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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

October 2024 Supplementary Examinations

Programme: B.E.

Semester: II

Branch: EEE/ECE/MD/EIE/ETE

Duration: 3 hrs.

Course Code: 23MA2BSMES/22MA2BSMES

Max Marks: 100

Course: Mathematical Foundation for Electrical Stream – 2

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT – 1			CO	PO	Marks
1	a)	Evaluate $\int_{x=0}^1 \int_{y=x}^{\sqrt{x}} xy \, dy \, dx$ by changing the order of integration.	1	1	6
	b)	Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ using $\Gamma(n) = 2 \int_0^{\infty} e^{-x^2} x^{2n-1} dx$.	1	1	7
	c)	Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0, \frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ using triple integration.	2	1	7
OR					
2	a)	Evaluate the integral $\int_{-a}^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2 + y^2} dy \, dx$ by changing into polar coordinates.	1	1	6
	b)	Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz \, dy \, dx$.	1	1	7
	c)	Show that $\int_0^{\pi/2} \sqrt{\sin \theta} d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin \theta}} d\theta = \pi$.	1	1	7
UNIT – 2					
3	a)	Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $P(1, -2, -1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$.	2	1	6
	b)	Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Also, find ϕ such that $\vec{F} = \nabla\phi$.	2	1	7
	c)	Apply Green's theorem to evaluate $\oint_C (xy + y^2)dx + x^2 dy$ where 'C' is the closed curve of the region bounded by $y = x$ and $y = x^2$.	2	1	7
UNIT – 3					
4	a)	Let $V = R^3$ be the vector space of the ordered triplets of real numbers over the field of real numbers. Verify which of the following are subspaces of R^3 . i. $W = \{(x_1, x_2, x_3) x_1 + x_2 + x_3 = 0\}$. ii. $W = \{(a, b, c) a \geq 0\}$.	1	1	6

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

	b)	Find the basis and dimension of the subspace spanned by the subset $W = \left\{ \begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix}, \begin{pmatrix} 2 & -4 \\ -5 & 7 \end{pmatrix}, \begin{pmatrix} 1 & -7 \\ -5 & 1 \end{pmatrix} \right\}$ of the vector space of all 2×2 matrices over \mathbb{R} .	1	1	7												
	c)	Find the matrix of linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x, y, z) = (2x, 4y + 5z)$ with respect to the bases $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ for \mathbb{R}^3 and $C = \{(1, 1), (0, 1)\}$ for \mathbb{R}^2 .	1	1	7												
OR																	
5	a)	Show that the set $B = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is a basis of the vector space $V_3(\mathbb{R})$.	1	1	6												
	b)	Let \mathbb{R}^+ be the set of all positive real numbers. Define vector addition as $u + v = uv \forall u, v \in \mathbb{R}^+$ and scalar multiplication by $k \cdot u = u^k \forall u \in \mathbb{R}^+, k \in \mathbb{R}$. Show that \mathbb{R}^+ is a vector space over the field of real numbers.	1	1	7												
	c)	Let $T: V \rightarrow W$ be a linear transformation defined by $T(x, y, z) = (x + y, x - y, 2x + z)$. Find the range, null space, rank, nullity and hence verify the rank nullity theorem.	1	1	7												
UNIT - 4																	
6	a)	Find a real root of the equation $3x = \cos x + 1$ correct to 4 decimal places using Newton-Raphson method.	1	1	6												
	b)	From the following table, estimate the number of students who obtained marks between 40 and 45.	2	1	7												
		<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Marks</td> <td>30-40</td> <td>40-50</td> <td>50-60</td> <td>60-70</td> <td>70-80</td> </tr> <tr> <td>No. of students</td> <td>31</td> <td>42</td> <td>51</td> <td>35</td> <td>31</td> </tr> </table>	Marks	30-40	40-50	50-60	60-70	70-80	No. of students	31	42	51	35	31			
Marks	30-40	40-50	50-60	60-70	70-80												
No. of students	31	42	51	35	31												
	c)	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Weddle's rule taking seven ordinates.	1	1	7												
UNIT - 5																	
7	a)	Employ Taylor series method to obtain an approximate value of y at $x = 0.2$ for the differential equation $y' = 2y + 3e^x$, $y(0) = 0$ by considering terms up to fourth degree.	1	1	6												
	b)	Apply Runge-Kutta fourth order method to find $y(0.2)$ for the differential equation $\frac{dy}{dx} = x + y$, $y(0) = 1$ taking $h = 0.2$.	1	1	7												
	c)	Compute $y(0.4)$ using Milne's predictor-corrector method, given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$ and $y(0.3) = 1.5049$.	1	1	7												
