Matrices

10/08/18

The study of matrices is applied in the study of design of experiments, multivariant anylsis, electrical Network, production Process etc.

elementary row transformation:-

(i) Interchange any two rows $(R_i \leftrightarrow R_j)$

(ii) Multiplication of any row by non-zero constant (i.e. $R_i \leftrightarrow kR_i$) $(k \neq 0)$

(iii) Addition to any row by a constant multipel of any other row (i.e $R_i \to R_i + kR_j$) $(k \neq 0)$

Echelon form:-

A non-zero matrix 'E' is said to be in echelon form if it satisfies the following conditions.

(i) All zero rows are below non-zero rows.

(ii) The first non-zero entry of any non-zero row is '1'

Note:- If $E = [a_{ij}]$ is said to be in echelon form if $a_{ij} = 0$ if $i > j$

e.g. $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ | main diagonal ke niche sab zero hona chaiye tab usko echelon form kha sakte hai

Que Reduce the following matrices to the echelon form

(i) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_1$

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{bmatrix}$ $R_3 \to R_3 - R_2$

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ This is in echelon form.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 0 & 2 & 3 \\ 8 & -1 & -5 \\ & -5 & -7 \end{bmatrix} \quad R_3 \to R_3 - 5R_2$$

$$\sim \begin{bmatrix} 0 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & 0 & \cancel{18} \end{bmatrix}$$ This is echolon form.

(3) Let

$$A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 5 & 6 & 7 & 8 \\ 1 & 1 & 1 & 1 \\ 6 & 6 & 6 & 6 \\ 11 & 11 & 11 & 11 \end{bmatrix} \longrightarrow \begin{matrix} R_3 \to \frac{1}{6} R_3 \\ R_4 \to \frac{1}{11} R_4 \end{matrix}$$

$$\sim \begin{bmatrix} 5 & 6 & 7 & 8 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - R_2 \\ R_4 \to R_4 - R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & -1/5 & -2/5 & -3/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ This is in echolon form.

(4)

$$\begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -1.5 \end{bmatrix} \quad R_2 \to R_2 - 2R_1$$

$$\sim \begin{bmatrix} 4 & 2 & 3 \\ 0 & 0 & 0 \\ -2 & -1 & -1.5 \end{bmatrix} \quad R_1 \to \tfrac{1}{2} R_1$$

$$\begin{bmatrix} 2 & 1 & 1.5 \\ 0 & 0 & 0 \\ -2 & -1 & -1.5 \end{bmatrix} \quad R_3 \to R_3 + R_1$$

$$\sim \begin{bmatrix} 2 & 1 & 1.5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

RANK of Matrix.

Note:- Equivalent matrices:

Two matrices of same order are said to be equivalent if one of these can be obtained by the application of a finite no. of successive elementory transformation to other.

i.e – A is equivalent to B $\Rightarrow A \sim B$

Rank of Matrix:-

– The no. of non-zero rows in the echolon form of a matrix 'A' is the rank of the matrix 'A'

Problem:- find the rank of the matrix by reducing into a echolon form.

(i)

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & -2 & 3 & -2 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

– This is in echolon form.

– There are two Non-zero row.

Rank of 'A' = 2. i.e $\rho(A) = 2$.

(2) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$ $R_4 \to R_4 - R_3$

$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_4 \to R_4 - R_2$

$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 \leftrightarrow R_3$

This is in echelon form there are three non-zero rows the rank of matrices is '3'

Q3 find 'b' if the rank of

$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ is 3

Soln.

Let. $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ leading to echelon form

$R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - bR_1$

$R_4 \to R_4 - 9R_1$

$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 2-b & 2+b & 2 \\ 0 & 0 & b+9 & 3 \end{bmatrix}$

$R_4 \to R_4 - 3R_2$

$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 2-b & 2+b & 2 \\ 0 & 0 & b+6 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2-b & 2+b & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & b+6 & 0 \end{bmatrix}$ $R_2 \leftrightarrow R_3$

This is in echelon form since the rank of matrices is '3' Then the value of b is $= -6, 2$.

Qu:- find the rank of

Let $A: \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ $\left[1 \;\; 3 \;\; 4 \;\; 3 \right.$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - R_1$

$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

There are three no-zero rows here

Rank of 'A' (is) 3

⟹ Linear System of equation:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_{n} = b_m$$

$$AX = B.$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ \vdots \\ b_m \end{bmatrix}$$

$\rho(A) = \rho(A:B) \longrightarrow$ consistence

$\rho(A) \neq \rho(A:B) \longrightarrow$ Non-consistent.

⟹ Linear system of eqn:-

m linear eqn in n unknowns is of the form

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}\right\} -(1)$$

where $x_1, x_2 \cdots x_n$ are unknowns a's & b's are known costants.

writing.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

If ~~atleat~~ atleast of one of the b's are non zero then the system is non-homogenious system

Otherwise it is a homogeneous system.

→ A set of values $x_1, x_2 \cdots x_n$ satisfying all eqn in (1) is called a soln.

→ A system of linear eqn is said to be consistent if it possesses a soln and inconsistent if it does not posesses a soln.

→ The matrix obtained by appending to 'A' an extra column consisting of elements of b is called augmented matrix.

i.e
$$[A:B] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & : & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & : & b_2 \\ \vdots & & & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} & : & b_m \end{bmatrix}$$

→ A non-homogenious system $AX = B$ is said to be consistent $\rho(A) = \rho(A:B)$

⟹ Gauss elimination method.

To solve non-homogenious system $AX = B$

Step 1:- write augmented matrix $[A:B]$ and reduce to echelon form.

Step 2:- ~~Reduce [A:B]~~ find $\rho(A)$ and $\rho(A:B)$

If $\rho(A) = \rho(A:B)$ the system is consistent.

If $\rho(A) \neq \rho(A:B)$ then the system is inconsistent.

Step 3:- If the system is consistent

(i) $\rho(A) = \rho(A:B) = r = n$ (no. of unknown)

then the system has unique solⁿ

(ii) If $\rho(A:B) = \rho(A) = r$
$< n$ (no. of unknown)

then the system has infinitely many solⁿ

(since $n-r$ unknown can be chosen arbitrarily)

(i) Solve

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 32$$

Matrix form is $AX = B$

where $A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$

Augmented matrix is

$$[A:B] = \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 0 & 11/2 & -27/2 & 11/2 \\ 0 & 22 & -54 & 27 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - \frac{3}{2}R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$7 \cdot \frac{21}{2}$

$$\sim \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 0 & 11/2 & -27/2 & 11/2 \\ 0 & 0 & 0 & 5 \end{array}\right] \quad R_3 \to R_3 - 4R_2$$

$\rho(A) = 2$ $\quad \rho(A:B) = 3$

$\rho(A:B) \neq \rho(A)$

Therefore the system is inconsistent.

2. Solve

$$2x + y = z$$
$$2x + 5y - 7z = 52$$
$$x + y + z = 9$$

$A = \begin{bmatrix} 2 & 1 & -1 \\ 2 & 5 & -7 \\ 1 & 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 52 \\ 9 \end{bmatrix}$

Augmented matrix is

$$A:B = \left[\begin{array}{ccc:c} 2 & 1 & -1 & 0 \\ 2 & 5 & -7 & 52 \\ 1 & 1 & -1 & 9 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 2 & 1 & -1 & 0 \\ 0 & 4 & -6 & 52 \\ 0 & 1/2 & -1/2 & -9 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - \frac{R_1}{2} \end{array}$$

$1 - \frac{1}{2}$

$-1 + \frac{1}{2}$

$$\sim \left[\begin{array}{ccc:c} 2 & 1 & -1 & 0 \\ 0 & 4 & -6 & 52 \\ 0 & 0 & -9/4 & 5/2 \end{array}\right] - (1) \quad R_3 \to R_3 - \frac{1}{8}R_2$$

$\rho(A) = \rho(A:B) = 3 =$ (No. of unknown)

The system is consistent & having unique solⁿ.

