U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February / March 2024 Semester End Main Examinations

Programme: B.E. | **Semester: II**
Branch: ME, IEM, AS, CH | **Duration: 3 hrs.**
Course Code: 22MA2BSMME | **Max Marks: 100**
Course: Mathematical Foundation for Mechanical Engineering Stream-2

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

		UNIT - I	***CO***	***PO***	**Marks**
1	a)	Evaluate $\iint_R xy(x+y)\,dxdy$, where R is the region bounded by the parabola $y=x^2$ and the line $y=x$.	CO1	PO1	**6**
	b)	Find the mass of a lamina in the form of the cardioid $r=a(1+\cos\theta)$ whose density at any point varies as the square of its distance from the initial line.	CO2	PO1	**7**
	c)	Using $\Gamma(n)=2\int_0^\infty e^{-x^2}x^{2n-1}dx$, prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.	CO1	PO1	**7**
		OR			
2	a)	Change the order of integration and hence evaluate $\int_0^1\int_{e^x}^{e}\frac{dy\,dx}{\log_e y}$.	CO1	PO1	**6**
	b)	Find the volume of the sphere $x^2+y^2+z^2=a^2$.	CO2	PO1	**7**
	c)	Prove that $\int_0^1\frac{x^2\,dx}{\sqrt{1-x^4}}\times\int_0^1\frac{dx}{\sqrt{1-x^4}}=\frac{\pi}{4}$.	CO1	PO1	**7**
		UNIT - II			
3	a)	Find the directional derivatives of $\phi(x,y,z)=x^2yz+4xz^2$ at the point $P(1,2,-1)$ in the direction of PQ where $Q(3,-3,-2)$.	CO1	PO1	**6**
	b)	A fluid motion is given by $\vec{V}=(y+z)\hat{i}+(z+x)\hat{j}+(x+y)\hat{k}$. show that the motion is irrotational and find its scalar potential.	CO1	PO1	**7**
	c)	Find the work done in moving a particle in the force field $\vec{F}=(2y+3)\hat{i}+(xy)\hat{j}+(yz-x)\hat{k}$ along the curve defined by $x=2t^2,\ y=t$ and $z=t^3$ from the point $(0,0,0)$ to the point $(2,1,3)$.	CO2	PO1	**7**

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

<table>
<tr><td></td><td></td><td></td><td>UNIT - III</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Form the partial differential equation by eliminating arbitrary constants from $z = a(x+y) + b(x-y) + abt + c$.</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Solve the partial differential equation $\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y} + a$ by direct integration method.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Derive the one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, where $u(x,t)$ denotes the temperature at a point x at time t.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Form the partial differential equation by eliminating arbitrary function from $z = f(x+y)g(x-y)$.</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Solve: $x^2(y^3 - z^3)p + y^2(z^3 - x^3)q = z^2(x^3 - y^3)$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Solve the partial differential equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$ by the method of separation of variables.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - IV</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Apply Newton-Raphson method to find the root of the equation $3\sin x - 2x + 5 = 0$ near $x = 3$ correct to four decimal places.</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Distance in nautical miles of the visible horizon for given heights in meters above the surface of the earth are given by the following table:
<table>
<tr><td>$x(heights)$</td><td>100</td><td>150</td><td>200</td><td>250</td><td>300</td><td>350</td><td>400</td></tr>
<tr><td>$y(distance)$</td><td>12</td><td>16</td><td>21</td><td>27</td><td>36</td><td>50</td><td>72</td></tr>
</table>
Find the value of y when $x = 225$ meters by using appropriate interpolation formula.</td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Simpson's rule to evaluate $\int_0^{0.6} e^{-x^2}\, dx$ by taking 10 ordinates.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - V</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Apply Taylor's series method to solve the initial value problem $\frac{dy}{dx} = x^2 + y$, $y(0) = 10$ at $x = 0.1$ by considering up to fourth degree.</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Runge-Kutta method of fourth-order to solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ at $x = 0.2$ by taking $h = 0.2$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>If $y' = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.04$ and $y(0.3) = 2.09$, find $y(0.4)$ by applying Milne's predictor-corrector method.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
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