1) $F[f(x)] = F(s)$. $\rightarrow$ Fourier transform.
↳ operator ↳ function

$$F^{-1}[F(s)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds = f(x) \rightarrow \text{Inverse fourier transform.}$$

Fourier series decomposes a periodic function into a discrete set of contributions in terms of one fundamental frequency. Fourier transform provides a continuous frequency resolution of a possibly non-periodic function.

Q→ Find the fourier transform of $f(x) = \begin{cases} xe^{-x} & x>0 \\ 0 & x<0 \end{cases}$

$$F[f(x)] = \int_{-\infty}^{\infty} f(x)e^{isx}dx = \int_{-\infty}^{0} f(x)e^{isx}dx + \int_{0}^{\infty} f(x)e^{isx}dx$$

$$F[f(x)] = 0 + \int_0^{\infty} xe^{-x}e^{isx}dx = \int_0^{\infty} xe^{-(1-is)x}dx.$$

$$= \left[\frac{xe^{-(1-is)x}}{-(1-is)} - \frac{(1)\,e^{-(1-is)x}}{(1-is)^2}\right]_0^{\infty}$$

$$= \left[\frac{0-0}{-(1-is)} - \left(\frac{0-e^0}{(1-is)^2}\right)\right]$$

$$= \frac{1}{(1-is)^2}$$

Q→ Obtain fourier transform of
$$f(x) = \begin{cases} \sin x & 0<x<\pi \\ 0 & \text{otherwise} \end{cases}$$

$$F[f(x)] = \int_{-\infty}^{\infty} f(x)e^{isx}dx = \int_{-\infty}^{0} f(x)e^{isx}dx + \int_0^{\pi} f(x)e^{isx}dx$$
$$+ \int_{\pi}^{\infty} f(x)e^{isx}dx.$$

$$F[f(x)] = \int_0^{\pi} \sin x \, e^{isx} dx.$$

$$= \left(\frac{e^{isx}}{\cancel{a^2}(is)^2 + \cancel{1}^2} \left[\cancel{isx}\; is \sin x - \cos isx \right] \right)_0^{\pi}$$

$$= \left(\frac{e^{isx}}{1 - s^2} \left[is \sin x - \cos \cancel{s} x \right] \right)_0^{\pi}$$

$$= \left(\frac{e^{\pi is}}{1-s^2} \left(\overset{1}{\cancel{\cos x}} \right) - \frac{1}{1-s^2}(-1) \right)$$

$$= \cancel{\frac{e^{\pi is}}{1-}} \; \frac{1}{1-s^2} \left(\cancel{e^{\pi}} \right.$$

$$= \frac{1}{1-s^2} \left(e^{\pi is} + 1 \right)$$

$$= \frac{e^{\pi is} + 1}{1 - s^2}$$

Q→ $f(x) = \begin{cases} x & |x| \le a \\ 0 & |x| > a \end{cases}$ $\pm x \le a$ $\therefore -a \le x \le a$ $x \le a$, $-x \le a \Rightarrow -a \le x$, $x \ge -a$

$$F[f(x)] = \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$= \int_{-\infty}^{-a} f(x) e^{isx} dx + \int_{-a}^{a} f(x) e^{isx} dx + \int_{a}^{\infty} f(x) e^{isx} dx$$

$$F[f(x)] = \int_{-a}^{a} x e^{isx} dx$$

$$= \left(\frac{x e^{isx}}{is} - \frac{e^{isx}}{(is)^2} \right)_{-a}^{a}$$

$$= \left[\frac{x e^{isx}}{is} + \frac{e^{isx}}{s^2} \right]_{-a}^{a}$$

$$= \left[\frac{a e^{isa}}{is} + \frac{e^{isa}}{s^2} + \frac{a e^{-isa}}{is} - \frac{e^{-isa}}{s^2} \right]$$

$$= \frac{a}{is} \left(e^{ias} + e^{-ias} \right) + \frac{1}{s^2} \left(e^{ias} - e^{-ias} \right)$$

$$= \frac{a}{is} 2\cos(as) + \frac{1}{s^2} 2i\sin(as)$$

Q→ Obtain the fourier transform of $f(x) = \begin{cases} 1-|x| & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$

$F[f(x)]$ →

& hence deduce that ~~integral~~ $\int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$

$$F[f(x)] = \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$= \int_{-\infty}^{-1} \cancel{f(x) e^{isx}} dx + \int_{-1}^{1} f(x) e^{isx} dx + \int_{1}^{\infty} \cancel{f(x) e^{isx}} dx$$

$1-|x|$ is even in the interval, $-1 \leq x \leq 1$

$$f(x) = \begin{cases} 1-x & 0 < x < 1 \\ 1+x & -1 < x < 0. \end{cases}$$

$$= \int_{-1}^{1} f(x) e^{isx} dx = \int_{-1}^{1} f(x)(\cos(sx) + i\sin(sx)) dx$$

$$= 2 \int_0^1 f(x) \cos(sx) + 0$$

$$= 2\int_0^1 (1-x)\cos(sx)\,dx = 2\left[(1-x)\frac{\sin(sx)}{s} - (-1)\left\{-\frac{\cos(sx)}{s^2}\right\}\right]_0^1$$

$$F(s) = 2\left\{\frac{1-\cos(s)}{s^2}\right\}$$

Inverse fourier transform.

$$f(x) = F^{-1}[F(s)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(s)e^{-isx}\,ds.$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2\{1-\cos(s)\}}{s^2}e^{-isx}\,ds.$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\cancel{2\times} 2\sin^2\frac{s}{2}}{4\times\frac{s^2}{4}}e^{-isx}\,ds$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\sin^2\frac{s}{2}}{\frac{s^2}{4}}e^{-isx}\,ds.$$

when $x=0$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{4\sin^2\frac{s}{2}}{s^2}\,ds = f(0)$$

$$1 = \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{\sin^2 s/2}{s^2}\,ds.$$

$$\frac{\pi}{2} = \int_{-\infty}^{\infty} \frac{\sin^2 s/2}{s^2}\,ds.$$

$t = s/2 \qquad ds = 2dt$

$$\cancel{\frac{\pi}{2} = \int_{-\infty}^{\infty} \frac{\sin^2 s/2}{2t}}$$

$$\frac{\pi}{2} = \int_{-\infty}^{\infty} \frac{\sin^2 t}{(2t)^2} \times 2dt$$

$$\frac{\pi}{2} = \int_{-\infty}^{\infty} \frac{\sin^2 t \, dt}{2t^2} \rightarrow \text{Even fn.}$$

$$\frac{\pi}{2} = 2\int_{0}^{\infty} \frac{\sin^2 t}{2t^2} dt$$

$$\frac{\pi}{2} = \int_{0}^{\infty} \frac{\sin^2 t \, dt}{t^2}$$

✓ → Obtain $F[f(x)]$

$$f(x) = \begin{cases} a^2 - x^2 & |x| \leq a. \\ 0 & |x| > a \end{cases}$$

Hence evaluate (a) $\displaystyle\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cdot dx$

(b) $\displaystyle\int_0^{\infty} \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\left(\frac{x}{2}\right) dx =$

$$F[f(x)] = \int_{-\infty}^{-a} f(x) e^{isx} dx + \int_{-a}^{a} f(x) e^{isx} dx + \int_{a}^{\infty} f(x) e^{isx} dx$$

$$F[f(x)] = \int_{-a}^{a} f(x) e^{isx} dx \quad = \int_{-a}^{a} f(x)(\cos(sx) + i\sin(sx)) dx$$

$$= \cancel{\int_{-a}^{a} (a^2 - x^2) e^{isx} dx} = 2\int_0^a f(x)\cos sx + 0$$

$$= \cancel{2\int_0^a (a^2 - x^2) e^{isx} dx.} = 2\int_0^a (a^2 - x^2)\cos sx$$

$$= \cancel{2\int_0^a a^2 e^{isx}} = 2\left[(a^2 - x^2)\left(\frac{\sin sx}{s}\right) - (-2x)\left(\frac{-\cos sx}{s^2}\right)\right.$$

$$\left. + (-2)\left(\frac{-\sin sx}{s^3}\right)\right]_0^a$$

$$2\left[\cancel{(a^2-x^2)\left(\frac{\sin sx}{s}\right)} - 2x\left(\frac{\cos sx}{s^2}\right) + 2\left(\frac{\sin sx}{s^3}\right)\right]_0^a$$

$$2\left[-2a\left(\frac{\cos sa}{s^2}\right) + 2\left[\frac{\sin sa}{s^3}\right]\right]$$

$$= 4\left[\frac{\sin sa - sa\cos sa}{s^3}\right]$$

$$: 4\left[\frac{\sin sa - sa\cos sa}{s^3}\right)$$

I.F.T $\quad F^{-1}\{F(s)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(s)e^{-isx}\,ds = f(x) \qquad F(s) = \frac{odd}{odd} = even$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(s)\{\cos(sx) - i\sin(sx)\}\,ds$$

$$= \frac{1}{2\pi}\times 2\int_0^{\infty} F(s)\cos(sx)\,dx$$

$$f(x) : \frac{1}{\pi}\int_0^{\infty} -4\left\{\frac{as\cos(as) - \sin(as)}{s^3}\right\}\cos sx\,dx$$

$a : 1$ & $x = 0$

$$f(0) : -\frac{4}{\pi}\int_0^{\infty}\frac{s\cos s - \sin s}{s^3}\,ds$$

$$1 - 0 : -\frac{4}{\pi}\int_0^{\infty}\frac{s\cos s - \sin s}{s^3}\,ds$$

$$\cancel{-\frac{\pi}{4} : \int_0^{\infty} s\cos}$$

$$-\frac{\pi}{4} : \int_0^{\infty}\frac{s\cos s - \sin s}{s^3}\,ds$$

$s \to x$.

$$\Rightarrow \int_0^\infty \frac{x\cos x - \sin x}{x^3}\,dx = -\pi/4$$

$a = 1$ & $x = 1/2$

$$f(1/2) = -\frac{4}{\pi}\int_0^\infty \frac{s\cos(s) - \sin(s)}{s^3}\cos\left(\frac{s}{2}\right) ds$$

$$1 - \left(\frac{1}{2}\right)^2 = -\frac{4}{\pi}\int_0^\infty \frac{s\cos(s) - \sin(s)}{s^3}\cos\frac{s}{2}\, ds$$

