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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2023 Supplementary Examinations

Programme: B.E. **Semester: III**
Branch: BT / CH **Duration: 3 hrs.**
Course Code: 19MA3BSAPM **Max Marks: 100**
Course: Applied Mathematics **Date: 15.09.2023**

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

1 a) Show that the equations $-2x+y+z=a$; $x-2y+z=b$ and $x+y-2z=c$ do not have a solution unless $a+b+c=0$. **6**

b) Perform three iterations of the Gauss-Seidal iteration method to approximate the solution of the system of equations $-x+6y+27z=85$; $2x+15y+6z=72$ and $54x+y+z=110$. **7**

c) Obtain the eigenvalues and the corresponding eigenvectors of the matrix **7**

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

UNIT - II

2 a) Obtain a real root correct to three decimal places of the equation $x\log_{10} x=1.2$ near $x=2.5$ using Newton- Raphson method. **6**

b) A survey conducted in a slum reveals the following information as classified below **7**

Income per day (Rs)	Under 100	100-200	200-300	300-400	400-500
Number of persons	20	45	115	210	115

Estimate the number of persons having income less than Rs. 450 per day using appropriate interpolation formula.

c) Evaluate $\int_0^1 \frac{dx}{1+x}$ by using Simpson's $\frac{1}{3}^{rd}$ rule for $n=6$. **7**

OR

3 a) Apply Lagrange's interpolation to find $f(1.6)$ from the given data **6**

x	1.2	2.0	2.5	3.0
$f(x)$	1.36	0.58	0.34	0.20

b) Apply Newton's forward formula to find the cubic polynomial satisfying **7**
$f(0)=0,\ f(2)=4,\ f(4)=56,\ f(6)=204,\ f(8)=496$ and $f(10)=980$.

c) Apply Runge-Kutta method of order four to find an approximate value of y at **7**
$x=0.1$, given that $y'=x+y^2$ with $y(0)=1$ and $h=0.1$.

UNIT - III

4 a) Find the Fourier series of the periodic function $f(x)=|x|$ over the interval **6**
$(-\pi,\pi)$.

b) Find the Fourier series of the periodic function $f(x)=\begin{cases}2-x & \text{in } 0\le x\le 4\\ x-6 & \text{in } 4\le x\le 8\end{cases}$. **7**

c) Find the Fourier cosine transform of the function e^{-x^2}. **7**

OR

5 a) Find the inverse Fourier sine transform of $\frac{e^{-as}}{s}$, $a>0$. **6**

b) Find the Fourier series of the periodic function $f(x)=x(2\pi-x)$ in $(0,2\pi)$. **7**

c) Find the Fourier transform of $e^{-a^2x^2}$, $a>0$ and hence deduce that $e^{-x^2/2}$ is self-reciprocal with respect to Fourier transform. **7**

UNIT - IV

6 a) Derive the explicit formula for the solution of one-dimensional wave equation **6**
$u_{tt}=c^2u_{xx}$.

b) Solve the parabolic equation $u_t=4u_{xx}$ subjected to $u(0,t)=0=u(8,t)$, $t\ge 0$ **7**
and $u(x,0)=4x-\frac{x^2}{2}$, up to two time-levels taking $h=1$ and $k=1/8$.

c) Solve numerically the equation $u_t=u_{xx}$ under the conditions **7**
$u(0,t)=0=u(1,t)$, $t\ge 0$ and $u(x,0)=\sin\pi x$, $0\le x\le 1$ using Schmidt method up to two time-levels taking $h=1/4$ and $k=1/96$.

UNIT - V

7 a) Find the extremal of the functional $\int_0^1 \left(x + y + (y')^2 \right) dx = 0$ under the conditions $y(0) = 1$ and $y(1) = 2$. **6**

b) Derive the Euler's equation of the variational problem in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'} \right) = 0$. **7**

c) A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a Catenary. **7**

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