The unique solⁿ

eq^n form of (i) is

$2x + y - z = 0$ —(1)'

$4y - 6z = 52$ —(2)

$\frac{9}{7} z = \frac{5}{2}$ —(3)

from (3) $z = 10/9$

from (2) $y = \frac{44}{3}$

from (1)' $x = -61/9$

Ques. Solve.

$x + 2y + z = 3$

$2x + 3y + 2z = 5$

$3x - 5y + 5z = 2$

$3x + 9y - z = 4$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & -5 & 5 \\ 3 & 9 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 5 \\ 2 \\ 4 \end{bmatrix}$$

Solve the system :-

$2x + 4y + z = 3$

$3x + 2y - 2z = -2$

$x - y + z = 6$

Matrix form is $AX = B$

where $A = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 2 & -2 \\ 1 & -1 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$

Augmented matrix $[A:B] = \left[\begin{array}{ccc:c} 2 & 4 & 1 & 3 \\ 3 & 2 & -2 & -2 \\ 1 & -1 & 1 & 6 \end{array}\right]$

$\sim \left[\begin{array}{ccc:c} 1 & -1 & 1 & 6 \\ 3 & 2 & -2 & -2 \\ 2 & 4 & 1 & 3 \end{array}\right]$ $R_1 \leftrightarrow R_3$

$\sim \left[\begin{array}{ccc:c} 1 & -1 & 1 & 6 \\ 0 & 5 & -5 & -20 \\ 0 & 6 & -1 & -9 \end{array}\right]$ $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$

$\sim \left[\begin{array}{ccc:c} 1 & -1 & 1 & 6 \\ 0 & 5 & -5 & -20 \\ 0 & 0 & 5 & 15 \end{array}\right]$ —(1) $R_3 \to R_3 - \frac{6}{5} R_2$

This is in echelon form

$\rho(A) = 3$ $\rho(A:B) = 3$

$\rho(A) = \rho(A:B)$ Then this system is said to be consistence & is unique.

eq^n (1) is written as

$x - y + z = 6$ —(2)

$5y - 5z = -20$ —(3)

$5z = 15$ —(4)

By backward substitution

from (4) $z = 3$

(3), $y = -1$

$x = 2$

(i) If $\lambda = 5$ & $\mu \neq 9$

then $\rho(A) = 2$

& $\rho(A:B) = 3$

$\rho(A) \neq \rho(A:B)$

$\therefore$ then the system has no sol^n

(ii) If $\lambda \neq 5$ & $\mu \neq 9$ (be any value)

$\rho(A) = 3$ $\rho(A:B) = 3$

$\rho(A) = \rho(A:B) = 3$ the system is consistent and has unique sol^n

(iii) If $\lambda = 5$ & $\mu = 9$

$\rho(A) = 2$ $\rho(A:B) = 2$

$\rho(A) = \rho(A:B) = 2$ the system is consistent and has infinitely many sol^n.

Qu. for what value of K the eq^n

$$x + y + z = 1$$
$$2x + y + 4z = k$$
$$4x + y + 10z = k^2$$

have a sol^n solve completely in each case.

Augmented matrix

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & k \\ 4 & 1 & 10 & k^2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & -3 & 2 & k^2-2k \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & -1 & & \\ & & & \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & 0 & & \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & -3 & 6 & k^2-4 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & 0 & 0 & k^2-3k+2 \end{array}\right] - (1) \quad R_3 \to R_3 - 3R_2$$

This is in echelon form.

$\rho(A) = 2$

for the system to be consistent $\rho(A:B) = 2$

for this $k^2 - 3k + 2 = 0$

solving $k = 1$ & 2

(i) $k = 1$

eq^n (1) given

$x + y + z = 1$ — (3)

$-y + 2z = -1$ — 4

we chosen $3 - 2 = 1$ unknown arbitirally let us chose $z = k$ arbitirally

from (4) $y = 2z + 1 = 2k + 1$

from (3) $x = 1 - y - z$

$= 1 - 2k_1 - 1 - k_1$

$= -3k_1$

(ii) $k = 2$

eq^n (1) give.

$x + y + z = 1 \to (5)$

$-y + 2z = 0 \to (6)$

we choose arbitrarily ie $z = k_2$ be arbitraly.

from (6) $y = 2k_2$

$x = 1 - 2k_2 - k_2$

$= 1 - 3k_2$

(i) If $\lambda = 5$ & $\mu \neq 9$

then $\rho(A) = 2$

& $\rho(A:B) = 3$

$\rho(A) \neq \rho(A:B)$

$\therefore$ then the system has no solⁿ

(ii) If $\lambda \neq 5$ & $\mu \neq 9$ (be any value)

$\rho(A) = 3$ $\rho(A:B) = 3$

$\rho(A) = \rho(A:B) = 3$ the system is consistent and has unique solⁿ

(iii) If $\lambda = 5$ & $\mu = 9$

$\rho(A) = 2$ $\rho(A:B) = 2$

$\rho(A) = \rho(A:B) = 2$ the system is consistent and has infinitely many solⁿ.

Q. for what value of K the eqⁿ

$$x + y + z = 1$$
$$2x + y + 4z = k$$
$$4x + y + 10z = k^2$$

have a solⁿ solve completely in each case.

Augmented matrix

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 2 & 1 & 4 & : & k \\ 4 & 1 & 10 & : & k^2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 4R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & -1 & 2 & : & k-2 \\ 0 & -3 & 2 & : & k^2-2k \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & -1 & 2 & : & k-2 \\ 0 & -3 & 6 & : & k^2-4 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & -1 & 2 & : & k-2 \\ 0 & 0 & 0 & : & k^2-3k+2 \end{bmatrix} \quad R_3 \to R_3 - 3R_2 \quad \text{—(1)}$$

This is in order for $\rho(A) = 2$

for the system to be consistent $\rho(A:B) = 2$

for that $k^2 - 3k + 2 = 0$

solving $k = 1$ & 2

(i) $k = 1$

eqⁿ (1) then

$x + y + z = 1$ —(3)

$-y + 2z = -1$ —(4)

we choose $3 - 2 = 1$ unknown arbitrary

let us choose $z = k_1$ arbitrary

from (4) $y = 2z + 1 = 2k_1 + 1$

from (3) $x = 1 - y - z$

$= 1 - 2k_1 - 1 - k_1$

$= -3k_1$

(ii) $k = 2$

eqⁿ (1) give

$x + y + z = 1 \to$ (5)

$-y + 2z = 0 \to$ (6)

we choose subtract arbitrary i.e. $z = k_2$ arbitrarily.

from (6) $y = 2k_2$

$x = 1 - 2k_2 - k_2$

$= 1 - 3k_2$

Que

Solve the system

$x_1 - 3x_2 + 4x_3 = -4$

$3x_1 - 7x_2 + 7x_3 = -8$

$-4x_1 + 6x_2 - x_3 = 7$

$[A:B]$ $\begin{bmatrix} 1 & -3 & 4 & : -4 \\ 3 & -7 & 7 & : -8 \\ -4 & 6 & -1 & : 7 \end{bmatrix}$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 + 4R_1$