$s \to x$

$$\frac{3}{4} = -\frac{4}{\pi}\int_0^\infty \frac{x\cos x - \sin x}{x^3}\,dx$$

$$\frac{-3\pi}{16} = \int_0^\infty \frac{x\cos x - \sin x}{x^3}\,dx$$

Q→ $F[e^{-a^2x^2}] = F[\exp(-a^2x^2)]$

$$= \int_{-\infty}^{\infty} e^{-a^2x^2} e^{isx}\,dx$$

$$= \int_{-\infty}^{\infty} e^{-(a^2x^2 - isx)}\,dx \qquad [\Gamma$$

PROPERTIES OF FOURIER TRANSFORM

(i) LINEAR PROPERTY.

$$F[af(x) + bg(x)] = aF[f(x)] + bF[g(x)]$$

$$\Rightarrow F[af(x) + bg(x)] = \int_{-\infty}^{\infty} \{af(x) + bg(x)\} e^{isx}\,dx$$

$$= a\int_{-\infty}^{\infty} f(x)e^{isx} + b\int_{-\infty}^{\infty} g(x)e^{isx}\,dx$$

$$= aF[f(x)] + bF[g(x)]$$

(ii) DAMPING AND SCALING PROPERTY

$$F[f(ax)] = \frac{1}{a}F\left(\frac{s}{a}\right)$$

(iii) SHIFTING PROPERTY.

$$F[f(x-a)] = e^{ias}F(s)$$

$$\Rightarrow \quad F[f(x-a)] = \int_{-\infty}^{\infty} f(x-a)e^{isx}\,dx$$

$x-a = t \Rightarrow dx = dt$

$x \to -\infty \Rightarrow t \to -\infty$

$x \to \infty \Rightarrow t \to -\infty$

$$F[f(x-a)] = \int_{-\infty}^{\infty} f(t)e^{is(t+a)}\,dt = e^{ias}\int_{-\infty} f(t)e^{ist}\,dt = e^{ias}F[f(x)]$$

(b) $F[e^{iax}f(x)] = F(s+a)$

$$\Rightarrow F[e^{iax}f(x)] = \int_{-\infty}^{\infty} e^{iax}f(x)e^{isx}\,dx = \int_{-\infty}^{\infty} f(x)e^{i(s+a)x}\,dx = F(s+a)$$

(iv) MODULATION PROPERTY

$$F[f(x)\cos(ax)] = F\left[f(x)\left\{\frac{e^{iax}+e^{-iax}}{2}\right\}\right] = \frac{1}{2}F[e^{iax}f(x)] + \frac{1}{2}F[e^{-iax}f(x)]$$

$$= \frac{1}{2}F(s+a) + \frac{1}{2}F(s-a) = \frac{F(s+a)+F(s-a)}{2}$$

Q→ Find the fourier transform of $f(x) = \begin{cases} 1 & |x| \le a \\ 0 & |x| > a \end{cases}$

and hence evaluate (a) $\int_0^{\infty} \frac{\sin(ax)\cos\lambda x}{x}\,dx$ (b) $\int_0^{\infty} \frac{\sin(ax)}{x}\,dx$

(c) $F\left[f(x)\left\{1+\cos\left(\frac{\pi x}{a}\right)\right\}\right]$

$$F[f(x)] = \int_{-\infty}^{\infty} f(x)e^{isx}\,dx = \int_{-\infty}^{-a} 0\,dx + \int_{-a}^{a} 1e^{isx}\,dx + \int_{a}^{\infty} 0\,dx = \left.\frac{e^{isx}}{is}\right|_{-a}^{a}$$

$$= \frac{e^{isa}-e^{is(-a)}}{is} = \frac{e^{ias}-e^{-ias}}{is} = \frac{2i\sin(as)}{is} = \frac{2\sin(as)}{s}$$

IFT:

$$F^{-1}[F(s)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(s)e^{isx}\,ds = f(x)$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\sin(as)}{s}\{\cos(sx) - i\sin(sx)\}\,ds$$

$$= \frac{2}{\pi}\int_0^{\infty} \frac{\sin(as)\cos(sx)}{s}\,ds.$$

$x = \lambda$

$$\int_0^{\infty} \frac{\sin(as)\cos(\lambda s)}{s}\, ds = \frac{\pi}{2} f(\lambda) \quad \therefore = \begin{cases} \pi/2 & \text{if } |\lambda| \le a \\ 0 & \text{if } |\lambda| > a \end{cases}$$

$$\therefore \int_0^{\infty} \frac{\cos(ax)\cos(\lambda x)}{x}\, dx \quad = \begin{cases} \pi/2 & \text{if } |\lambda| \le a \\ 0 & \text{if } |\lambda| > a \end{cases}$$

$$\lambda = 0 \Rightarrow \int_0^{\infty} \frac{\sin ax}{x}\, dx = \pi/2$$

$$F\left[f(x)\left\{1 + \cos\left(\frac{\pi x}{a}\right)\right\}\right] = F[f(x)] + F\left[f(x)\cos\left(\frac{\pi x}{a}\right)\right]$$

$$= F(s) + \frac{1}{2}\left[F\left(s + \frac{\pi}{a}\right) + F\left(s - \frac{\pi}{a}\right)\right] = \frac{2\sin(as)}{s} + \frac{\sin\{a(s + \pi/a)\}}{s + \pi/a}$$

$$+ \frac{\sin\{a(s - \pi/a)\}}{s - \pi/a}$$

$$= \frac{2\sin(as)}{s} + a\left\{\frac{\sin(\pi + as)}{as + \pi} - \frac{\sin(\pi - as)}{as - \pi}\right\}$$

→ Find the transform of $e^{-|x|}$

$$F[f(x)] = \int_{-\infty}^{\infty} f(x) e^{isx}\, dx \qquad f(x) = \begin{cases} e^{x} & x \ge 0 \\ e^{-x} & x < 0 \end{cases}$$

$$F[f(x)] = \int_{-\infty}^{0} e^{-(-x)} e^{isx}\, dx + \int_0^{\infty} e^{-x} e^{isx}\, dx$$

$$= \int_{-\infty}^{0} e^{(is+1)x}\, dx + \int_0^{\infty} e^{-(1-is)x}\, dx$$

$$= \left.\frac{e^{(1+is)x}}{1+is}\right]_{-\infty}^{0} + \left.\frac{e^{-(1-is)x}}{-(1-is)}\right|_0^{\infty}$$

$$= \frac{1}{1+is} + \frac{1}{1-is} = \frac{2}{1+s^2}$$

Q → $F[e^{-a^2x^2}] = F[\exp(-a^2x^2)]$ where $a>0$ & hence ST $e^{-x^2/2}$ is self reciprocal wrt fourier transform.

$$= \int_{-\infty}^{\infty} e^{-a^2x^2} e^{isx}\, dx$$

$$= \int_{-\infty}^{\infty} e^{-(a^2x^2 - isx)}\, dx \qquad \left[\Gamma\left(\tfrac{1}{2}\right) = 2\int_0^{\infty} e^{-t^2} dt = \sqrt{\pi}\right.$$

$$a^2x^2 - isx = (ax)^2 - isx = \underset{A}{(ax)^2} - 2\underset{A}{ax}\underset{B}{\left(\frac{is}{2a}\right)} + B^2 - B^2$$

$$a^2x^2 - isx = (ax)^2 - 2(ax)\left(\frac{is}{2a}\right) + \left(\frac{is}{2a}\right)^2 - \left(\frac{is}{2a}\right)^2$$

$$= \left(ax - \frac{is}{2a}\right)^2 + \frac{s^2}{4a^2}$$

$$F[e^{-a^2x^2}] = \int_{-\infty}^{\infty} \exp\left[-\left\{\left(ax - \frac{is}{2a}\right)^2 + \frac{s^2}{4a^2}\right\}\right] dx$$

$$= \int_{-\infty}^{\infty} \exp\left\{-\left(ax - \frac{is}{2a}\right)^2\right\} \exp\left(\frac{-s^2}{4a^2}\right) dx$$

$$F[e^{-a^2x^2}] = \exp\left(\frac{-s^2}{4a^2}\right)\int_{-\infty}^{\infty} \exp\left\{-\left(ax - \frac{is}{2a}\right)^2\right\} dx$$

$$ax - \frac{is}{2a} = t \Rightarrow a\,dx = dt$$

$$x \to -\infty \Rightarrow t \to -\infty$$

$$x \to \infty \Rightarrow t \to \infty$$

$$F[e^{-a^2x^2}] = \exp\left(\frac{-s^2}{4a^2}\right)\int_{-\infty}^{\infty} e^{-t^2}\frac{dt}{a}.$$

$$= \frac{1}{a} e^{-s^2/4a^2}\int_{-\infty}^{\infty} e^{-t^2} dt = \frac{1}{a} e^{-s^2/4a^2}\left[2\int_0^{\infty} e^{-t^2} dt\right)$$

$\cancel{\frac{1}{a}} \int$

$$= \frac{1}{a} e^{-(s^2/4a^2)} \Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{a} e^{-(s^2/4a^2)}$$

(a) $F[e^{-x^2/2}] = ?$

$e^{-(x^2/2)} = e^{-(a^2x^2)} \Rightarrow a^2 = 1/2$

let $a = \frac{1}{\sqrt{2}}$ then

$$F[e^{-x^2/2}] = \frac{\sqrt{\pi}}{1/\sqrt{2}} e^{-s^2/2}$$

$$F[e^{-(x^2/2)}] = k e^{-(s^2/2)}$$

$$T[f(x)] = k f(s).$$

such fns are said to be self-reciprocal wrt transformation T.

$\therefore e^{-x^2/2}$ is a self reciprocal wrt the fourier transform.

) Hence find FT of

(i) $e^{-2(x-3)^2}$ (ii) $e^{-x^2} \cos 3x$.

