U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January / February 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: Chemical Engineering **Duration: 3 hrs.**
Course Code: 21MA3BSAPM **Max Marks: 100**
Course: Applied Mathematics

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	*PO*	Marks
		UNIT - 1			
1	a)	Find the rank of the matrix $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ by reducing it to the echelon form.	*1*	*1*	**6**
	b)	Apply Gauss-seidel iterative method to approximate the solution of the system of equations $27x + 6y - z = 85$, $6x + 15y + 2z = 72$ and $x + y + 54z = 110$. Carry out three iterations, taking the initial approximation to the solution as $(2, 3, 2)$.	*1*	*1*	**7**
	c)	Test for consistency and solve the following system of equations: $x + y + z = -3$, $3x + y - 2z = -2$ and $2x + 4y + 7z = 7$.	*1*	*1*	**7**
		OR			
2	a)	Apply Gauss elimination method to solve the system of equations $2x + y + 4z = 12$, $4x + 11y - z = 33$ and $8x - 3y + +2z = 20$.	*1*	*1*	**6**
	b)	Find the values λ and μ for which the following system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ has (i) a unique solution, (ii) infinitely many solutions and (iii) no solution.	*1*	*1*	**7**
	c)	Find all the eigenvalues and the corresponding eigenvectors of the following matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$.	*1*	*1*	**7**
		UNIT - 2			
3	a)	Apply Newton-Raphson method to find the real root of the equation $3x = cos\ x\ + 1$, near to 0.5, correct to three decimal places.	*1*	*1*	**6**
	b)	Apply Newton's appropriate interpolation formula to evaluate y (8) from y (1) = 24, y (3) = 120, y (5) = 336, y (7) = 720.	*1*	*1*	**7**

<table>
<tr><td></td><td></td><td>c)</td><td>Evaluate $\int_{4}^{5.2} \log_e x \, dx$ by taking six equal strips and by using Simpson’s 1/3 rule.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>The following table gives the melting point of an alloy of lead and zinc, where t is the temperature in °C and p is the % of lead in the alloy:
<table>
<tr><td>P</td><td>60</td><td>70</td><td>80</td><td>90</td></tr>
<tr><td>t</td><td>226</td><td>250</td><td>276</td><td>304</td></tr>
</table>
Find the melting point of the alloy containing 84% of lead by applying Newton's interpolation formula.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Lagrange's Interpolation formula to find the value of y when $x = 5$, for the following data:
<table>
<tr><td>x</td><td>1</td><td>3</td><td>4</td><td>6</td></tr>
<tr><td>y</td><td>-3</td><td>9</td><td>30</td><td>132</td></tr>
</table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply fourth order Runge-Kutta method to solve the initial value problem $\frac{dy}{dx} = 3e^x + 2y, y(0) = 0$ at $x = 0.1$ by taking $h = 0.1$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Obtain the Fourier series of the periodic function $f(x) = x^2$ in $(-\pi, \pi)$.</td><td>2</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Obtain the Fourier series of the periodic function $f(x) = e^x$ in $(0, 2\pi)$.</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the Fourier sine transform of the function
$$f(x) = \begin{cases} 4x & 0 < x < 1 \\ 4 - x & 1 < x < 4 \\ 0 & x > 4 \end{cases}.$$</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Obtain the Fourier series of the periodic function $f(x) = |x|$ in $(-\pi, \pi)$.</td><td>2</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Obtain the Fourier series of the periodic function $f(x) = x + x^2$ in $(-l, l)$.</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the inverse Fourier sine transform of $\frac{e^{-as}}{s}, a > 0$.</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Derive Crank-Nicolson formula to solve numerically the one-dimensional heat equation $u_t = c^2 u_{xx}$.</td><td>1</td><td>1</td><td>6</td></tr>
</table>

		b)	Find the numerical solution of the parabolic equation $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ when $u(0,t) = 0, u(4,t) = 0$ and $u(x,0) = x(4-x)$ by taking $h = 1, k = 1$. Find the values up to $t = 3$.	1	1	**7**
		c)	Apply Schmidt method to solve the equation $u_t = u_{xx}$, subject to the conditions $u(0,t) = 0 = u(1,t), t \geq 0$ and $u(x,0) = \sin \pi x$, $0 \leq x \leq 1$. Carry out the computations for two -time levels taking $h = \frac{1}{3}, k = \frac{1}{36}$.	1	1	**7**
			OR			
8		a)	Derive the numerical solution of one-dimensional wave equation $u_{tt} = c^2 u_{xx}$.	1	1	**6**
		b)	Apply three-level formula to solve the wave equation $u_{tt} = 4u_{xx}$ subject to the boundary conditions $u(0,t) = 0 = u(4,t), t \geq 0$ and the initial conditions $u_t(x,0) = 0$ and $u(x,0) = x(4-x)$ by taking $h = 1,\ k = \frac{1}{2}$. Carry out the computations upto two-time levels.	1	1	**7**
		c)	Evaluate the pivotal values of the equation $u_{xx} = 0.0625 u_{tt}$ taking $h = 1$ and $k = \frac{1}{4}$ for $0 \leq t \leq 0.5$. The boundary conditions are $u(0,t) = 0 = u(5,t), t \geq 0$; $u_t(x,0) = 0$ and $u(x,0) = x^2(5-x)$.	1	1	**7**
			UNIT - 5			
9		a)	Derive the Euler's equation of the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.	3	1	**6**
		b)	Find the extremal of the functional $I = \int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x) dx$.	3	1	**7**
		c)	Show that the shape of the heavy cable which hangs freely under gravity between two fixed points is a Catenary.	3	1	**7**
			OR			
10		a)	Prove that the geodesics on a plane are straight lines.	3	1	**6**
		b)	Find the path in which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity.	3	1	**7**
		c)	Find the extremal of the functional $I = \int_0^{\pi/2} \left(y^2 - (y')^2 + 4y\cos x\right) dx$ under the end conditions $y(0) = 0$; $y\left(\frac{\pi}{2}\right) = 0$.	3	1	**7**

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