$\sim \begin{bmatrix} 1 & -3 & 4 & : -4 \\ 0 & 2 & -5 & : 4 \\ 0 & -6 & 15 & : -9 \end{bmatrix}$

-12
-16
12

Que. S.T the eq^n

$3x + 4y + 5z = a$

$4x + 5y + 6z = b$

$5x + 6y + 7z = c$

do not have sol^n unless

$a + c = 2b$

sol^n. Augmented matrix is

$[A:B] = \begin{bmatrix} 3 & 4 & 5 & : a \\ 4 & 5 & 6 & : b \\ 5 & 6 & 7 & : c \end{bmatrix}$

$R_2 \to R_2 - \frac{4}{3}R_1$

$R_3 \to R_3 - \frac{5}{3}R_1$

$\sim \begin{bmatrix} 3 & 4 & 5 & : a \\ 0 & -1/3 & -2/3 & : b - \frac{4a}{3} \\ 0 & -2/3 & -4/3 & : c - \frac{5a}{3} \end{bmatrix}$

$R_3 \to R_3 - 2R_2$

$\sim \begin{bmatrix} 3 & 4 & 5 & : a \\ 0 & -1/3 & -2/3 & : b - 4a/3 \\ 0 & 0 & 0 & : c - 2b + a \end{bmatrix}$

$\rho(A) = 2$

for the system to be consistent

$\rho(A:B) = 2$ for this $c - 2b + a = 0$

$a + c = 2b$.

Thus the system have sol^n when $a + c = 2b$.

Que. Test for consistency

$-2x + y + z = a$

$x - 2y + z = b$

$x + y - 2z = c$

where a, b, c are constants.

sol^n Augmented matrix is

$[A:B] : \begin{bmatrix} -2 & 1 & 1 & : a \\ 1 & -2 & 1 & : b \\ 1 & 1 & -2 & : c \end{bmatrix}$

$$\sim \begin{bmatrix} -2 & 1 & 1 & : & a \\ 0 & -3/2 & 3/2 & : & b+a/2 \\ 0 & 3/2 & -3/2 & : & c+a/2 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 + \frac{1}{2}R_1 \\ R_3 \to R_3 + \frac{1}{2}R_1 \end{array}$$

$$\sim \begin{bmatrix} -2 & 1 & 1 & : & a \\ 0 & -3/2 & 3/2 & : & b+a/2 \\ 0 & 0 & 0 & : & a+b+c \end{bmatrix} \quad R_3 \to R_3 + R_2$$

$\rho(A) = 2$

$\therefore$ for the system to be Consistent

$\rho(A : B) = 2$ for this $a+b+c=0$.

The system has infinitely many soln.

⇒ L U - factorization :-

* — *

Let A is any $n \times n$ matrix then it can be factorised as $A = LU$ where L is the lower triangular matrix with one's in the main diagonal of order $n \times n$ and U is the upper triangular matrix which is got by reducing the given matrix A to echelon form without changing any row.

To find U :-

→ Reduce the matrix 'A' to echelon from without interchanging the row. This echelon form is 'U'

To find L :-

→ write all main diagonal element as 1 and all above element as '0'

→ if the operation is $R_j \to R_j - kR_i$ where k is called as multiplier.

→ in each position below the main diagonal of 'L' place the multiplier 'k' used to introduce zero in that posn of U.

Qs. find the L-U factorization of

Let $A = \begin{bmatrix} 4 & 3 & -1 \\ -2 & -4 & 5 \\ 1 & 2 & 6 \end{bmatrix}$

$$\begin{array}{l} R_2 \to R_2 + \frac{1}{2}R_1 \\ R_3 \to R_3 - \frac{1}{4}R_1 \end{array}$$

$$\sim \begin{bmatrix} 4 & 3 & -1 \\ 0 & -5/2 & 9/2 \\ 0 & 5/4 & 25/4 \end{bmatrix} \quad R_3 \to R_3 + \frac{1}{2}R_2$$

$$\sim \begin{bmatrix} 4 & 3 & -1 \\ 0 & -5/2 & 9/2 \\ 0 & 0 & 34/4 \end{bmatrix} \approx U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/4 & -1/2 & 1 \end{bmatrix}$$

$A = LU$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/4 & -1/2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & -1 \\ 0 & -5/2 & 9/2 \\ 0 & 0 & 34/4 \end{bmatrix}$$

Qus. 2.

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

$R_2 \to R_2 - \frac{1}{2}R_1$

$R_3 \to R_3 - \frac{3}{2}R_1$

$$\sim \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & -7/2 & 1/2 \end{bmatrix}$$

$$\sim U \to \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} \qquad R_3 \to R_3 + 7R_2$$

$1 - \quad 2 - \frac{3}{2} \quad 3 - \frac{1}{2} \quad 1 - \frac{9}{2} \quad 2 - \frac{3}{2} \quad \frac{1}{2} - \frac{35}{2} \quad \frac{1}{2} + \frac{35}{2}$

$A = LU$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix}$$

Qus. (3)

Let $A \cdot \begin{bmatrix} 1 & 1 & 0 & 4 \\ 2 & -1 & 5 & 0 \\ 5 & 2 & 1 & 2 \\ -3 & 0 & 2 & 6 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & -3 & 5 & -8 \\ 0 & -3 & 1 & -18 \\ 0 & 3 & 2 & 18 \end{bmatrix} \qquad \begin{aligned} &R_2 \to R_2 - 2R_1 \\ &R_3 \to R_3 - 5R_1 \\ &R_4 \to R_4 + 3R_1 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & -3 & 5 & -8 \\ 0 & 0 & -4 & -10 \\ 0 & 0 & 7 & 10 \end{bmatrix} \qquad \begin{aligned} &R_3 \to R_3 - R_2 \\ &R_4 \to R_4 + R_2 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & -3 & 5 & -8 \\ 0 & 0 & -4 & -10 \\ 0 & 0 & 0 & -\frac{30}{4} \end{bmatrix} \simeq U. \qquad R_4 \to R_4 + \frac{7}{4}R_3$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 5 & 1 & 1 & 0 \\ -3 & -1 & -7/4 & 1 \end{bmatrix}$$

$A = LU.$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 5 & 1 & 1 & 0 \\ -3 & -1 & -7/4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & -3 & 5 & -8 \\ 0 & 0 & -4 & -10 \\ 0 & 0 & 0 & -30/4 \end{bmatrix}$$

#

To solve the system of eq^n by L-U factorization method. consider the system.

$AX = B$ — (1)

decompose into $LUX = B \to (2)$ $\because A = LU$

Take $UX = Y \to (3)$

then eq^n (2) $LY = B \to (4)$

By solving the pair of eq^n we get 'X' first we solve $LY = B$ to get Y by forward substitution and then solve $UX = Y$ to get X by backward substitution

Que. Solve by LU-factorization

(i) $3x + 2y + 7z = 4$

$2x + 3y + z = 5$

$3x + 4y + z = 7$

Sol.