(i) $f(x) = e^{-2x^2} \Rightarrow f(x-3) = e^{-2(x-3)^2}$

$F[f(x-3)] = e^{i3s} F(s)$ $[\because F[f(x-a)] = e^{ias} F(s)]$

$F(s) = F[f(x)] = F[e^{-2x^2}] = \frac{\sqrt{\pi}}{\sqrt{2}} e^{-s^2/8}$ $\left[\because a^2 = 2 \text{ \& } a = \sqrt{2} \ (a>0)\right.$

Q→ Find the inverse fourier transform of $e^{-a|s|}$

I.F.T

$$F^{-1}[F(s)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)\, e^{-isx}\, ds = f(x)$$

$$f(x) = \frac{1}{2\pi}\left[\int_{-\infty}^{0} e^{-a(-s)} e^{-isx}\, ds + \int_{0}^{\infty} e^{-as} e^{-isx}\, ds\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\infty}^{0} e^{(a-ix)s}\, ds + \int_{0}^{\infty} e^{-(a+ix)s}\, ds\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{(a-ix)s}}{(a-ix)}\Bigg|_{-\infty}^{0} - \frac{e^{-(a+ix)s}}{a+ix}\Bigg|_{0}^{\infty}\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{a+ix} - 0 - \left[0 - \frac{1}{a-ix}\right]\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{a-ix} + \frac{1}{a+ix}\right]$$

$$= \frac{1}{2\pi}\left[\frac{2a}{a^2+x^2}\right]$$

$$f(x) = \frac{a}{\pi(a^2+x^2)}$$

Q→ Find IFT of e^{-s^2}.

$$F^{-1}[F(s)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)\, e^{-isx}\, ds = f(x)$$

~~$f(x) = \frac{1}{2\pi}\int$~~ $= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-s^2} e^{-isx}\, dx$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-(s^2 + isx)} ds = f(x)$$

wkt $2\int_0^{\infty} e^{-t^2} dt = \Gamma(1/2) = \sqrt{\pi}$

$$s^2 + isx = s^2 + s(ix) = s^2 + 2(s)\left(\frac{ix}{2}\right)$$

$$= s^2 + 2(s)\left(\frac{ix}{2}\right) + \left(\frac{ix}{2}\right)^2 - \left(\frac{ix}{2}\right)^2$$

$$= \left(s^2 + \frac{ix}{2}\right) + \frac{x^2}{4}$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\left\{\left(s + \frac{ix}{2}\right)^2 + \frac{x^2}{4}\right\}} ds$$

$$\cancel{= \frac{1}{2\pi} e^{-x^2/4} \int_{-\infty}^{\infty} e^{-s + ix}}$$

$$= \frac{1}{2\pi} e^{-x^2/4} \int_{-\infty}^{\infty} e^{-\left(s + \frac{ix}{2}\right)^2} ds.$$

$s + \frac{ix}{2} = t \Rightarrow ds = dt$

$s \to \pm\infty \Rightarrow t \to \pm\infty.$

$$f(x) = \frac{1}{2\pi} e^{-(x^2/4)} \int_{-\infty}^{\infty} e^{-t^2} dt.$$

$e^{-t^2} \to$ is an even fn

$$= \frac{1}{2\pi} e^{-(x^2/4)} \left[2\int_0^{\infty} e^{-t^2} dt\right]$$

$$= \frac{1}{2\pi} e^{-(x^2/4)} \Gamma(1/2)$$

$$f(x) = \frac{1}{2\pi} e^{-(x^2/4)} \sqrt{\pi}$$

Infinite fourier cosine transform

$$F_c[f(x)] = \int_0^\infty f(x)\cos(sx)\,dx$$

Infinite fourier sine transform

$$F_s[f(x)] = \int_0^\infty f(x)\sin(sx)\,dx$$

$$F_c^{-1}[F_c(s)] = \frac{2}{\pi}\int_0^\infty F_c(s)\cos(sx)\,ds$$

$$F_s^{-1}[F_s(s)] = \frac{2}{\pi}\int_0^\infty F_s(s)\sin x(sx)\,ds.$$

Q→ Fourier cosine transform of $f(x) = \begin{cases} \cos(x) & 0 < x < a \\ 0 & x \geq a \end{cases}$

or infinite fourier cosine

$$F_c[f(x)] = \int_0^\infty f(x)\cos(sx)\,dx$$

$$= \int_0^a f(x)\cos(sx)\,dx + \int_a^\infty f(x)\cos(sx)\,dx$$

$$= \int_0^a \cos x\cos(sx)\,dx$$

$$= \frac{1}{2}\int_0^a \cos((s+1)x) + \cos((s-1)x)\,dx.$$

$$= \frac{1}{2}\left[\frac{\sin((s+1)x)}{s+1} + \frac{\sin((s-1)x)}{s-1}\right]_0^a$$

$$= \frac{1}{2}\left[\frac{\sin((s+1)a)}{s+1} + \frac{\sin((s-1)a)}{s-1}\right]$$

obtain fourier cosine transform of $f(x) = \begin{cases} x & \\ 2-x & 1<x<2 \\ 0 & \text{otherwise} \end{cases}$

or infinite fourier cosine.

$$F_c[f(x)] = \int_0^\infty f(x)\cos(sx)\,dx$$

$$= \int_0^1 x\cos(sx)\,dx + \int_1^2 (2-x)\cos(sx)\,dx$$

$$= x\frac{\sin(sx)}{s} + \frac{\cos(sx)}{s^2}\Big|_0^1 + (2-x)\frac{\sin(sx)}{s} - (-1)\left(-\frac{\cos(sx)}{s^2}\right)\Big|_1^2$$

$$= \frac{\sin(s)}{s} + \frac{\cos(s)}{s^2} - \frac{1}{s^2} + \left[-\frac{\sin s}{s} - \left(\frac{\cos(2s)}{s^2} - \frac{\cos s}{s^2}\right)\right]$$

$$= \cancel{\frac{\sin s}{s}} + \frac{\cos s}{s^2} - \frac{1}{s^2} - \cancel{\frac{\sin s}{s}} - \frac{\cos(2s)}{s^2} + \frac{\cos s}{s^2}$$

$$= \frac{2\cos(s)}{s^2} - \frac{\cos(2s)}{s^2} + \cancel{\frac{\cos s}{s}}$$

$$= \frac{2\cos(s)}{s^2} - \frac{\cos(2s)}{s^2} - \frac{1}{s^2}$$

$$= \frac{1}{s^2}\left(2\cos(s) - \cos(2s) - 1\right)$$

$$= \frac{2\cos(s)}{s^2} - \frac{2\cos^2(s)}{s^2}$$

$$= \frac{2\cos(s)}{s^2}\left(1 - \cos(s)\right) = \cancel{\frac{4\cos(s)}{s^2} \times \cos^2\left(\frac{s}{2}\right)}$$

$$= \frac{4\cos(s)}{s^2} \times \sin^2\left(\frac{s}{2}\right)$$

Q→ find fourier sine transform of $f(x) = \begin{cases} 4x & 0 < x < 1 \\ 4-x & 1 < x < 4 \\ 0 & x > 4 \end{cases}$

Sol $F_S[f(x)] = \int_0^\infty f(x) \sin(sx)\, dx$

$= \int_0^1 4x \sin(sx)\, dx + \int_1^4 (4-x) \sin(sx)\, dx$

$= \cancel{4x(-\sin}$

$= \left[4x\left(\frac{-\cos(sx)}{s}\right) - 4\left(\frac{-\sin(sx)}{s^2}\right)\right]_0^1$

$+ \left[4-x\left(\frac{-\cos(sx)}{s}\right) - (-1)\left(\frac{-\sin(sx)}{s^2}\right)\right]_1^4$

$= 4\left(\frac{-1}{s}\right) + \cancel{4x}$

$= -\frac{4\cos(s)}{s} + \frac{4\sin(s)}{s^2} + \left[\frac{3\cos(s)}{s} - \left[\frac{\sin 4s}{s^2} - \frac{\sin s}{s^2}\right]\right]$

$= -\frac{4\cos(s)}{s} + \frac{4\sin(s)}{s^2} + \frac{3\cos(s)}{s} - \frac{\sin 4s}{s^2} + \frac{\sin s}{s^2}$

$= \frac{5\sin(s)}{s^2} - \frac{\cos(s)}{s} - \frac{\sin 4s}{s^2}$

If $f(x) = e^{-ax}$, $a>0$, find the fourier cosine transform & hence deduce ..

(a) Fourier cosine transform of xe^{-ax}

(b) " sine " " "

(c) the value of $\int_0^\infty \frac{\cos(\alpha x)}{a^2+\alpha^2} d\alpha$

$$F_c[f(x)] = \int_0^\infty f(x)\cos(sx)\,dx$$

$$= \int_0^a f(x)\cos(sx)\,dx + \int_0^\infty f(x)\cos(sx)\,dx$$

$$= \int_0^\infty e^{-ax}\cos(sx)\,dx$$

$$= \frac{e^{-ax}}{a^2+s^2}\left[-a\cos(sx) + s\sin(sx)\right]\Big|_0^\infty$$

$$= -\frac{1}{a^2+s^2}\left[-a\right]$$

$$\int_0^\infty e^{-ax}\cos(sx)\,dx = \frac{a}{a^2+s^2} = F(a,s)$$

$$* \quad \frac{\partial F}{\partial a} = \int_0^\infty \frac{\partial}{\partial a}\left\{e^{-ax}\cos(sx)\right\}dx$$

$$\frac{(s^2+a^2) - a(2a)}{(s^2+a^2)^2} = \int_0^\infty (-x)e^{-ax}\cos(sx)\,dx$$

$$\frac{s^2-a^2}{(s^2+a^2)^2} = -\int_0^\infty (xe^{-ax})\cos(sx)\,dx$$

$$F_c[xe^{-ax}] = \frac{a^2-s^2}{(s^2+a^2)^2}$$

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(b) $$\frac{\partial F_c}{\partial s} = \int_0^\infty \frac{\partial}{\partial s}\{e^{-ax}\cancel{\cos}(sx)\}dx$$

$$\frac{-a(2s)}{(s^2+a^2)^2} = \int_0^\infty e^{-ax}(\sin sx)\times -x\,dx$$

$$\frac{\cancel{2a} +2as}{(s^2+a^2)^2} = \int_0^\infty x e^{-ax}\sin(sx)\,dx$$

$$F_s[xe^{-ax}] = \frac{+2as}{(s^2+a^2)^2}$$

(c) Inverse fourier cosine transform is:

$$f(x) = F_c^{-1}[F_c(s)] = \frac{2}{\pi}\int_0^\infty F_c(s)\cos(sx)dx$$

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{a}{s^2+a^2}\cos(sx)dx$$

put $x=\lambda$ $$\int_0^\infty \frac{\cos(s\lambda)}{s^2+a^2}ds = \frac{\pi}{2a}f(\lambda) = \frac{\pi}{2a}e^{-a\lambda}$$

$$\cancel{\int_0^\infty \frac{\cos(sx)}{s^2+a^2}} \quad \int_0^\infty \frac{\cos(\lambda x)}{x^2+a^2}dx = \frac{\pi}{2a}e^{-\lambda a}$$

Q→ Obtain infinite fourier series transform of $e^{-|x|}$ and hence show that $\int_0^\infty \frac{x\sin(mx)}{1+x^2}dx = \frac{\pi}{2}e^{-m}$, $m>0$.

