>0.9896</td><td>0.9589</td><td>0.9089</td><td>0.8415</td></tr></table>Apply the Simpson's rule to find the volume of the solid formed.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>8</td><td>a)</td><td>Find an approximate root of the equation $x\log_{10}(x) = 1.2$ near $x = 1.5$ using Newton-Raphson method. Perform three iterations.</td><td>1</td><td>1</td><td>6</td></tr>
</table>

<table>
<tr><td></td><td></td><td>b)</td><td>Distance y in nautical miles of the visible horizon for given heights x in metres above the surface of the earth are given by the following table:
<table><tr><td>x</td><td>100</td><td>150</td><td>200</td><td>250</td><td>300</td><td>350</td><td>400</td></tr><tr><td>y</td><td>12</td><td>16</td><td>21</td><td>27</td><td>36</td><td>50</td><td>72</td></tr></table>Find the value of y when $x = 225$ metres by applying Newton's forward interpolation formula.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Given that
<table><tr><td>x</td><td>4</td><td>4.2</td><td>4.4</td><td>4.6</td><td>4.8</td><td>5</td><td>5.2</td></tr><tr><td>$y = \log_e x$</td><td>1.3863</td><td>1.4351</td><td>1.4816</td><td>1.5261</td><td>1.5686</td><td>1.6094</td><td>1.6487</td></tr></table>Evaluate $I = \int_4^{5.2} \log x dx$ by using Weddle's rule.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>9</td><td>a)</td><td>Solve by Taylor's series method the equation $\frac{dy}{dx} = \log_e(xy), y(1) = 2$ for $y(1.1)$ by considering up to third degree terms.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Intensity of radiation is directly proportional to the amount of remaining radioactive substance. The differential equation is $\frac{dy}{dt} = -ky$, where $k = 0.01, t_0 = 0, y_0 = 100g$. Determine how much substance will remain at the moment $t = 25$ sec by Modified Euler's method with $h = 25$. Perform two iterations.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Solve the differential equation $\frac{dy}{dx} = \frac{y - x}{y + x}, y(0) = 1$ at $x = 0.2$ taking $h = 0.2$ by Runge Kutta method of fourth order.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>10</td><td>a)</td><td>Employ Taylor's series method to obtain approximate value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$ by considering up to third degree terms.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply modified Euler's method to compute $y(0.1)$, given $\frac{dy}{dx} = x^2 + y$ with the initial condition $y(0) = 1$ by taking $h = 0.1$. Perform two iterations.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Milne's Predictor-corrector method to find $y(2)$ given $\frac{dy}{dx} = \frac{2y}{x}$, $x \neq 0$, $y(1) = 2$, $y(1.25) = 3.13$, $y(1.5) = 4.5$ and $y(1.75) = 6.13$.</td><td>1</td><td>1</td><td>7</td></tr>
</table>

******