Let $A = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix}$ $R_2 \to R_2 - \frac{2}{3}R_1$

$R_3 \to R_3 - R_1$

$\sim \begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 2 & -6 \end{bmatrix}$ $R_3 \to R_3 - \frac{6}{5}R_2$

$\sim \begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix} = U$

$L = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix}$

System = $AX = B \to (1)$

$LUX = B \to (2)$

Take $UX = Y$ — (3)

Then 2 is $LY = B$ — 4

Taking $LY = B$

$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$

$y_1 = 4$

$\frac{2}{3}y_1 + y_2 = 5 \Rightarrow y_2 = 7/3$

$y_1 + \frac{6y_2}{5} + y_3 = 7 \Rightarrow y_3 = 1/5$

Now we have $UX = Y$

$\begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7/3 \\ 1/5 \end{bmatrix}$

$3x + 2y + 7z = 4$ — (5)

$\frac{5}{3}y - \frac{11}{3}z = \frac{7}{3}$ — (6)

$-\frac{8}{5}z = \frac{1}{5} \to (7)$

from (7) $z = -\frac{1}{8}$.

fm (6) $\frac{5}{3}y - \frac{11}{3}\left(-\frac{1}{8}\right) = \frac{7}{3}$

$\frac{5}{3}y + \frac{11}{24} = \frac{7}{3}$

$\frac{5}{3}y = \frac{7}{3} - \frac{11}{24}$

$\frac{5}{3}y = \frac{56-11}{24}$

$\frac{5}{3}y = \frac{45}{24}$ $y = \frac{\cancel{45}^{9}}{\cancel{24}_{8}} \times \frac{\cancel{3}}{\cancel{5}}$

$y = 9/8$

fm (5) $x = 7/8$.

Que.

$3x_1 + x_2 + x_3 = 4$

$x_1 + 2x_2 + 2x_3 = 3$

$2x_1 + x_2 + 3x_3 = 4$

Let $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ $R_2 \to R_2 - \frac{1}{3}R_1$

$R_3 \to R_3 - \frac{2}{3}R_1$

$\sim \begin{bmatrix} 3 & 1 & 1 \\ 0 & 5/3 & 5/3 \\ 0 & 1/3 & 7/3 \end{bmatrix}$

$\sim \begin{bmatrix} 3 & 1 & 1 \\ 0 & 5/3 & 5/3 \\ 0 & 0 & 2 \end{bmatrix} \approx U$ $R_3 \to R_3 - \frac{1}{5}R_2$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 2/3 & 1/5 & 1 \end{bmatrix}$$

System is $AX = B$ —(1)
decomposing into $LUX = B \rightarrow$ (2)
Take $UX = Y$ —(3)
Then (2) is $LY = B$ —(4)

Taking $LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 2/3 & 1/5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix}$$

$y_1 = 4$

$\frac{1}{3}y_1 + y_2 = 3 \qquad y_2 = 3 - \frac{4}{3} = \frac{5}{3}$

$\frac{2}{3}y_1 + \frac{y_2}{5} + y_3 = 4 \qquad y_3 = 1.$

Now we have $UX = Y$

$$\begin{bmatrix} 3 & 1 & 1 \\ 0 & 5/3 & 5/3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5/3 \\ 1 \end{bmatrix}$$

$3x_1 + x_2 + x_3 = 4$

$\frac{5}{3}x_2 + \frac{5}{3}x_3 = \frac{5}{3}$

$2x_3 = 1$

$x_3 = \frac{1}{2} \qquad x_2 = \frac{1}{2} \qquad \underline{x_1 = 1}$

calculation

$\frac{5}{3}x_2 + \frac{5}{6} = \frac{5}{3}$

$\frac{5}{3}x_2 = \frac{5}{3} - \frac{5}{6} = \frac{10-5}{6} = \frac{\cancel{4}2}{\cancel{6}3}$

$x_2 = \frac{2}{3} \times \frac{3}{5}$

$x_2 = 2/.$

Ques. $10x + y + z = 12$
$2x + 10y + z = 13$
$2x + 2y + 10z = 14$

$\boxed{x = y = z = 1}$

Let $A = \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix}$

$R_2 \rightarrow R_2 - \frac{R_1}{5}$
$R_3 \rightarrow R_3 - \frac{R_1}{5}$

$$= \begin{bmatrix} 10 & 1 & 1 \\ 0 & 49/5 & 4/5 \\ 0 & 9/5 & 49/5 \end{bmatrix}$$

$10 - \frac{1}{5}$, $1 - \frac{1}{5}$, $2 - \frac{1}{5}$, $10 - \frac{1}{5}$, $\frac{36}{49}$

$R_3 \rightarrow R_3 - \frac{9R_2}{49}$

$$\Rightarrow \begin{bmatrix} 10 & 1 & 1 \\ 0 & 49/5 & 4/5 \\ 0 & 0 & \frac{2221}{245} \end{bmatrix} \sim U$$

$R_3 \rightarrow R_3 - \frac{9}{49}$

$\frac{36}{245}$

$\frac{49}{5} - \frac{36}{245}$

245

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1 & 0 \\ 1/5 & 9/49 & 1 \end{bmatrix}$$

$AX = B$ —(1)
$LUX = B \rightarrow 2$
$UX = Y$
$LY = B.$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1 & 0 \\ 1/5 & 9/49 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 14 \end{bmatrix}$$

$y_1 = 12$

$\frac{y_1}{5} + y_2 = 13$

$y_2 = 13 - \frac{12}{5}$

$y_2 = \frac{53}{5}$

65, $\frac{12}{53}$

$\frac{y_1}{5} + \frac{9y_2}{49} + y_3 = 14.$

$\frac{12}{5} + \frac{9 \times 53}{49 \times 5} + y_3 = 14 \qquad \frac{473}{49}$

$y_3 =$

(4) $x+2y+3z=14$

$2x+3y+4z=20$

$3x+4y+z=14$ $\quad [x=1, y=2, z=3]$

(5) $3x_1-6x_2-3x_3=-3$

$-2x_1+6x_3=-22$

$-4x_1+7x_2+4x_3=3$ $\quad x=-28, y=-7, z=-13$

Solⁿ

(4)

⇒ Gauss-Seidel Iterative method

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \Rightarrow x_1 = \frac{1}{a_{11}}[b_1 - a_{12}x_2 - a_{13}x_3]$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \Rightarrow x_2 = \frac{1}{a_{22}}[b_2 - a_{21}x_1 - a_{23}x_3]$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \Rightarrow x_3 = \frac{1}{a_{33}}[b_3 - a_{31}x_1 - a_{32}x_2]$

⇒ It is an Interative method to solve the system of eq^n approximately consider the system of eq^n.

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}\right\} \text{— (1)}$$

where the diagonal coefficient are not zero and are large compare to other element such a system is called diagonally dominant system.

→ The system of eq^n may be ~~the~~ re-written as

$$\left.\begin{aligned} x_1 &= \frac{1}{a_{11}}[b_1 - a_{12}x_2 - a_{13}x_3] \\ x_2 &= \frac{1}{a_{22}}[b_2 - a_{21}x_1 - a_{23}x_3] \\ x_3 &= \frac{1}{a_{33}}[b_3 - a_{31}x_1 - a_{32}x_2] \end{aligned}\right\} \text{— (2)}$$

If $x_1(0)$, $x_2(0)$ & $x_3(0)$ are some rough initial gauss then the iterative formula to find 1st, 2nd, 3rd approximately for x_1, x_2 & x_3 is

$$\left.\begin{aligned} x_1^{(k+1)} &= \frac{1}{a_{11}}[b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}] \\ x_2^{(k+1)} &= \frac{1}{a_{22}}[b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}] \\ x_3^{(k+1)} &= \frac{1}{a_{33}}[b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}] \end{aligned}\right\} \text{— (3)}$$

The iteration process stop when the desired order of approximation is reached or two succesive iteration are ~~really the~~ nearly the same.