Ans. $$F_s[f(x)] = \int_0^\infty f(x)\sin(sx)dx.$$

$$f(x) = \begin{cases} e^{-x} & x>0 \\ e^{-(-x)} & x<0 \end{cases}$$

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$$F_c[f(x)] = \int_0^\infty e^{-x} \sin(sx)\,dx$$

$$= \left[\frac{e^{-x}}{1+s^2}\{(-1)\sin(sx) - s\cos(sx)\}\right]_0^\infty$$

$$= -\left[\frac{1}{1+s^2}[-s\cos]\right] = \frac{s}{1+s^2}$$

Inverse sine transfom $= F_s^{-1}[F_s(s)] = \frac{2}{\pi}\int_0^\infty F_s(s)\sin(sx)\,ds = f(x)$, $x>0$

$$= \frac{2}{\pi}\int_0^\infty F_s(s)\sin(sx)\,ds = f(x), \quad x>0$$

$$\int_0^\infty \frac{s}{1+s^2}\sin(sx)\,ds = \frac{\pi}{2}e^{-x}$$

$x = m$, $m>0$ $\quad \int_0^\infty \frac{s}{1+s^2}\sin(ms)\,ds = \frac{\pi}{2}e^{-m}$

$s \to x \Rightarrow \int_0^\infty \frac{x\sin(mx)}{1+x^2}\,dx = \frac{\pi}{2}e^{-m}$, $m>0$.

Q→ Obtain fourier sine transform of e^{-ax} $(a>0)$ & hence find the fourier ~~to~~ sine transform of $\frac{x}{a^2+x^2}$.

Ans $F_s[f(x)] = \int_0^\infty f(x)\sin(sx)\,dx$

$$= \int_0^\infty e^{-ax}\sin(sx)\,dx = \frac{e^{-ax}}{a^2+s^2}\{(-a)\sin(sx) - s\cos(sx)\}$$

$$= \frac{s}{a^2+s^2}.$$

Inverse fourier transform;

$$F_s^{-1}[F_s(s)] = \frac{2}{\pi}\int_0^\infty F_s(s)\sin(sx)\,ds = f(x)$$

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$$\frac{2}{\pi}\int_0^\infty \frac{s}{s^2+a^2}\sin(sx)\,ds = e^{-ax}$$

$$\int_0^\infty \frac{s}{s^2+a^2}\sin(sx)\,ds = \frac{\pi}{2}e^{-ax}$$

$$s \leftrightarrow x$$

$$\int_0^\infty \frac{x}{x^2+a^2}\sin(sx)\,dx = \frac{\pi}{2}e^{-as}.$$

Q→ ~~Q~~ obtain the fourier cosine transform of $\frac{1}{1+x^2}$ and hence deduce the fourier sine transform of $\frac{x}{1+x^2}$

$$F_c[f(x)] = \int_0^\infty \frac{1}{1+x^2}\cos(sx)\,dx = F_c(s) = g(s) \quad \text{(1)}$$

$$= \cancel{\left[\frac{1}{1+x^2}\left(\frac{\sin(sx)}{s}\right) - \tan^{-1}(x)\left(\frac{\cos(sx)}{s^2}\right)\right]_0^\infty}$$

$\tan^{-1}(\infty)$

$$g(s) = \int_0^\infty \frac{1}{1+x^2}\cos(sx)\,dx$$

$$\frac{dg(s)}{ds} = \frac{\partial}{\partial s}\int_0^\infty \frac{1}{1+x^2}\cos(sx)\,dx = \int_0^\infty \frac{1}{1+x^2}\frac{\partial(\cos(sx))}{\partial s}\,dx$$

$$\frac{dg}{ds} = \int_0^\infty \frac{1}{1+x^2}(-x\sin(sx))\,dx. \rightarrow \text{(1a)}$$

$$\frac{d^2g}{ds^2} = \int_0^\infty \frac{1}{1+x^2}\{-x^2\cos(sx)\}\,dx$$

$$\frac{d^2g}{ds^2} = \int_0^\infty \frac{1}{1+x^2}\{-x^2\cos(sx)\}\,dx.$$

$$\frac{dg}{ds} = -\int_0^\infty \frac{x}{1+x^2} \sin(sx)\, dx$$

Xly & ÷de by x.

$$\frac{dg}{ds} = -\int_0^\infty \frac{x^2}{x(1+x^2)} \sin(sx)\, dx$$

$$\frac{dg}{ds} = -\int_0^\infty \frac{x^2+1-1}{x(1+x^2)} \sin(sx)\, dx.$$

$$= -\left[\int_0^\infty \frac{1}{x} \sin(sx) - \int_0^\infty \frac{1}{x(1+x^2)} \sin(sx)\right]$$

$$\int_0^\infty \frac{1}{x(1+x^2)} \sin(sx)\, dx - \int_0^\infty \frac{1}{x} \sin(sx)\, dx.$$

$$\frac{dg}{ds} = -\frac{\pi}{2} + \int_0^\infty \frac{1}{x(x^2+1)} \sin(sx)\, dx \quad \rightarrow (2) \qquad \left[\because \int_0^\infty \frac{\sin(ax)}{x}\, dx = \begin{cases} \pi/2 & a>0 \\ -\pi/2 & a<0 \end{cases}\right.$$

$$I = \int_0^\infty \frac{1}{x(x^2+1)} \sin(sx)\, dx$$

$$\frac{d^2g}{ds^2} = \int_0^\infty \frac{\partial}{\partial s}\left\{\frac{1}{x(x^2+1)} \sin(sx)\right\} dx$$

$$= \int_0^\infty \frac{1}{x^2+1} \cos(sx)\, dx = g(s)$$

$$\frac{d^2g}{ds^2} = g(s). \qquad = \frac{d^2g}{ds^2} - g(s) = 0$$

~~$D^2(g(s)$~~ $D^2 g(s) - g(s) = 0.$

AE: $\quad m^2 - 1 = 0$

$\quad m = \pm 1$

$CF = C_1 e^{m_1 s} + C_2 e^{m_2 s}$

$g(s) = C_1 e^{s} + C_2 e^{-s}$. — ③

Conditions to solve C_1 and C_2.

$g(0) = \frac{\pi}{2} = \int_0^\infty \frac{1}{1+x^2}\,dx = \tan^{-1}x \Big|_0^\infty$

$g(0) = \pi/2$.

$S = 0$ in $\frac{dg}{ds} = -\frac{\pi}{2} + \int_0^\infty 0\,dx = -\pi/2$.

$S = 0$ in $g(s)$ $g(0) = C_1 + C_2 = \pi/2$. — ④

$\frac{d(3)}{ds} = \frac{dg}{ds} = C_1 e^{s} - C_2 e^{-s}$

$S = 0$ $\frac{dg}{ds}\Big|_{s=0} = C_1 - C_2$ — ⑤

$= -\pi/2$.

Solving ④ & ⑤

$C_1 = 0$.

$C_2 = \pi/2$

$g(s) = F_c(s) = F_c\left[\frac{1}{1+x^2}\right] = \frac{\pi}{2} e^{-s}$

From 1 a

$\int_0^\infty \frac{x}{1+x^2} \sin(sx)\,dx = -\frac{dg}{ds}$

$$F_s\left[\frac{x}{1+x^2}\right] = -\frac{dg}{ds} = -\frac{\pi}{2}(-e^{-s}) = \frac{\pi}{2}e^{-s}$$

$F_c[e^{-x^2}] = ?$ & hence find $F_s[xe^{-x^2}]$.

Ans: $F_c[e^{-x^2}] = \int_0^\infty e^{-x^2}\cos(sx)\,dx = g(s)$

$$g(s) = \int_0^\infty e^{-x^2}\cos(sx)\,dx \quad \text{—(1)}$$

$$\frac{dg}{ds} = \int_0^\infty e^{-x^2}(-x\sin(sx))\,dx.$$

$$\frac{dg}{ds} = \int_0^\infty (-xe^{-x^2})\sin(sx)\,dx.$$

$$\frac{dg}{ds} = \frac{1}{2}\int_0^\infty \underset{u}{\sin(sx)}\,\underset{v}{\{-2xe^{-x^2}\}}\,dx$$

$$\int -2xe^{-x^2}dx = e^{-x^2}$$

$$\frac{dg}{ds} = \frac{1}{2}\left\{\cancel{\sin(sx)e^{-x^2}\Big|_0^\infty} - \int_0^\infty s\cos(sx)e^{-x^2}dx.\right.$$

$$\frac{dg}{ds} = -\frac{s}{2}\int_0^\infty \cos(sx)e^{-x^2}dx$$

$$\frac{dg}{ds} = -\frac{s}{2}g(s)$$

$$\cancel{\left(D+\frac{s}{2}\right)g(s)=0}$$

~~Ans~~ A.E: $\cancel{m+s/2=0}$

$\cancel{m=-s/2}$

$$\frac{dg}{g(s)} = \frac{-s}{2} ds$$

$$\log g(s) = \cancel{\frac{2s}{}} \quad \frac{-s^2}{4} + \log c$$

$$\log g(s) = \log e^{-s^2/4} + \log c$$

$$g(s) = c e^{-s^2/4} \quad \text{--- (2)}$$

$s=0$ in (1) $\quad g(0) = \int_0^\infty e^{-x^2} dx \qquad \Gamma\left(\frac{1}{2}\right) = 2\int_0^\infty e^{-x^2} dx$

$$g(0) = \frac{\sqrt{\pi}}{2}$$

$s=0$ in (2) $\quad g(0) = c$

$$\therefore c = \frac{\sqrt{\pi}}{2}$$

$$g(s) = \frac{\sqrt{\pi}}{2} e^{-s^2/4}$$

$$g(s) = F[e^{-x^2}] = \frac{\sqrt{\pi}}{2} e^{-s^2/4}$$

Q→ Obtain the fourier sine transform of $\frac{x}{1+x^2}$.

