

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

October 2024 Supplementary Examinations

Programme: B.E.

Branch: EEE/ECE/MD/EIE/ETE

Course Code: 23MA2BSMES/22MA2BSMES

Course: Mathematical Foundation for Electrical Stream – 2

Semester: II

Duration: 3 hrs.

Max Marks: 100

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.		UNIT – 1	CO	PO	Marks
	1	a) Evaluate $\int_{x=0}^1 \int_{y=x}^{\sqrt{x}} xy \, dy \, dx$ by changing the order of integration.	1	1	6
		b) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ using $\Gamma(n) = 2 \int_0^{\infty} e^{-x^2} x^{2n-1} dx$.	1	1	7
		c) Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0, \frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ using triple integration.	2	1	7
		OR			
	2	a) Evaluate the integral $\int_{-a}^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2} \, dy \, dx$ by changing into polar coordinates.	1	1	6
		b) Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) \, dz \, dy \, dx$.	1	1	7
		c) Show that $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} \, d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin \theta}} \, d\theta = \pi$.	1	1	7
		UNIT – 2			
	3	a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $P(1, -2, -1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$.	2	1	6
		b) Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Also, find ϕ such that $\vec{F} = \nabla\phi$.	2	1	7
		c) Apply Green's theorem to evaluate $\oint_C (xy + y^2) \, dx + x^2 \, dy$ where 'C' is the closed curve of the region bounded by $y = x$ and $y = x^2$.	2	1	7
		UNIT – 3			
	4	a) Let $V = R^3$ be the vector space of the ordered triplets of real numbers over the field of real numbers. Verify which of the following are subspaces of R^3 . i. $W = \{(x_1, x_2, x_3) \mid x_1 + x_2 + x_3 = 0\}$. ii. $W = \{(a, b, c) \mid a \geq 0\}$.	1	1	6

	b)	Find the basis and dimension of the subspace spanned by the subset $W = \left\{ \begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix}, \begin{pmatrix} 2 & -4 \\ -5 & 7 \end{pmatrix}, \begin{pmatrix} 1 & -7 \\ -5 & 1 \end{pmatrix} \right\}$ of the vector space of all 2×2 matrices over \mathbb{R} .	1	1	7												
	c)	Find the matrix of linear transformation $T: R^3 \rightarrow R^2$ defined by $T(x, y, z) = (2x, 4y + 5z)$ with respect to the bases $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ for R^3 and $C = \{(1, 1), (0, 1)\}$ for R^2 .	1	1	7												
		OR															
5	a)	Show that the set $B = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is a basis of the vector space $V_3(\mathbb{R})$.	1	1	6												
	b)	Let \mathbb{R}^+ be the set of all positive real numbers. Define vector addition as $u + v = uv \forall u, v \in \mathbb{R}^+$ and scalar multiplication by $k.u = u^k \forall u \in \mathbb{R}^+, k \in \mathbb{R}$. Show that \mathbb{R}^+ is a vector space over the field of real numbers.	1	1	7												
	c)	Let $T: V \rightarrow W$ be a linear transformation defined by $T(x, y, z) = (x + y, x - y, 2x + z)$. Find the range, null space, rank, nullity and hence verify the rank nullity theorem.	1	1	7												
		UNIT – 4															
6	a)	Find a real root of the equation $3x = \cos x + 1$ correct to 4 decimal places using Newton-Raphson method.	1	1	6												
	b)	From the following table, estimate the number of students who obtained marks between 40 and 45. <table border="1" data-bbox="411 1108 1157 1187"> <tr> <td>Marks</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60-70</td><td>70-80</td></tr> <tr> <td>No. of students</td><td>31</td><td>42</td><td>51</td><td>35</td><td>31</td></tr> </table>	Marks	30-40	40-50	50-60	60-70	70-80	No. of students	31	42	51	35	31	2	1	7
Marks	30-40	40-50	50-60	60-70	70-80												
No. of students	31	42	51	35	31												
	c)	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Weddle's rule taking seven ordinates.	1	1	7												
		UNIT - 5															
7	a)	Employ Taylor series method to obtain an approximate value of y at $x = 0.2$ for the differential equation $y' = 2y + 3e^x$, $y(0) = 0$ by considering terms up to fourth degree.	1	1	6												
	b)	Apply Runge-Kutta fourth order method to find $y(0.2)$ for the differential equation $\frac{dy}{dx} = x + y$, $y(0) = 1$ taking $h = 0.2$.	1	1	7												
	c)	Compute $y(0.4)$ using Milne's predictor-corrector method, given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$ and $y(0.3) = 1.5049$.	1	1	7												