→ This method can be ~~genrali~~ genralised to the system in n-equation in 'n' unknown.

The method is known as gauss seidel method

Qus. $10x + y + z = 12$

$2x + 10y + z = 13$

$2x + 2y + 10z = 14.$

The given system is diagonaly dominant

re-writing the eq^n.

$x = \frac{1}{10}[12 - y - z]$

$y = \frac{1}{10}[13 - 2x - z]$

$z = \frac{1}{10}[14 - 2x - 2y]$

Let $x^{(0)} = y^{(0)} = z^{(0)} = 0$ be the initial guess

the iterative formula is

$$\left.\begin{aligned} x^{(k+1)} &= \frac{1}{10}[12 - y^{(k)} - z^{(k)}] \\ y^{(k+1)} &= \frac{1}{10}[13 - 2x^{(k+1)} - z^{(k)}] \\ z^{(k+1)} &= \frac{1}{10}[14 - 2x^{(k+1)} - 2y^{(k+1)}] \end{aligned}\right\} \text{-(1)}$$

— for $k=0$ in (1) the 1st approximation are.

$x' = \frac{1}{10}[12 - y^{(0)} - z^{(0)}]$

$x' = \frac{12}{10} = 1.2$

$$y^{(1)} = \frac{1}{10}\left[13 - 2x^{(1)} - z^{(10)}\right]$$
$$= \frac{1}{10}[13 - 2(1.2) - 0] = 1.06$$

$$z^1 = \frac{1}{10}\left[14 - 2x^{(1)} - 2y^{(1)}\right]$$
$$= \frac{1}{10}[14 - 2(2.2) - 2 \times 1.06]$$
$$= 0.948$$

for $k = 1$ in ① the 2^{nd} approximation are.

$$x^{(2)} = \frac{1}{10}\left[12 - y^{(1)} - z^{(1)}\right]$$
$$x^2 = \frac{1}{10}[12 - 1.06 - 0.948]$$
$$x^{(2)} = 0.999$$
$$y^{(2)} = \frac{1}{10}\left[13 - 2x^{(2)} - z^{(1)}\right]$$
$$= \frac{1}{10}[13 - 2 \times 0.999 - 0.948]$$
$$= 1.005$$
$$z^{(2)} = \frac{1}{10}\left[14 - 2x^2 - 2y^{(2)}\right]$$
$$= \frac{1}{10}[14 - 2 \times 0.999 - 2 \times 1.005]$$
$$z^{(2)} = 0.999$$

for $k = 2$ in ① 3^{rd} approximation.

$$x^{(3)} = \frac{1}{10}\left[12 - y^{(2)} - z^{(2)}\right] = 0.999$$
$$y^{(3)} = \frac{1}{10}\left[13 - 2x^{(3)} - z^{(2)}\right]$$
$$= \frac{1}{10}[13 - 2 \times 0.999 - 0.999]$$
$$= 1.0001$$
$$z^{(3)} = \frac{1}{10}\left[14 - 2x^{(3)} - 2y^{(3)}\right]$$
$$= \frac{1}{10}[14 - 2 \times 0.999 - 2 \times 1.0001]$$
$$z^3 = 1.00$$

1.998
2.0002

After 3^{rd} approximation the solution to the system is $x = 0.999$, $y = z = 1$.

Ques. Solve.
$$3x + 20y - z = -18$$
$$20x + y - 2z = 17$$
$$2x - 3y + 20z = 25$$

$\Rightarrow$ The given system is not diagonally dominant writing 2^{nd} eqn 1^{st} and 1^{st} eqn 2^{nd} the diagonally system is

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

$$x = \frac{1}{20}[17 - y + 2z]$$
$$y = \frac{1}{20}[-18 - 3x + z]$$
$$z = \frac{1}{20}[25 - 2x + 3y]$$

Let $x^{(0)} = y^{(0)} = z^{(0)} = 0$ be the initial guess.

The iterative formula is

$$\left.\begin{aligned} x^{(k+1)} &= \frac{1}{20}\left[17 - y^{(k)} + 2z^{(k)}\right] \\ y^{(k+1)} &= \frac{1}{20}\left[-18 - 3x^{(k+1)} + z^{(k)}\right] \\ z^{(k+1)} &= \frac{1}{20}\left[25 - 2x^{(k+1)} + 3y^{(k+1)}\right] \end{aligned}\right\} \text{— ①}$$

for $k = 0$ in ① the 1^{st} approximation are.

$$x^1 = \frac{1}{20}\left[17 - y^{(0)} + 2z^{(0)}\right] = 0.85$$
$$y^1 = \frac{1}{20}\left[-18 - 3x^{(1)} + z^{(0)}\right] = -1.0275$$
$$z^1 = \frac{1}{20}\left(25 - 2x^{(1)} + 3y^{(1)}\right)$$
$$= \frac{1}{20}[25 - 2 \times 0.85 + 3 \times (-1.0275)] = 1.011$$

3.0825

for $k=1$ in (1) the 2nd approximation

$$x^{(2)} = \frac{1}{20}\left[17 - y^{(1)} + 2z^{(1)}\right] =$$
$$= \frac{1}{20}\left[17 - (-1.0275) + 2(1.011)\right]$$
$$= 1.0024.$$

$$y^{(2)} = \frac{1}{20}\left[-18 - 3x^{(2)} + z^{(1)}\right]$$
$$= \frac{1}{20}\left[-18 - 3(1.0024) + 1.011\right]$$
$$= -0.999$$

$$z^{(2)} = \frac{1}{20}\left[25 - 2x^{(2)} + 3y^{(2)}\right] = 0.999$$

for $k=2$ in (1) 3rd approximation

$$x^{(3)} = \frac{1}{20}\left[17 - y^{(2)} + 2z^{(2)}\right] = 0.999$$

$$y^{(3)} = \frac{1}{20}\left[-18 - 3x^{(3)} + z^{(2)}\right] = -0.999$$

$$z^{(3)} = \frac{1}{20}\left[25 - 2x^{(3)} + 3y^{(3)}\right] = 0.999.$$

Que.

Solve by gauss seidel method. starting with (2, 2, -1)

$$5x_1 - x_2 + x_3 = 10$$
$$2x_1 + 4x_2 = 12$$
$$x_1 + x_2 + 5x_3 = -1$$

starting with (2, 2, -1)

Soln.

given system is diagonally dominant.

Rewriting $x_1 = \frac{1}{5}\left[10 + x_2 - x_3\right]$

$$x_2 = \frac{1}{4}\left[12 - 2x_1\right]$$
$$x_3 = \frac{1}{5}\left[-1 - x_2 - x_1\right]$$

The initial values given are.

$$x_1^{(0)} = 2 \quad x_2^{(0)} = 2 \quad x_3^{(0)} = -1$$

The iteration formula is

$$\left.\begin{aligned} x_1^{(k+1)} &= \frac{1}{5}\left[10 + x_2^{(k)} - x_3^{(k)}\right] \\ x_2^{(k+1)} &= \frac{1}{4}\left[12 - 2x_1^{(k+1)}\right] \\ x_3^{(k+1)} &= \frac{1}{5}\left[-1 - x_1^{(k+1)} - x_2^{(k+1)}\right] \end{aligned}\right\} - (1)$$

for $k=0$ in (1) the 1st approximation are

$$x_1^{(1)} = \frac{1}{5}\left[10 + x_2^{(0)} - x_3^{(0)}\right] = 2.6$$

$$x_2^{(1)} = \frac{1}{4}\left[12 - 2x_1^{(1)}\right] = \frac{1}{4}\left[12 - 2\times(2.6)\right]$$
$$= 1.7.$$

$$x_3^{(1)} = \frac{1}{5}\left[-1 - x_1^{(1)} - x_2^{(1)}\right]$$
$$= \frac{1}{5}\left[-1 - 2.6 - 1.7\right]$$
$$= -1.06.$$

for $k=1$ in (1) the 2nd approximation are.