$F_s\left[\frac{x}{1+x^2}\right] = F$

~~half~~ $\quad F_s[f(x)] = \int_0^\infty f(x)\sin(sx)dx = F_s(s)$

$$F_s(s) = \int_0^\infty \frac{x}{1+x^2}\sin(sx)dx \quad \text{--- (1)}$$

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$$= \int_0^\infty \frac{x^2}{x(1+x^2)} \sin(sx)\,dx = \int_0^\infty \frac{(x^2+1-1)}{x(1+x^2)} \sin(sx)\,dx$$

$$= \int_0^\infty \frac{\sin(sx)}{x}\,dx - \int_0^\infty \frac{1}{x(1+x^2)} \sin(sx)\,dx$$

$$= \frac{\pi}{2} - \int_0^\infty \frac{1}{x(1+x^2)} \sin(sx)\,dx \quad \text{--- (2)}$$

$$\frac{dF_s}{ds} = 0 - \int_0^\infty \frac{\partial}{\partial s} \left\{ \frac{1}{x(1+x^2)} \sin(sx) \right\} dx = -\int_0^\infty \frac{1}{x(1+x^2)} x\cos(sx)\,dx$$

$$\frac{dF_s}{ds} = -\int_0^\infty \frac{1}{1+x^2} \cos(sx)\,dx \quad \text{--- (3)}$$

$$\frac{d^2F_s}{ds^2} = -\int_0^\infty \frac{\partial}{\partial s} \left\{ \frac{1}{1+x^2} \cos(sx) \right\} dx.$$

$$\frac{d^2F_s}{ds^2} = -\int_0^\infty \frac{1}{1+x^2} (-x\sin(sx))\,dx = \int_0^\infty \frac{x}{1+x^2} \sin(sx)\,dx$$

$$\frac{d^2F_s(s)}{ds^2} = F_s(s).$$

$\cancel{D^2 F_s}$ $D^2F_s - F_s = 0$

$$(D^2-1)F_s = 0$$

AE: $m^2 - 1 = 0$

$m = \pm 1.$

$$F_s(s) = C_1 e^{s} + C_2 e^{-s} \quad \text{--- (4)}$$

Now $s=0$ in (2) $\Rightarrow F_s(0) = \pi/2$

$S=0$ in (3) $\Rightarrow \left.\frac{dF_s}{ds}\right|_{s=0} = -\int_0^\infty \frac{1}{1+x^2}\,dx = -\tan^{-1}x\Big]_0^\infty$

$$= -\pi/2.$$

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$S=0$ in (4) $\quad C_1 + C_2 = F_S(0) = \pi/2 \quad -(5)$

~~$S=0$ in (5)~~ $\quad C_1 - C_2 = F_S(0) = -\pi/2 . \quad -(6)$

$$\left.\frac{dF_s}{ds}\right|_{s=0}$$

Solving (5) & (6)

$\cancel{C_1 = \pi/4} \quad C_1 = 0$

$C_2 = \pi/2 .$

$$\therefore F_s\left[\frac{x}{1+x^2}\right] = \frac{\pi}{2} e^{-s}$$

H.W $\quad F_s\left[\frac{1}{x(a^2+x^2)}\right] = ?$ $\quad$ Ans $\rightarrow \frac{\pi}{2a^2}(1-e^{-s})$

Hint : $F_s(s) = CF + PI$

$(D^2-a^2) F_s = k \neq 0$

$k = ?$

Q $\rightarrow$ Obtain $F_s\left[\frac{e^{-ax}}{x}\right]$ $a>0$

$$F_s[f(x)] = \int_0^\infty f(x)\sin(sx)\,dx = \cancel{F_s d}\; F_s(s)$$

$$F_s = \int_0^\infty \frac{e^{-ax}}{x}\sin(sx)\,dx. \quad -(1)$$

$$\frac{dF_s}{ds} = \int_0^\infty \frac{e^{-ax}}{x}(\cos sx)\times x\,dx$$

$$\frac{dF_s}{ds} = \int_0^\infty e^{-ax}\cos(sx)\,dx$$

$$\frac{dF_s}{ds} = \frac{e^{-ax}}{a^2+s^2}\Big[-a\cos(sx) + s\sin(sx)\Big]_{x=0}^{\infty}$$

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$$= -\left[\frac{-a}{a^2+s^2}\right]$$

$$\frac{dF_s}{ds} = \frac{a}{a^2+s^2}$$

$$\frac{dF_s}{a} = \frac{ds}{a^2+s^2}.$$

$$\frac{F_s}{a} = \frac{1}{a}\tan^{-1}\left(\frac{s}{a}\right) + c$$

$$F_s = \tan^{-1}\left(\frac{s}{a}\right) + ac$$

$$F_s(s) = \tan^{-1}\left(\frac{s}{a}\right) + c'$$

$$F_s(0) = c'$$

$s=0$ in eqn (1)

$$F(s) = \int_0^\infty 0\,dx = 0 = c'$$

$$F_s(s) = \tan^{-1}\left(\frac{s}{a}\right)$$

Q→ Find the inverse fourier transform of $\frac{\sin(as)}{s}$, $a>0$.

$$F^{-1}[F(s)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(s)e^{-isx}ds = f(x)$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\sin(as)}{s}e^{-isx}ds$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\sin(as)}{s}\left(\cos(sx) - i\sin(sx)\right)dx$$

$$= \cancel{\frac{1}{2\pi}\int_{-\infty}^{\infty}} \cancel{\frac{-2}{2\pi}\int_0^\infty \frac{\sin(as)i\sin(sx)}{s}dx}$$

$$= \frac{1}{\pi}\int_0^\infty \frac{\sin(as) \times i\sin(sx)}{s}\, dx$$

$$\therefore \frac{1}{\pi}\int_0^\infty \frac{\sin(as)}{s}\cos(sx)\, ds.$$

$$f(x) = \frac{1}{2\pi}\left[\int_0^\infty \frac{\sin((a+x)s)}{s}\, ds + \int_0^\infty \frac{\sin((a-x)s)}{s}\, ds\right]$$

$$f(x) = \frac{1}{2\pi}\left[\int_0^\infty \frac{\sin((x+a)s)}{s}\, ds - \int_0^\infty \frac{\sin((x-a)s)}{s}\, ds\right]$$

Now : $\int_0^\infty \frac{\sin(As)}{s}\, ds = \begin{cases} \pi/2 & A>0 \\ -\pi/2 & A<0 \end{cases}$

$A = x+a$ or $A = x-a$

① ② ③

$-\infty \quad -a \quad 0 \quad a \quad \infty$

* Región ① $x < -a$

$x < -a \Rightarrow x + a < (-a+a) \Rightarrow x + a < 0$

$\cancel{x > a}$ $x < -a \Rightarrow x - a < -a - a \Rightarrow x - a < -2a$ $\rightarrow a > 0$

$\Rightarrow x - a < 0$

$$\therefore f(x) = \frac{1}{2\pi}\left[\left(-\frac{\pi}{2}\right) - \left(-\frac{\pi}{2}\right)\right] = 0$$

* Región ② $-a < x < a$

$-a < x \Rightarrow a - a < x + a \Rightarrow x + a > 0$

$x < a \Rightarrow x - a < a - a \Rightarrow x - a < 0$

$$f(x) = \frac{1}{2\pi}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \frac{1}{2}.$$

ion (3) $x > a$

$x + a > a + a \Rightarrow x + a > 2a \Rightarrow x + a > 0$

$x > a \Rightarrow x - a > a - a \Rightarrow x - a > 0$

$$f(x) = \frac{1}{2\pi}\left[\frac{\pi}{2} - \frac{\pi}{2}\right] = 0.$$

$$f(x) = \begin{cases} 0 & x < -a \\ 1/2 & -a < x < a \\ 0 & x > a. \end{cases}$$

Obtaine the inverse fourier cosine transform of

$$F_c(s) = \begin{cases} \frac{1}{2\pi}\left\{a - \frac{s}{2}\right\} & 0 < s < 2a \\ 0 & s > 2a \end{cases}$$

$$F_c^{-1}[F_c(s)] = \frac{2}{\pi}\int_0^\infty F_c(s)\cos(sx)\,ds$$

$$\cancel{= \frac{2}{\pi}\int_0^{2a} \frac{1}{2\pi}\, a\{}$$

$$\cancel{= \frac{2}{\pi}\int_0^{2a} \frac{1}{2\pi}\left\{a - \frac{s}{2}\right\} ds}$$

$$= \frac{2}{\pi}\int_0^{2a} \frac{1}{2\pi}\left\{a - \frac{s}{2}\right\}\cos(sx)\,ds.$$

$$= \frac{1}{\pi^2}\int_0^{2a}\left\{a - \frac{s}{2}\right\}\cos(sx)\,ds.$$

$$= \frac{1}{\pi^2}\left[\int_0^{2a} a\cos sx - \frac{s}{2}\cos(sx)\,ds\right]$$

$$= \frac{1}{\pi^2}\left[a\frac{\sin(sx)}{x}\Big|_0^{2a} - \left(\frac{s}{2}\left(\frac{\sin(sx)}{x}\right) - \frac{1}{2}\left(\frac{-\cos(sx)}{x}\right)\right)\Big|^{2a}\right.$$

$$= \frac{1}{\pi^2}\left[\int_0^{2a}\right.$$

$$= \frac{1}{\pi^2}\left[\frac{a\sin(sx)}{x}\Big|_0^{2a} - \left(\frac{s}{2}\left(\frac{\sin(sx)}{x}\right) - \frac{1}{2}\left(-\frac{\cos(sx)}{x}\right)\right)\right]_0^{2a}$$

$$= \frac{1}{\pi^2}\left[\frac{a\sin 2ax}{x} - \left[\frac{2a}{2}\left(\frac{\sin(2ax)}{x}\right) - \frac{1}{2}\left(\frac{1}{x} - \frac{\cos 2ax}{x}\right)\right.\right.$$

$$= \frac{1}{\pi^2}\left[\cancel{\frac{2a\sin(2ax)}{2ax}} - \cancel{\frac{2a\sin(2ax)}{2ax}} - \frac{1}{2x} + \frac{1}{2}\frac{\cos(2ax)}{x}\right)$$

$$= \frac{1}{2\pi^2}\left[\frac{\cos(2ax)}{x} + 1\right]$$

Q→ find inverse fourier cosine transform of $\frac{1}{1+s^2}$.

