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# B.M.S. College of Engineering, Bengaluru-560019

**Autonomous Institute Affiliated to VTU**

## September / October 2023 Semester End Main Examinations

**Programme: B.E.** **Semester: II**
**Branch: ME, IEM, AS, CH** **Duration: 3 hrs.**
**Course Code: 22MA2BSMME** **Max Marks: 100**
**Course: Mathematical Foundation for Mechanical Engineering Stream-2** **Date: 27.09.2023**

**Instructions**: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

**Important Note:** Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

| | | **UNIT – I** | ***CO*** | ***PO*** | **Marks** |
|---|---|---|---|---|---|
| 1 | a) | Evaluate $\iint_R x\,y\,dx\,dy$, if $R$ is the domain bounded by $x-axis$, ordinate $x=2a$ and the curve $x^2=4ay$. | *CO1* | *PO1* | **6** |
| | b) | Evaluate $\int_0^1\int_{\sqrt{x}}^1 \frac{y^2 dy\,dx}{\sqrt{y^4-x^2}}$ by changing the order of integration. | *CO1* | *PO1* | **7** |
| | c) | Show that $\int_0^\infty xe^{-x^8}dx \times \int_0^\infty x^2e^{-x^4}dx = \frac{\pi\sqrt{2}}{32}$. | *CO1* | *PO1* | **7** |
| | | **OR** | | | |
| 2 | a) | Evaluate $\int_{-3}^{3}\int_0^{\sqrt{9-x^2}} \sqrt{x^2+y^2}\,dy\,dx$ by changing into polar coordinates. | *CO1* | *PO1* | **6** |
| | b) | Compute the volume of the tetrahedron bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and the co-ordinate planes using triple integrals. | *CO2* | *PO1* | **7** |
| | c) | Derive the relationship between the Beta and Gamma function. | *CO1* | *PO1* | **7** |
| | | **UNIT – II** | | | |
| 3 | a) | Find the directional derivative of $x^2y^2z^2$ at the point $(1,1,-1)$ in the direction of the tangent to the curve $x=e^t$, $y=1+2\sin t$ and $z=t-\cos t$, where $-1\le t\le 1$. | *CO1* | *PO1* | **6** |
| | b) | $\overrightarrow{F}=(6xy+z^3)\hat{i}+(3x^2-z)j+(3xz^2-y)k$ is velocity vector field. Show that $\overrightarrow{F}$ is an irrotational and hence find its scalar potential $\phi(x,y,z)$, given $\phi(1,-1,1)=1$. | *CO2* | *PO1* | **7** |

| | | | | | |
|---|---|---|---|---|---|
| | c) | Apply Green's theorem to compute the work done $W = \oint_C (y - \sin x)\,dx + \cos x\,dy$, where $C$ is the triangle enclosed by the lines $y=0$, $x = \pi/2$ and $y = 2x/\pi$. | CO2 | PO1 | **7** |
| | | **UNIT – III** | | | |
| 4 | a) | Solve $x\frac{\partial^2 z}{\partial x^2} + \frac{\partial z}{\partial x} = 9x^2y^3$ by direct integration method. | CO1 | PO1 | **6** |
| | b) | Solve $x^2\left(y^3 - z^3\right)p + y^2\left(z^3 - x^3\right)q = z^2\left(x^3 - y^3\right)$. | CO1 | PO1 | **7** |
| | c) | Derive the one-dimensional heat equation in the form $u_t = c^2 u_{xx}$. | CO1 | PO1 | **7** |
| | | **OR** | | | |
| 5 | a) | Form a partial differential equation by eliminating arbitrary functions from $z = \frac{1}{r}[f(r-at) + g(r+at)]$. | CO1 | PO1 | **6** |
| | b) | Solve $z(z^2 + xy)(px - qy) = x^4$. | CO1 | PO1 | **7** |
| | c) | Apply the method of separation of variables to solve $\frac{\partial z}{\partial x} = 2\frac{\partial z}{\partial y} + z$ subjected to the condition $z(x,0) = 6e^{-3x}$. | CO1 | PO1 | **7** |
| | | **UNIT – IV** | | | |
| 6 | a) | Apply Newton-Raphson method to find a root of the function $f(x) = x\tan(x) + 1$ near $x = \pi$ correct to 4 decimal places. | CO1 | PO1 | **6** |
| | b) | Apply suitable interpolation formula to compute the viscosity of the oil $140^\circ C$ from the given data: (see table below) | CO2 | PO1 | **7** |
| | c) | The velocity $v(km/min)$ of a moped which starts from rest, is given at fixed intervals of time $t(\min)$ as follows: (see table below) Estimate the approximate distance covered in 20 minutes using suitable Simpson's rule. | CO2 | PO1 | **7** |
| | | **UNIT – V** | | | |
| 7 | a) | Compute the approximate value of $y$ at $x = 0.1$ and $x = 0.2$ by determining the Taylor series solution of $y' = x^2y - 1$ up to fourth degree term given $y(0) = 1$. | CO1 | PO1 | **6** |
| | b) | Find an approximate solution of $y' = x + \sin(y)$, $y(0) = 1$ at $x = 0.2$ by Runge-Kutta method of order 4 taking $h = 0.2$. | CO1 | PO1 | **7** |
| | c) | Find an approximate solution of $y' = xy + y^2$ at $x = 0.4$ using Milne's predictor corrector method given $y(0) = 1.0000$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$ and $y(0.3) = 1.5049$. | CO1 | PO1 | **7** |

Table for 6 b):

| Temperature $^\circ C$ | 110 | 130 | 160 | 190 |
|---|---|---|---|---|
| Viscosity | 10.8 | 8.1 | 5.5 | 4.8 |

Table for 6 c):

| $t(\min)$ | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| $v(km/min)$ | 10 | 18 | 25 | 29 | 32 | 20 | 11 | 5 | 2 | 0 |

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