$$x_1^{(2)} = \frac{1}{5}\left[10 + x_2^{(1)} - x_3^{(1)}\right] = 2.552.$$
$$= \frac{1}{5}\left[10 + 1.7 + 1.06\right]$$

$$x_2^{(2)} = \frac{1}{4}\left[12 - 2x_1^{(2)}\right] = 1.724.$$
$$\frac{1}{4}\left[12 - 2(2.552)\right]$$

$$x_3^{(2)} = \frac{1}{5}\left[-1 - x_1^{(2)} - x_2^{(2)}\right] = -1.0552.$$

for $k=2$ in (1) the 3rd appm are.

$$x_1^{(3)} = \frac{1}{5}\left[10 + x_2^{(2)} - x_3^{(2)}\right] = 2.555$$
$$x_2^{(3)} = \frac{1}{4}\left[12 - 2x_1^{(3)}\right] = 1.7225$$
$$x_3^{(3)} = \frac{1}{5}\left[-1 - x_1^{(3)} - x_2^{(3)}\right] = -1.0555$$

for $k=(3)$ in (1) the 4th approximation

$$x_1^{(4)} = \frac{1}{5}\left[10 + x_2^{(3)} - x_3^{(3)}\right] = 2.555$$
$$x_2^{(4)} = \frac{1}{4}\left[12 - 2x_1^{(4)}\right] = 1.7225$$
$$x_3^{(4)} = \frac{1}{5}\left[-1 - x_1^{(4)} - x_2^{(4)}\right] = -1.0555.$$

since 3rd approximation & 4 approximation the value of x_1, x_2, & x_3 are nearly same then we stop the process.

thus the ans is:- $x_1 = 2.555$

$x_2 = 1.722$

$x_3 = -1.0555$

Solve:- by gauss siedel method. satisfying these eqn.

$$2x_1 + x_2 + 9x_3 = 12$$
$$8x_1 + 2x_2 - 2x_3 = -8$$
$$x - 8x_2 + 3x_3 = -4$$

diagonally dominant system.

$$8x_1 + 2x_2 - 2x_3 = -8$$
$$x - 8x_2 + 3x_3 = -4$$
$$2x_1 + x_2 + 9x_3 = 12$$

$$x_1 = -\frac{1}{8}[-8 + 2x_3 - 2x_2]$$
$$x_2 = -\frac{1}{8}[-4 - 3x_3 - x]$$
$$x_3 = \frac{1}{9}[12 - 2x_1 - x_2]$$

The initial value are taken as.

$x_1^{(0)} = 0$ $x_2^{(0)} = 0$ $x_3^{(0)} = 0$

The iteration formula is

$$x_1^{(k+1)} = \frac{1}{8}[-8 - 2x_2^{(k)} + 2x_3^{(k)}]$$
$$x_1' = \frac{1}{8}[-8] = -1$$
$$x_2^{(k+1)} = -\frac{1}{8}[-4 - x_1^{(k+1)} - 3x_3^{(k)}]$$
$$x_2' = 0.375$$
$$x_3^{(k+1)} = \frac{1}{9}[12 - 2x_1^{(k+1)} - x_2^{(k+1)}]$$
$$x_3^{(1)} = 1.5138$$

for k = 1 in (1) the 2nd approxm.

diagonal must > than remaining

$$x_1^{(2)} = \frac{1}{8}[-8 - 2x_2^{(1)} + 2x_3^{(1)}]$$
$$x_1^{(2)} = -0.7153$$
$$x_2^{(2)} = -\frac{1}{8}[-4 - x_1^{(2)} - 3x_3^{(1)}]$$
$$x_2^{(2)} = 0.9782$$
$$x_3^{(2)} = \frac{1}{9}[12 - 2x_1^{(2)} - x_2^{(2)}]$$
$$x_3^{(2)} = 1.3836$$

for k=2 in (1) the 3rd appxn are.

$$x_1^{(3)} = \frac{1}{8}[-8 - 2x_2^{(2)} + 2x_3^{(2)}]$$
$$= \frac{1}{8}[-8 - 2(0.9782) + 2(1.3836)]$$
$$x_1^{(3)} = -0.8986$$
$$x_2^{(3)} = -\frac{1}{8}[-4 - x_1^{(3)} - 3x_3^{(2)}]$$
$$x_2^{(3)} = 0.906$$
$$x_3^{(3)} = \frac{1}{9}[12 - 2x_1^{(3)} - x_2^{(3)}]$$
$$x_3^{(3)} = 1.432.$$

Thus $x_1 = -0.899$

$x_2 = 0.906$

$x_3 = 1.432.$

Eigen value & Eigen vector:-

⇒ ~~Eigen values & eigen vector~~

⇒ Let 'A' be a given square matrix of order 'n'. Suppose there exist non-zero column matrix x of order 'n' & a real or complex no. λ such that $AX = \lambda X$ then, X is called an eigen vector of A & λ is called eigne value of 'A'.

Characteristic eqn:

$$AX = \lambda X$$

$$AX = \lambda I X$$

$$[A - \lambda I]X = 0 \longrightarrow (1)$$

characteristic eqn is $|A - \lambda I| = 0$

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{n1} & a_{n2} & a_{n2} & \cdots & a_{nn}-\lambda \end{vmatrix}$$

$$a_0\lambda^n + a_1\lambda^{n-1} + \cdots + a_{n-1}\lambda + a_n = 0$$

Solving we get eigne value.

$\lambda_1, \lambda_2 \cdots \lambda_n$.

Procedure:-

To find Eigen value or eigen vectors:-

Step:-1 — write characteristic eqn $|A - \lambda I| = 0$
Solving we get eign value $\lambda_1, \lambda_2 \cdots \lambda_n$

Step:-2 Take Non-Zero eigen vector $X[x_1, x_2 \cdots x_n]^T$
write $[A - \lambda I]X = 0$ — (1)

Step 3:- Taking $\lambda = \lambda_1$ in (1)
Solve the system of eqn (1)
— to get eigin vector X_1 corresponding to λ_1
Similarly Taking $\lambda = \lambda_2, \lambda = \lambda_3 \cdots \lambda = \lambda_n$ in (1)
— for distinct eigen values
— find eigne vectors $X_2, X_3 \cdots X_n$.

Que.
Find eigne values & eigne vector for the matrix

Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

Characteristic eqn $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & 0 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(2-\lambda)(2-\lambda) - (2-\lambda)1 = 0$$

$$= \cancel{(2-\lambda)[4-2\lambda-2\lambda+\lambda^2]}\ (2-\lambda)[(2-\lambda)(2-\lambda) - 1]$$

$\lambda = 1, 2, 3$ is the eigne value.

~~values~~

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be the eigne vector

Satisfying $[A - \lambda I]X = 0$

$$\begin{bmatrix} 2-\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & 0 & 2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \rightarrow (1)$$

for $\lambda = 1$ in (1)

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

We can choose one unknown arbitrary.