$$F_c^{-1}\left[\frac{1}{1+s^2}\right] = \frac{2}{\pi}\int_0^{\infty}\frac{1}{1+s^2}\cos(sx)\,ds = f(x) \quad \text{—(1)}$$

$$\frac{df}{dx} = \frac{2}{\pi}\int_0^{\infty}\frac{1}{1+s^2}\frac{\partial}{\partial x}\cos(sx)\,ds.$$

$$= \frac{2}{\pi}\int_0^{\infty}\frac{1}{1+s^2}\times -s\sin(sx)\,ds$$

$$\frac{d^2f}{dx^2} = \frac{2}{\pi}\int_0^{\infty}\frac{-s^2}{1+s^2}\times\cos(sx)\,ds.$$

$$= \frac{2}{\pi}\int_0^{\infty}\frac{-s^2-1+1}{1+s^2}\cos(sx)\,ds = \frac{2}{\pi}\int_0\frac{1}{1+s^2}\cos sx\,ds - \frac{2}{\pi}\int_0^{\infty}\cos(sx)\,ds.$$

$$= -\frac{2}{\pi}\int_0^{\infty}\frac{s^2}{s(1+s^2)}\sin(sx)\,ds$$

$$= -\frac{2}{\pi}\int_0^{\infty}\frac{s^2+1-1}{s(1+s^2)}\sin(sx)\,ds$$

$$= -\frac{2}{\pi}\left[\int_0^{\infty}\frac{\sin(sx)}{s} - \frac{\sin(sx)\,ds}{s(1+s^2)}\right]$$

$$= -\frac{2}{\pi}\left[\frac{\pi}{2} - \int_0^{\infty}\frac{\sin(sx)\,ds}{s(1+s^2)}\right]$$

$$\frac{df}{dx} = \int_0^{\infty}\frac{\sin(sx)}{s(\cancel{1}+s^2)}\,ds - 1 \quad \leftarrow ②$$

$$\frac{d^2f}{dx^2} = \int_0^{\infty}\frac{\cos(sx)\times s\,ds}{s(1+s^2)}$$

$$= \int_0^{\infty}\frac{\cos(sx)\,ds}{(1+s^2)}$$

$$\frac{d^2f}{dx^2} = f(x)$$

$$\frac{d^2f}{dx^2} - f(x) = 0$$

$$D^2f(x) - f(x) = 0$$

$$AE:\ m^2 - 1 = 0$$

$$m = \pm 1$$

$$CF = f(x) = C_1e^{x} + C_2e^{-x}.$$

$s{:}\ x=0$ in ① $\quad f(0) = \frac{2}{\pi}\int_0^{\infty}\frac{ds}{1+s^2} = \frac{2}{\pi}\tan^{-1}\Big|_0^{\infty} = 1.$

$x=0$ in ② $\quad \frac{df(0)}{dx} = 0 - 1$

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CF: $f(0) = C_1 + C_2 = 1$

$\frac{df(0)}{dx} = C_1 - C_2 = -1$

Solving, $C_2 = 1$

$\therefore f(x) = e^{-x}$

Q→ $F_c^{-1}\left[\frac{\sin(as)}{s}\right] = ?$ $a > 0$

$F_c^{-1}\left[\frac{\sin(as)}{s}\right] = \cancel{\frac{2}{\pi}\int_0^\infty \frac{1}{1+s^2}}$

$= \frac{2}{\pi}\int_0^\infty \frac{\sin(as)}{s}\cos(sx)\,ds = f(x)$

$\cancel{= \frac{2}{\pi}\int_0^\infty}$

$= \frac{2}{\pi \times 2}\int_0^\infty \cancel{\frac{\sin((x-a)s) - \sin((x+a)s)}{s}\,ds}$

$= \frac{1}{\pi}\int_0^\infty$

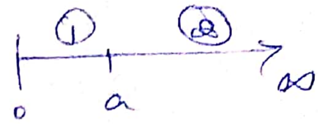
$= \frac{2}{\pi \times 2}\left[\int_0^\infty \frac{\sin((a+x)s)}{s} + \int_0^\infty \frac{\sin((a-x)s)}{s}\,ds\right]$

$= \frac{1}{\pi}\left[\int_0^\infty \frac{\sin((x+a)s)}{s}\,ds - \int_0^\infty \frac{\sin((x-a)s)}{s}\,ds\right]$

$$\int_0^\infty \frac{\sin As}{s}\,ds = \begin{cases} \pi/2 & A>0 \\ -\pi/2 & A<0. \end{cases}$$

Now $F_c^{-1}[F_c(s)] = f(x), \quad x \geq 0$

$\Rightarrow x+a > a \Rightarrow x+a>0$



$$\therefore f(x) = \frac{1}{\pi}\left[\frac{\pi}{2} - \int_0^\infty \frac{\sin((x-a)s)}{s}\,ds\right]$$

* Region 1 $\quad 0<x<a$

$x<a \Rightarrow x-a<a-a \Rightarrow x-a<0$

$$f(x) = \frac{1}{\pi}\left\{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right\} = 1$$

* Region 2 $\quad x>a \Rightarrow x-a>a-a \Rightarrow x-a>0$

$$f(x) = \frac{1}{\pi}\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = 0$$

$$F_c^{-1}\left[\frac{\sin(as)}{s}\right] = \begin{cases} 1 & 0<x<a \\ 0 & x>a \end{cases}, \quad a>0$$

Q → $F_s^{-1}\left[\dfrac{s}{1+s^2}\right]$.

$$F_s^{-1}\left[\frac{s}{1+s^2}\right] = \frac{2}{\pi}\int_0^\infty \frac{s}{1+s^2}\sin(sx)\,ds = f(x)$$

$$= \frac{2}{\pi}\int_0^\infty \frac{s^2+1-1}{s(s^2+1)}\sin(sx)\,ds$$

This is correct

$$= \frac{2}{\pi}\left[\int_0^\infty \frac{\sin(sx)}{s} - \int_0^\infty \frac{1}{s(s^2+1)}\sin(sx)\,ds\right]$$

$$= \frac{2}{\pi}\left[\frac{\pi}{2} - \int_0^\infty \frac{\sin(sx)}{s(s^2+1)}\,ds\right]$$

$$\cancel{f(x) = 1 - \int_0^\infty \frac{\sin(sx)}{s(s^2+1)}\,ds}$$

$$\cancel{\frac{df}{dx} = -\int_0^\infty \frac{\cos(sx)}{s^2+1}\,ds.}$$

$$\cancel{\frac{df}{dx} = \frac{2}{\pi}\int_0^\infty \frac{s^2}{1+s^2}\cos(sx)\,dx.}$$

$$f(x) = 1 - \frac{2}{\pi}\int_0^\infty \frac{\sin(sx)}{s(s^2+1)}\,ds \quad -(1)$$

$$\frac{df}{dx} = -\frac{2}{\pi}\int_0^\infty \frac{\cos(sx)}{(s^2+1)}\,ds \quad -(2)$$

$$\frac{d^2f}{dx^2} = \frac{2}{\pi}\int_0^\infty \frac{s\sin(sx)}{(s^2+1)}\,dx.$$

$$\frac{d^2f}{dx^2} = f(x).$$

$$D^2f(x) - f(x) = 0.$$

AE: $m = \pm 1$

CF: $f(x) = C_1e^{x} + C_2e^{-x}$

$x=0$ in (1) $f(0) = 1$

$x=0$ in (2) $\frac{df(0)}{dx} = -1$

$C_1 + C_2 = 1$

$C_1 - C_2 = -\pi/2 - 1$

Solving $2C_1 = 1 + \pi/2$

$C_1 = \frac{1}{2} + \frac{\pi}{4}$

① $\frac{1}{2} + \frac{\pi}{4} + C_2 = 1$

$C_2 = \frac{1}{2} - \frac{\pi}{4}$.

$f(x) = \frac{1}{2} + \frac{\pi}{4}$.

$2C_1 = 0$

$C_1 = 0 \qquad C_2 = 1$

$f(x) = e^{-x}$

Q→ $F_S^{-1}\left[\frac{e^{-as}}{s}\right]$ → Ans $\frac{2}{\pi}\tan^{-1}\left(\frac{x}{a}\right)$

$F_S^{-1}\left[\frac{e^{-as}}{s}\right] = \frac{2}{\pi}\int_0^\infty \frac{e^{-as}}{s}\sin(sx)\,ds = f(x),\ a>0$

$\frac{df(x)}{dx} = \frac{2}{\pi}\int_0^\infty e^{-as}\cos(sx)\,ds$

$= \frac{2}{\pi}\left[\frac{e^{-as}}{a^2+x^2}\left[-a\cos(sx) + s\sin(sx)\right]\right]_0^\infty$

$\cancel{\frac{2}{\pi}\left[\frac{e^{-as}}{a^2}\right]}\ \cancel{\frac{2}{\pi}\left[\frac{e^{-as}}{a^2}x - a\right]} = \cancel{\frac{2}{\pi}\left[\frac{e^{-as}}{a}\right]}$

$= \frac{2}{\pi}\left[\frac{1}{a^2+x^2}x - a\right]$

Q→ Solve $\int_0^\infty f(x)\cos(sx)\,dx =$

CONVOLUTION

If $f(x)$ $g(x)$ $(-\infty, \infty)$

$$f * g(x) = \int_{-\infty}^{\infty} f(x)\, g(x-u)\, du$$

$$f * g(x) = g * f(x)$$

Convolution th- for fourier transform

If $F[f(x)] = F(s)$ and $F[g(x)] = G(s)$

$$F[f * g(x)] = F(s)G(s)$$

Q→ $\frac{i}{(1+s^2)^2}$ if $\frac{2}{1+s^2}$ $e^{-|x|}$

$f(x) = e^{-|x|}$ $\quad F(s) = \frac{2}{1+s^2}$

$f(x) = g(x)$. $\quad F(s) = G(s)$

By convolution th.

$$F[f * g(x)] = \cancel{F(s)g(s)}\ F(s)G(s)$$

$$\therefore F^{-1}[F(s)G(s)] = f * g(x)$$

$$F^{-1}\left[\left(\frac{2}{1+s^2}\right)^2\right] = \int_{-\infty}^{\infty} f(u)\, g(x-u)\, du$$

$$F^{-1}\left[\frac{4}{(1+s^2)^2}\right] = \int_{-\infty}^{\infty} e^{-|u|} e^{-|x-u|}\, du$$

$$\frac{4}{i}F^{-1}\left[\frac{i}{(1+s^2)^2}\right] = \int_{-\infty}^{\infty} e^{-|u|-|x-u|}\,du$$

$$F^{-1}\left[\frac{i}{(1+s^2)^2}\right] = \frac{i}{4}\int_{-\infty}^{\infty} e^{-|u|-|x-u|}\,du.$$

Q) Find inverse fourier transform of $\frac{1}{(a^2+s^2)(b^2+s^2)}$ using convolution theorem.

