U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January / February 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: Chemical Engineering and Biotechnology **Duration: 3 hrs.**
Course Code: 19MA3BSAPM **Max Marks: 100**
Course: Applied Mathematics

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	*PO*	Marks
		UNIT - 1			
1	a)	Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$.	*1*	*1*	**6**
	b)	Apply Gauss-Seidel method to approximate the solution of the system of linear equations $20x + y - 2z = 17$, $3x + 20y - z = -18$ and $2x - 3y + 20z = 25$. Perform three iterations.	*1*	*1*	**7**
	c)	Test the consistency of the system of equations $x - 2y + 3z = 2$, $3x - y + 4z = 4$ and $2x + y - 2z = 5$ and hence solve if consistent.	*1*	*1*	**7**
		OR			
2	a)	Solve the system of linear equations $2x + y + 4z = 12$, $4x + 11y - z = 33$ and $8x - 3y + 2z = 20$ by Gauss elimination method.	*1*	*1*	**6**
	b)	Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.	*1*	*1*	**7**
	c)	Investigate the values of λ and μ such that the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ may have (i). a unique solution (ii). infinitely many solutions and (iii). No solution.	*1*	*1*	**7**
		UNIT - 2			
3	a)	Apply Newton-Raphson method to approximate root of the equation $3x = \cos x + 1$ near $x = 0$. Perform three iterations.	*1*	*1*	**6**
	b)	Apply Lagrange's interpolation formula to approximate $f(1.5)$ from the data given below. x: 0, 1, 2, 5 $f(x)$: 2, 3, 12, 147	*1*	*1*	**7**
	c)	Apply Simpson's 1/3rd rule to evaluate $\int_0^1 \frac{1}{1+x}\,dx$ by taking 7 ordinates. Hence deduce the value of $\log_e 2$.	*1*	*1*	**7**
		OR			

	4	a)	Apply appropriate interpolation formula to estimate the number of students who have obtained marks between 40 and 45 marks from the data given below. Marks: 30-40, 40-50, 50-60, 60-70, 70-80 No of students: 31, 42, 51, 35, 31	1	1	**6**
		b)	Apply the Runge-Kutta method of fourth order to compute $y(0.2)$ given $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$.	1	1	**7**
		c)	Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\, dx$ using the Trapezoidal rule by taking seven ordinates.	1	1	**7**
			UNIT - 3			
	5	a)	Obtain the Fourier series expansion of the periodic function $f(x) = 2x - x^2$ over the interval $(0, 2l)$.	2	1	**6**
		b)	The intensity of an alternating current after passing through a rectifier is given by $i(x) = \begin{Bmatrix} I_0 \sin x & \text{for} & 0 \le x \le \pi \\ 0 & \text{for} & \pi \le x \le 2\pi \end{Bmatrix}$ where I_0 is the maximum current and the period is 2π. Express $i(x)$ as a Fourier series.	2	1	**7**
		c)	Find the inverse Fourier transform of $F(s) = e^{-s}$.	2	1	**7**
			OR			
	6	a)	Obtain the Fourier series expansion of the periodic function $f(x)$ over the interval $(0, 2l)$. Where $f(x) = \begin{Bmatrix} l - x & \text{for} & 0 < x < l \\ 0 & \text{for} & l < x \le 2l \end{Bmatrix}$	2	1	**6**
		b)	Obtain the Fourier series expansion of the periodic function $f(x) = \begin{Bmatrix} -\pi & \text{for} & -\pi < x < 0 \\ x & \text{for} & 0 < x < \pi \end{Bmatrix}$.	2	1	**7**
		c)	Find the Fourier sine transform of $f(x) = e^{-\lvert x \rvert}$.	2	1	**7**
			UNIT - 4			
	7	a)	Derive the finite difference formula to solve the one-dimensional wave equation $u_{tt} = c^2 u_{xx}$.	1	1	**6**
		b)	Approximate the solution of the boundary value problem $u_t = u_{xx}$ under the conditions $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin \pi x$, $0 \le x \le 1$ using Schmidt method taking $h = 0.2$ and $\alpha = 1/2$.	1	1	**7**
		c)	Evaluate the pivotal values of the equation $u_{tt} = 16 u_{xx}$ taking $h = 1$ up to $t = 1.25$. The boundary conditions are $u(0,t) = 0$, $u(5,t) = 0$; $u_t(x,0) = 0$ and $u(x,0) = x^2(5 - x)$.	1	1	**7**
			OR			
	8	a)	Derive the expression for Crank-Nicolson method to solve the one-dimensional heat equation $u_t = c^2 u_{xx}$.	1	1	**6**
		b)	Solve the wave equation $u_{tt} = u_{xx}$, given that $u_t(x,0) = 0$, $u(0,t) = 0$, $u(1,t) = 0$ and $u(x,0) = \sin \pi x$ for $0 \le x \le 1$, by taking $h = 0.25$ and $k = 0.2$. Compute up to two-time levels.	1	1	**7**

		c)	Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 < x < 5, t \geq 0$ given that $u(x,0) = 20$, $u(0,t) = 0$ and $u(5,t) = 100$. Compute u for two time-levels with $h = 1$ and $\alpha = \frac{1}{2}$.	*1*	*1*	**7**
			UNIT - 5			
9		a)	Find the extremal of the functional $\int_{x_0}^{x_1}(1 + x^2 y')y' dx$.	*3*	*1*	**6**
		b)	Solve the variational problem $\delta \int_0^1 (x + y + y'^2)\, dx = 0$ under the conditions $y(0) = 1$ and $y(1) = 2$.	*3*	*1*	**7**
		c)	Find the path in which a particle, in the absence of friction will slide from one point to another in the shortest time under the action of gravity.	*3*	*1*	**7**
			OR			
10		a)	Derive Euler's equation $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.	*3*	*1*	**6**
		b)	A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a catenary.	*3*	*1*	**7**
		c)	Find the extremal of the functional $\int_{x_0}^{x_1}(y^2 + y'^2 - 2y\ \sin x)dx$. under the conditions $y(0) = y\left(\frac{\pi}{2}\right) = 0$.	*3*	*1*	**7**

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