$x + z = 0$ - (2)

$y = 0$

$z = k_1$

from (2) $x = -k_1$

$\therefore$ eigne vector $X_1 = \begin{bmatrix} -k_1 \\ 0 \\ k_1 \end{bmatrix} = k_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ correspondig of $\lambda = 1$

for $\lambda = 2$ in (1)

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x = 0$

$z = 0$

Let $y = k_2$ be chosen arbitraty.

eigne vector in $x_2 = \begin{bmatrix} 0 \\ k_2 \\ 0 \end{bmatrix}$ correspondig to $\lambda = 2$

for $\lambda = 3$ in (1)

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

we can choose one unknown arbitraty.

$-x + z = 0 \rightarrow (2)$

$-y = 0$

$z = k_3$ be choosen.

eigne vector of $x_3 = \begin{bmatrix} k_3 \\ 0 \\ k_3 \end{bmatrix} = k_3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ correspondig to $\lambda = 3$

Ques. find eigne value of eigne vectors of matrix.

Let $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

Characteristic eqn(i)

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -3-\lambda & -7 & -5 \\ 2 & 4-\lambda & 3 \\ 1 & 2 & 2-\lambda \end{vmatrix}$$

$\Rightarrow (-3-\lambda)\left[(4-\lambda)(2-\lambda) - 6\right] + 7\left[2(2-\lambda) - 3\right] - 5\left[4 - (4-\lambda)\right]$

$\Rightarrow (-3-\lambda)\left[8 - 4\lambda - 2\lambda + \lambda^2 - 6\right] + 7\left[4 - 2\lambda - 3\right] - 5\left[\cancel{4} - \cancel{4} + \lambda\right]$

$\Rightarrow (-3-\lambda)\left[2 - 6\lambda + \lambda^2\right] + 7\left[-2\lambda + 1\right] - 5\left[\lambda\right]$

$\Rightarrow (-3-\lambda)\left[2 - 6\lambda + \lambda^2\right] - 14\lambda + 7 - 5\lambda$

$\Rightarrow -6 + 18\lambda - 3\lambda^2 - 2\lambda + 6\lambda^2 - \lambda^3 - 14\lambda + 7 - 5\lambda$

$\Rightarrow -\lambda^3 - 3\lambda^2 - 3\lambda + 1 = 0$

$\lambda^3 + 3\lambda^2 + 3\lambda - 1 = 0$

$\lambda^3 + 3\lambda(\lambda + 1) - 1 = 0$

$(\lambda - 1)^3 = 0$

$\lambda = 1, 1, 1$

for $\lambda = 1$ in (1)

$$\begin{bmatrix} -4 & -7 & -5 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -4 & -7 & -5 \\ 0 & -1/2 & 1/2 \\ 0 & 1/4 & -1/4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -4 & -7 & -5 \\ 0 & -1/2 & 1/2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-4x - 7y - 5z = 0 \rightarrow (2)$

$-\frac{1}{2}y + \frac{1}{2}z = 0 \rightarrow (3)$

$z = k$ be chosen arbitrarily from (3) $y = k$.

from (2) $x = -3k$.

eigne vector $X = \begin{bmatrix} -3k \\ k \\ k \end{bmatrix} = k \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$

Corresponding to $\lambda = 1$.

Q6 find the eigne value & eigne vector.

let $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Characteristic eqn

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)[(5-\lambda)(1-\lambda)-1]$

$(1-\lambda)[(5-\lambda)(1-\lambda) - 1] - 1[(1-\lambda) - 3] + 3[1 - (5-\lambda)(3)]$

$\cancel{(1-\lambda)[5 - 5\lambda - \lambda + \lambda^2] - 1[1-\lambda-3] + 3[1 - 15 + 3\lambda]}$

$\cancel{(1-\lambda)}[5 - 6\lambda + \lambda^2] - 1 + \lambda + 3 + 3 - 45 + 9\lambda$

$5 - 6\lambda + \lambda^2 - 5\lambda + 6\lambda^2 - \lambda^3 + \lambda + 2 + 9\lambda - 42$

$\cancel{-\lambda^3 + 7\lambda^2 + 4\lambda - 35 = 0}$

$\cancel{\lambda^3 - 7\lambda^2 - 4\lambda + 35 = 0}$

$\lambda^3 - 7\lambda^2 + 6\lambda + 36 = 0$

$\lambda = -2, 3, 6$ is the eigne value.

let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be the eigne vector.

& satisfying $[A - \lambda I]X = 0$

$$\begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad -(1)$$

for $\lambda = -2$ in (1)

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \longrightarrow \begin{bmatrix} 3 & 1 & 3 \\ 0 & 20/3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_1$

$3x + y + 3z = 0$

$x + 7y + z = 0$

$y = k$

$\cancel{3x + k_1 + 3z = 0}$

$\cancel{x + 7k_1 + z = 0}$ $\cancel{-(2) \text{ multi by } (3)}$

$\cancel{3x + k_1 + 3z = 0}$

$3x + y + 3z = 0 \; (2)$

$\frac{20}{3}y = 0$

$y = 0$

$z = k_1$

from (2) $x = -k_1$

eigne vector $x_1 = k_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ Corresponding to $\lambda = -2$

for $\lambda = 3$

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Step:-2 find eigne vector e_1, e_2, e_3 corresponding to eigne value $\lambda_1, \lambda_2, \lambda_3$

Step:-3:- write modal matrix $P = [e_1 \, e_2 \, e_3]_{3\times 3}$

Step:-4 → find P^{-1}

Step:-3 → obtained diagonal matrix $D = P^{-1} A P$

$$= \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Problems:- Diagonalise the matrix

(i) Let $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

characteristic eqn is $|A - \lambda| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(5-\lambda)(1-\lambda) - 1] - 1[(1-\lambda) - 3] + 3[1 - (5-\lambda)(3)]$$

$$5 - \lambda - 5\lambda + \lambda^2 (1-\lambda) - 1 - 1[(1-\lambda) - 3)] + 3[1 - (5-\lambda)(3)]$$

eigne value are $-2, 3, 6$

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be a eigne vector

Satisfying $[A - \lambda I]x = 0$

$$\begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad - (1)$$

for $\lambda = -2$ in (2)

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\simeq \begin{bmatrix} 3 & 1 & 3 \\ 0 & 20/3 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad R_2 \to R_2 - \tfrac{1}{3}R_1, \quad R_3 \to R_3 - R_1$$

$3x + y + 3z = 0$ — (2)

$\frac{20}{3} y = 0 \Rightarrow y = 0$

let $z = k_1$

from (2) $3x + 0 + 3k_1 = 0$

$x_1 = -k_1$

eigne vector Correspondy to $X_1 = k_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $\lambda = -2$

(ii) for $\lambda = 3$ in (1)

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad R_2 \to R_2 + \tfrac{1}{2}R_1, \quad R_3 \to R_3 + \tfrac{3}{2}R_1$$

$$\simeq \begin{bmatrix} -2 & 1 & 3 \\ 0 & 5/2 & 5/2 \\ 0 & 5/2 & 5/2 \end{bmatrix} \qquad R_3 \to R_3 - R_2$$

$$\simeq \begin{bmatrix} -2 & 1 & 3 \\ 0 & 5/2 & 5/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$-2x + y + 3z = 0$ (3)

$\frac{5}{2}(y + z) = 0$ — (4)

$z = k_2$ be chosen arbitrary.

from (iv) $y = -k_2$.

from (3) $x = k_2$

eigne vector is $x_2 = k_2 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ Colum to 3.