Ans $F\{e^{-a|x|}\}$ Consider $F(s) = \frac{1}{a^2+s^2}$ $G(s) = \frac{1}{b^2+s^2}$

$$F[e^{-a|x|}] = \int_{-\infty}^{\infty} e^{-a|x|} e^{isx}\,dx = 2\int_0^{\infty} e^{-ax}\cos(sx)\,dx$$

$$= \frac{2a}{a^2+s^2}$$

$$f(x) = F^{-1}\left[\frac{1}{a^2+s^2}\right] = \frac{e^{-a|x|}}{2a}$$

$$\Rightarrow g(x) = F^{-1}\left[\frac{1}{b^2+s^2}\right] = \frac{e^{-b|x|}}{2b}$$

By convolution theorem

$$F[f*g(x)] = F(s)G(s)$$

$$\therefore F^{-1}[F(s)G(s)] = f*g(x)$$

$$F^{-1}\left[\left(\frac{2}{1+s^2}\right)^2\right] = \int_{-\infty}^{\infty} f(u)\,g(x-u)\,du.$$

$$F^{-1}\left[\frac{1}{(a^2+s^2)(b^2+s^2)}\right] = \int_{-\infty}^{\infty} \frac{e^{-a|u|}}{2a} \times \frac{e^{-b|x-u|}}{2b}\,du$$

$$= \frac{1}{4ab}\int_{-\infty}^{\infty} e^{-a|u|-b|x-u|}\,du.$$

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Q→ Apply convolution theorem to find inverse fourier transform of $e^{-s^2/2}$.

Ans: Let $F(s)G(s) = e^{-s^2/2}$

By convolution theorem,

$$F[f*g(x)] = F(s)G(s)$$

$$\cancel{F^{-1}\{G(}\; F^{-1}[F(s)G(s)] = f*g(x) = \int_{-\infty}^{\infty} f(u)g(x-u)\,du$$

$$F(s)G(s) = e^{-s^2/2} \Rightarrow F(s) = G(s) = \left(e^{-s^2/2}\right)^{1/2} = e^{-s^2/4}$$

$$f(x) = F^{-1}\left[e^{-s^2/4}\right]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-s^2/4}\, e^{-isx}\,ds$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-(s^2/4 + isx)}\,dx.$$

$$\frac{s^2}{4} + isx = \left(\frac{s}{2}\right)^2 + 2\times\frac{s}{2}\times ix + (ix)^2 - (ix)^2$$

$$= \left(\frac{s}{2} + ix\right)^2 + x^2.$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\left\{\left(\frac{s}{2}+ix\right)^2 + x^2\right\}}\,ds$$

$$= \frac{1}{2\pi}\, e^{-x^2}\int_{-\infty}^{\infty} e^{-\left(\frac{s}{2}+ix\right)^2}\,ds$$

$$\frac{s}{2} + ix = t \Rightarrow ds = 2dt \qquad \left[\int_{-\infty}^{\infty} e^{-t^2}\,dt = \sqrt{\pi}.\right.$$

$$f(x) = \frac{1}{2\pi}\, e^{-x^2}\int_{-\infty}^{\infty} e^{-t^2}\,2dt = \frac{e^{-x^2}}{\pi}\int_{-\infty}^{\infty} e^{-t^2}\,dt = \frac{e^{-x^2}}{\pi}\times\sqrt{\pi}$$

$$f(x) = \frac{e^{-x^2}}{\sqrt{\pi}} = g(x).$$

$$F^{-1}\left[e^{-s^2/4}\, e^{-s^2/4}\right] = \int_{-\infty}^{\infty} \frac{e^{-u^2}}{\sqrt{\pi}} \times \frac{e^{-(x-u)^2}}{\sqrt{\pi}}\, du$$

$$F^{-1}\left[e^{-s^2/4}\right] = \frac{1}{\pi}\int_{-\infty}^{\infty} e^{-\{u^2+(x-u)^2\}}\, du.$$

Now $u^2 + (x-u)^2 = u^2 + x^2 + u^2 - 2ux = 2u^2 - 2ux + x^2$.

$$= (\sqrt{2}u)^2 - \frac{2\sqrt{2}u\,x}{\sqrt{2}} + x^2$$

$$= (\sqrt{2}u)^2 - 2(\sqrt{2}u)\frac{x}{\sqrt{2}} + \left(\frac{x}{\sqrt{2}}\right)^2 - \left(\frac{x}{\sqrt{2}}\right)^2 + x^2$$

$$= \left(\sqrt{2}u \mp \frac{x}{\sqrt{2}}\right)^2 + \frac{x^2}{2}.$$

$$F^{-1}\left[e^{-s^2/2}\right] = \frac{1}{\pi}\int_{-\infty}^{\infty} e^{-\left\{\left(\sqrt{2}u \mp \frac{x}{\sqrt{2}}\right)^2 + \frac{x^2}{2}\right\}}\, du$$

$$= \frac{1}{\pi}\, e^{-x^2/2} \int_{-\infty}^{\infty} e^{-\left(\sqrt{2}u - \frac{x}{2}\right)^2}\, du.$$

$\sqrt{2}u - \frac{x}{2} = t \Rightarrow \sqrt{2}\,du = dt \Rightarrow du = \frac{dt}{\sqrt{2}}$

$$= \frac{1}{\pi}\, e^{-x^2/2} \int_{-\infty}^{\infty} e^{-t^2} \frac{dt}{\sqrt{2}}.$$

$$= \frac{1}{\sqrt{2}\,\pi}\, e^{-x^2/2} \times \int_{-\infty}^{\infty} e^{-t^2}\, dt = \frac{1}{\sqrt{2}\,\pi}\, e^{-x^2/2} \times \sqrt{\pi}$$

$$= \frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}.$$

P.T.O.

PARSEVAL'S IDENTITY

If $F[f(x)] = F(s)$ & $F[g(x)] = G(s)$

then (a) $\frac{1}{2\pi}\int_{-\infty}^{\infty} F(s)\bar{G}(s)\,ds = \int_{-\infty}^{\infty} f(x)\bar{g}(x)\,dx$

(b) $\frac{1}{2\pi}\int_{-\infty}^{\infty} |F(s)|^2\,ds = \int_{-\infty}^{\infty} |f(x)|\,dx.$

Q→ Apply Parseval's identity for Fourier transform on

$$f(x) = \begin{cases} 1 & |x| \le a \\ 0 & |x| > a \end{cases}$$ & hence prove that

$$\int_0^{\infty} \frac{\sin^2(ax)}{x^2}\,dx = \frac{\pi a}{2}.$$

$$F[f(x)] = \int_{-\infty}^{\infty} f(x)e^{isx}\,dx = \int_{-\infty}^{-a} 0\,e^{isx}\,dx + \int_{-a}^{a} e^{isx}\,dx + \int_{a}^{\infty} 0\,e^{isx}\,dx$$

$$= \left.\frac{e^{isx}}{is}\right|_{x=-a}^{a} = \frac{1}{is}\left(e^{isa} - e^{is(-a)}\right)$$

$$= \frac{2i\sin(as)}{is} = \frac{2\sin(as)}{s}.$$

Let $f(x) = g(x) \Rightarrow F(s) = G(s)$

By Parseval's identity

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} |F(s)|^2\,ds = \int_{-\infty}^{\infty} |f(x)|^2\,dx$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{2\sin(as)}{s}\right)^2 ds = \int_{-\infty}^{-a} |f(x)|^2\,dx + \int_{-a}^{a} |f(x)|^2\,dx + \int_{a}^{\infty} |f(x)|^2\,dx$$

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$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\sin^2(as)}{s^2}ds = 0+\int_{-a}^{a}dx+0 = x\Big]_{-a}^{a} = a-(-a)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\sin^2(as)}{s^2}ds = 2a$$

$$\cancel{\int_{-\infty}^{\infty} 4\sin}\ \int_{-\infty}^{\infty}\frac{\sin^2(as)}{s^2}ds = a\pi$$

$$2\int_{0}^{\infty}\frac{\sin^2(as)}{s^2}ds = a\pi$$

$$\int_{0}^{\infty}\frac{\sin^2(as)}{s^2}ds = \frac{\pi a}{2}$$

$$\int_{0}^{\infty}\frac{\sin^2(ax)}{x^2}dx = \frac{\pi a}{2}$$

Q.10 → Evaluate $\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^3}\right)^2 dx$. by applying Parvesal's identity on $f(x) = \begin{cases} a^2-x^2 & |x|\le a. \\ 0 & |x|>a \end{cases}$

PARVESAL'S IDENTITY FOR FOURIER COSINE TRANSFORM.

$$\frac{2}{\pi}\int_0^{\infty} F_c(s)\,G_c(s)\,ds = \int_0^{\infty} f(x)g(x)\,dx$$

$$\frac{2}{\pi}\int_0^{\infty}(F_c(s))^2 ds = \int_0^{\infty}(f(x))^2 dx$$

PARVESAL'S IDENTITY FOR FOURIER SINE TRANSFORM.

$$\frac{2}{\pi}\int_0^{\infty} F_s(s)\,G_s(s)\,ds = \int_0^{\infty} f(x)g(x)\,dx$$

$$\frac{2}{\pi}\int_0^{\infty}(F_s(s))^2 ds = \int_0^{\infty}(f(x))^2 dx$$

Q→ ST $\int_0^\infty \frac{dx}{(a^2+x^2)(b^2+x^2)} = \frac{\pi}{2ab(a+b)}$ using Parvesals identity for cosine / sine transform. $a, b > 0$

Ans. wkt $F_c[e^{-ax}] = \frac{a}{a^2+s^2}$

Let $F_c(s) = \frac{1}{a^2+s^2}$ $G_c(s) = \frac{1}{b^2+s^2}$

∴ $f(x) = \frac{e^{-ax}}{a}$ ~~g(x) = e^{-b}~~ $g(x) = \frac{e^{-bx}}{b}$ $a, b > 0$

Parvesal's identity is

$$\frac{2}{\pi}\int_0^\infty F_c(s)\, G_c(s)\, ds = \int_0^\infty f(x) g(x)\, dx.$$

$$\frac{2}{\pi}\int_0^\infty \frac{ds}{(a^2+s^2)(b^2+s^2)} = \int_0^\infty \frac{e^{-ax}}{a}\cdot\frac{e^{-bx}}{b}\, dx$$

$$= \frac{\pi}{2ab}\int_0^\infty e^{-(a+b)x}\, dx.$$

$$\int_0^\infty \frac{ds}{(a^2+s^2)(b^2+s^2)} = \frac{\pi}{2ab(a+b)}$$

$$\int_0^\infty \frac{dx}{(a^2+x^2)(b^2+x^2)} = \frac{\pi}{2ab(a+b)}$$

Q→ Evaluate $\int_0^\infty \frac{x^2}{(x^2+a^2)^2}\, dx$ using Parvesal's identity.

