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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2023 Semester End Main Examinations

Programme: B.E. | **Semester: III**
Branch: CH/BT | **Duration: 3 hrs.**
Course Code: 19MA3BSAPM | **Max Marks: 100**
Course: APPLIED MATHEMATICS | **Date: 10.04.2023**

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

1 a) Perform three iterations of Gauss-Seidel iteration method to approximate the solution of the system of equations $27x+6y-z=85;\ 6x+15y+2z=72$ and $x+y+54z=110$. **6**

b) Find $'b'$ if the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ is 3. **7**

c) Determine the eigenvalues and the corresponding eigenvectors of the matrix $A=\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ **7**

UNIT - II

2 a) Apply Newton- Raphson method to find a real root correct to three decimal places of the equation $x\sin x+\cos x=0$ near $x=\pi$. **6**

b) Estimate the number of students who have scored less than 85 marks using the following data. **7**

Marks	0-20	20-40	40-60	60-80	80-100
Number of Students	41	62	65	50	17

c) Evaluate $\int_0^6 \frac{\sin x}{x}dx$ using Trapezoidal rule with seven ordinates. **7**

OR

3 a) The observed values of a function are respectively 168, 120, 72 and 63 at the four positions 3, 7, 9 and 10 of the independent variable. What is the best estimate you can give for the value of the function at the position 8 of the independent variable? **6**

b) Compute the area bounded by the curve $y = f(x)$, x – axis and the extreme ordinates from the following table using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule. **7**

x	0	1	2	3	4	5	6
y	0	2	2.5	2.3	2	1.7	1.5

c) Apply Runge-Kutta method of fourth order to find an approximate value of y at $x = 0.1$, given that $\frac{dy}{dx} = x + y^2$ with $y(0) = 1$ and $h = 0.1$. **7**

UNIT - III

4 a) Obtain the Fourier series of the periodic function $f(x) = \sin(mx)$, where m is neither zero nor an integer over the interval $(-l, l)$. **6**

b) Obtain the Fourier series of the periodic function $f(x) = e^{-x}$ over the interval $(0, 2\pi)$. **7**

c) Find the Fourier transform of $f(x) = \begin{cases} 1-|x| & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$ and hence deduce that $\int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$. **7**

OR

5 a) Find the inverse Fourier cosine transform and the inverse Fourier sine transform of the function e^{-as}, $a > 0$. **6**

b) Obtain the Fourier series of the periodic function $f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$. **7**

c) Find the Fourier transform of $e^{-a^2 x^2}$, $a > 0$ and hence deduce that $e^{-x^2/2}$ is self-reciprocal with respect to the Fourier transform. **7**

UNIT - IV

6 a) Derive the Schmidt's explicit two-level formula for the solution of the one-dimensional heat equation $u_t = c^2 u_{xx}$ and hence obtain the Bendre- Schmidt formula. **6**

b) Solve the wave equation $u_{tt} = 4u_{xx}$ with $0 \le x \le 1$ subject to $u(0,t) = 0 = u(1,t)$; $u(x,0) = \begin{cases} \frac{5x}{3} & \text{for } 0 \le x \le \frac{3}{5} \\ \frac{5(1-x)}{2} & \text{for } \frac{3}{5} \le x \le 1 \end{cases}$ and $u_t(x,0) = 0$ by taking $h = \frac{1}{5}$ and $k = \frac{1}{10}$ up to two time-levels. **7**

c) Solve numerically the parabolic equation $u_t = u_{xx}$ subject to the conditions $u(0,t) = 0 = u(1,t)$ and $u(x,0) = \sin(\pi x)$ with $0 \le x \le 1$. Carry out computations for $0 \le t \le 0.1$ by taking $h = 0.2$ and $k = 0.02$. **7**

UNIT - V

7 a) Obtain the extremal of $I = \int_0^1 \left(x + y + (y')^2 \right) dx$ with $y(0) = 1$ and $y(1) = 2$. **6**

b) Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'} \right) = 0$. **7**

c) A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a Catenary. **7**

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