Consider $F_s[e^{-ax}] = \frac{s}{s^2+a^2}$

$$\frac{2}{\pi}\int_0^\infty \{F_s(s)\}^2 ds = \int_0^\infty \{f(x)\}^2 dx$$

$$\frac{2}{\pi}\int_0^\infty \left(\frac{s}{s^2+a^2}\right)^2 ds = \int_0^\infty (e^{-ax})^2 dx$$

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INVERSE OF A MATRIX

$I = A^{-1} = \frac{adj A}{|A|}$

$AA^{-1} = I$

$(AB)^{-1} = B^{-1}A^{-1}$.

$$\int_0^{\infty} \frac{s^2}{(s^2+a^2)^2} ds = \frac{\pi}{2} \int_0^{\infty} e^{-2ax} dx = \frac{\pi}{4a}$$

Q→ Apply Parseval's identity for cosine/sine transform on $f(x) = \begin{cases} 1 & 0<x<1 \\ 0 & x>1 \end{cases}$ and evaluate $\int_0^{\infty} \left(\frac{1-\cos x}{x}\right)^2 dx$ and $\int_0^{\infty} \frac{\sin^4 x}{x^2} dx$.

~~Ans $F_c[f(x)] = \int_0^{\infty} f(x$~~ Cosine transform is not applied here.

Ans $F_s[f(x)] = \int_0^{\infty} f(x) \sin sx \, dx$. $\int_0^1 \sin(sx) dx + 0$

$$= -\frac{\cos(sx)}{s}\Big]_{x=0}^{1} = -\frac{1}{s}(\cos(s)-1)$$

$$= \frac{1-\cos(s)}{s}$$

Applying Parseval's identity,

$$\frac{2}{\pi}\int_0^{\infty} \{F_s(s)\}^2 ds = \int_0^{\infty} \{f(x)\}^2 dx$$

$$\Rightarrow \frac{2}{\pi}\int_0^{\infty} \left(\frac{1-\cos s}{s}\right)^2 ds = \int_0^{\infty} (f(x))^2 dx = \int_0^1 1^2 dx + 0 = x|_0^1$$

$$= 1$$

$$\int_0^{\infty} \left(\frac{1-\cos s}{s}\right)^2 ds = \pi/2 \Rightarrow \int_0^{\infty} \left(\frac{1-\cos x}{x}\right)^2 dx = \pi/2$$

$$\int_0^{\infty}\left(\frac{1-\cos x}{x}\right)^2 dx = \int_0^{\infty} \frac{\left(2\sin^2\frac{x}{2}\right)^2}{x^2} = 4\int \frac{\sin^4\frac{x}{2}}{x^2} dx \qquad \frac{x}{2} \Rightarrow t \quad dx = 2dt.$$

$$4\int_0^{\infty} \frac{\sin^4 t}{(2t)^2} 2dt = \frac{\pi}{2}. \quad \therefore \int_0^{\infty} \frac{\sin^4 t}{t^2} dt = \pi/4.$$

$$\Rightarrow \int_0^{\infty} \frac{\sin^4 x}{x^2} dx = \pi/4$$

COFACTOR MATRIX.

A matrix with elements that are the cofactors, term-by-term of a given square matrix.

Eg: Cofactor of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$ will be

Coffactor of each element:

$A_{11} = \begin{vmatrix} 4 & 5 \\ 0 & 6 \end{vmatrix} = 24$ $\quad A_{12} = -\begin{vmatrix} 0 & 5 \\ 1 & 6 \end{vmatrix} = 5$ $\quad A_{13} = \begin{vmatrix} 0 & 4 \\ 1 & 0 \end{vmatrix} = -4$

$A_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 6 \end{vmatrix} = -12$ $\quad A_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 6 \end{vmatrix} = 3$ $\quad A_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = 2$

$A_{31} = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = -2$ $\quad A_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = -5$ $\quad A_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} = 4$

$$\text{Cofactor of } A = \begin{bmatrix} 24 & 5 & -4 \\ -12 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix}$$

~~ADJOINT OF~~ Cofactor of each element,

$$A_{ij} = (-1)^{i+j} M_{ij}.$$

ADJOINT OF A MATRIX.

Adjoint of a matrix A is the transpose of cofactor matrix of A.

$$\text{Cofactor of } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

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TYPES OF MATRIX

1. SYMMETRIC & SKEW SYMMETRIC MATRICES

A square matrix $A = [a_{ij}]$ is said to be symmetric when $a_{ij} = a_{ji}$ for all i and j.

Eg: $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

If $a_{ij} = -a_{ji}$ for all i & j so that all the diagonal elements are zero, then the matrix is called skew symmetric.

Eg: $\begin{bmatrix} 0 & h & -g \\ -h & 0 & f \\ g & -f & 0 \end{bmatrix}$

2. TRIANGULAR MATRIX. [Echelon form of matrix] → not exactly.

$\begin{bmatrix} a & h & g \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix}$ $\qquad$ $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & -5 & 4 \end{bmatrix}$

TRANSPOSE OF A MATRIX

The matrix obtained from any given matrix A, by interchanging rows and columns is called the transpose of A and is denoted by A'.

~~ADJOINT~~ Eg. For a symmetric matrix $A' = A$ and for skew symmetric matrix $A' = -A$

Every square matrix can be expressed as a sum of a symmetric & a skew symmetric matrix.

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

The transpose of the product of two matrices is the product of their transpose taken in the reverse order

$(AB)' = B'A'$.

MINOR OF A ~~KH~~ DETERMINANT (MATRIX)

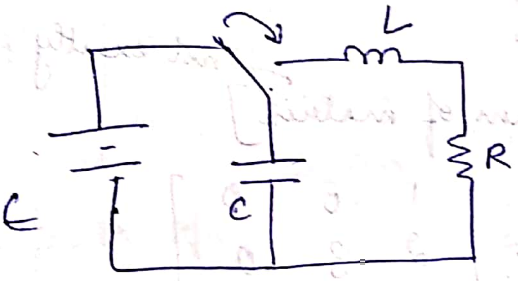
— Value computed from the determinant of a square matrix which is obtained ~~by~~ after crossing out a row & a column corresponding to the element that is considered. Denoted as M_{ij}.

H.W

If $f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$ evaluate $\int_0^{\infty} \sin ax$.

S.T $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt$ and $\int_0^{\infty} \frac{\sin^4 t}{t^4} dt = \frac{\pi}{3}$ using

Parseval's identity for fourier transform.



$q(0) = q_0$

Requirement $Q(0.05) = 0.01 q_0$

$$L\frac{di}{dt} + Ri + \frac{q}{c} = 0$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$$

$L = 5H$

$C = 10^{-4}F$

$$q = e^{\alpha t}\{C_1 \cos\beta t + C_2 \sin\beta t\}$$

$$= q_0 e^{-Rt/2L} \cos\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\, t.$$

$$0.01 q_0 = q_0 e^{-0.005R} \cos\{\sqrt{2000 - 0.01R^2})0.05\}$$

$$f(R) = e^{-0.005R} \cos\{0.05\sqrt{2000 - 0.01R^2}\} - 0.01$$

schoology access code

CPWQ-RMXZ-GBVS8

Anthony Christy Melson

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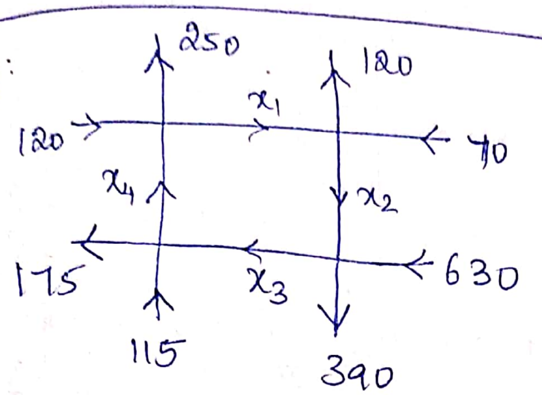
Internal choice for Unit 3 and Unit 5.

Eg:



Eqns: $x_1+120 = x_4+250$

$x_1+70 = x_2+120$

$x_3+630 = x_2+390$

$x_3+175 = x_4+115$

$x_1-x_4 = 130$

$x_1-x_2 = 50$

$x.$

Eqns: $120+x_4 = 250+x_1$

$x_2+120 = x_1+70$

$x_2+630 = x_3+390$

$x_3+115 = x_4+175$

$x_1-x_4 = -130$ — ①

$x_1-x_2 = 50$ — ②

$x_2-x_3 = -240$ — ③

$x_3-x_4 = 60$ — ④

Solving ① & ② $-x_2+x_4 = 180$. — ⑤

solving ⑤ & ③ $-x_3+x_4 = -60$

0

foot
cu. yard
1 cord of
...ch of stone
...wood is equivalent
...ng, 4 feet wide and
...rch of stone or
...et long 1½ feet
...igh.