(iii) for $\lambda = 6$ in (1)

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad R_2 \to R_2 + \tfrac{1}{5}R_1, \quad R_3 \to R_3 + \tfrac{3}{5}R_1$$

$$\begin{bmatrix} -5 & 1 & 3 \\ 0 & -4/5 & 8/5 \\ 0 & 8/5 & -16/5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & 1 & 3 \\ 0 & -4/5 & 8/5 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad R_3 \to R_3 + 2R_2$$

let $z = k_3$

$-5x + y + 3z = 0$ — (5)

$-\frac{4}{5} y + \frac{8}{5} z = 0$ — (6)

for (6) $y = 2k_3$

from (5) $x = k_3$

eigne vector is $x_3 = k_3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ eigne vector corresponding to $\lambda = 6$

Modal matrix.

$$P = [x_1 \; x_2 \; x_3] = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{6}\begin{bmatrix} -3 & 0 & 3 \\ 2 & -2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$D = P^{-1} A P$

$= \frac{1}{6}\begin{bmatrix} -3 & 0 & 3 \\ 2 & -2 & 2 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} P \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

Problem:- find Diagonalise the matrix

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$

Characteristic eqn $|A - \lambda I| = 0$

$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 1 \\ -4 & 4 & 3-\lambda \end{vmatrix} = 0$

$(1-\lambda)\left[(2-\lambda)(3-\lambda) - 4\right] - 1\left[(3-\lambda)0 - (-4)\right] + 1\left(4(2-\lambda)\right)$

$(1-\lambda)\left(6 - 2\lambda - 3\lambda + \lambda^2\right) - 4 \; \cancel{} - 4 + (8 - 4\lambda)$

$6 - 2\lambda - 3\lambda + \lambda^2 - 6\lambda + 2\lambda^2 + 3\lambda^2 + \lambda^3 - \cancel{8} + \cancel{8} - 4\lambda$

$\lambda^3 + 6\lambda^2 - 15\lambda + 6 = 0$

$\boxed{\lambda = 1, 2, 3}$

Let $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be the eigne vector.

Satisfying $[A - \lambda I]x = 0$

$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ -4 & 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

Only in LU we can't change row

$R_3 \to R_3 \leftrightarrow R_1$

$\begin{bmatrix} -4 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\begin{bmatrix} -4 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad R_3 \to R_3 - R_2$

$-4x + 4y + 2z = 0$ — ②

$y + z = 0$ — ③

$z = k_1$ (let)

from ③ $y = -k_1$

from ② $x = \frac{-k_1}{2}$

eigne vector $x_1 = k_1\begin{bmatrix} -\frac{1}{2} \\ -1 \\ 1 \end{bmatrix}$ corresponding to $\lambda = 1$

for $\lambda = 2$.

$\begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ -4 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y \\ z \end{bmatrix} = 0$

$R_3 \to R_3 - 4R_1$

$\begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$x_2 = k_2\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \lambda = 2$

$x_3 = k_3\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \lambda = 3$

$-x + y + z = 0$

$z = 0$

Let $y = k$

Modal matrix $P = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ -2 & 0 & 1 \end{bmatrix}$

$$P^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 4 & -3 & -1 \\ -2 & 2 & 1 \end{bmatrix}$$

$$P^{-1}AP = [P^{-1}][A][P]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = D.$$

Qs. Diagonalise the matrix

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Characteristic eq^n $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 2 & 0 \\ 2 & 5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(5-\lambda)(3-\lambda) - 2(5-\lambda)(3-\lambda) = 0$$

$$(2-\lambda)(15-5\lambda-3\lambda+\lambda^2) - 2(15-5\lambda-3\lambda+\lambda^2)$$

$$\cancel{30} - 10\lambda - 6\lambda + 2\lambda^2 - 15\lambda + 5\lambda^2 + 3\lambda^2 - \lambda^3 - \cancel{30} + 10\lambda + 6\lambda - \lambda^2$$

$$-\lambda^3 + 9\lambda^2 - 21\lambda = 0$$

$$\lambda^3 - 9\lambda^2 + 21\lambda = 0$$

$$\lambda(\lambda^2 - 9\lambda + 21) = 0 \qquad \lambda = 1, 3, 6 \text{ are eigne value.}$$

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be eigne vector

satisfying $[A - \lambda I]X = 0$

$$\begin{bmatrix} 2-\lambda & 2 & 0 \\ 2 & 5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \rightarrow (1)$$

(i) for $\lambda = 1$ in (1)

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad R_2 \to R_2 - 2R_1$$

$x + 2y = 0$ — (2)

$2z = 0 \Rightarrow z = 0$

Let $y = k_1$ be chosen arbitrarily.

from (2) $x = -2y_1$

$= -2k_1$

eigne vector is $X_1 = k_1 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ corresponding to $\lambda = 1$

(ii) for $\lambda = 3$ in (1)

$$\begin{bmatrix} -1 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & 2 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad R_2 \to R_2 + 2R_1$$

Let $z = k_2$ be chosen arbitrarily.

$-x + 2y = 0$ — (3)

$6y = 0 \Rightarrow y = 0$

from (3) $x = 2y = 0$

$X_2 = k_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ in the eigne vector corresponding to $\lambda = 3$

(iii) for $\lambda = 6$ in (1)

$$\begin{bmatrix} -4 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -4 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-4x + 2y = 0 \rightarrow (4)$

$-3z = 0 \Rightarrow z = 0$

Let $y = k_3$ be chosen arbitrarily.

from (4) $x = \frac{1}{2} y_1$

$= \frac{1}{2} k_3$

Eigen vector in x_3 : $k_3 \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix} = k_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ corresponding to $\lambda = 6$.

Modal matrix $P = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$

$P^{-1} = \begin{bmatrix} -0.4 & 0.2 & 0 \\ 0 & 0 & 1 \\ 0.4 & 0.8 & 0 \end{bmatrix}$

$D = [P^{-1}][A][P]$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

Que Diagonalise the Matrix

(i) $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Characteristic eqn

$|A - \lambda I| = 0$

4, 1, -1

$\begin{vmatrix} 2-\lambda & 1 & -1 \\ 1 & 1-\lambda & -2 \\ -1 & -2 & 1-\lambda \end{vmatrix}$ —(1)

$[(2-\lambda)(1-\lambda)(1-\lambda)-(4)] - 1(1-\lambda) - (-1)(-2) - 1(-2)(1-\lambda)(-1)$

$(2-\lambda)(1-\lambda-\lambda+\lambda^2-4) -1+\lambda-2+2+1-\lambda$

$2-2\lambda-2\lambda+2\lambda^2-8-\lambda+\lambda^2+\lambda^2-\lambda^3+4\lambda-1+\lambda-\cancel{1}+\cancel{1}+1-\lambda$

$-\lambda^3+4\lambda^2+\lambda-4=0$

$x_1 = 4, -1, 1$

for $\lambda = 1$ in eqn (1)

$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & -2 \\ -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$R_1 \leftrightarrow R_3$

$\begin{bmatrix} -1 & -2 & 0 \\ 1 & 0 & -2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$R_2 \to R_2 + R_1$

$R_3 \to R_3 + 2R_1$

error

$\begin{bmatrix} -1 & -2 & 0 \\ 0 & 0 & -2 \\ 0 & -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

Note:- The Charateristic Polynomial of 3×3 matrix is $\lambda^3 - P_1\lambda^2 + P_2\lambda + P_3 = 0$

where P_1 = Sum of the diagonal elements

P_2 = sum of minors of ______

$P_3 = |A|$

Sol^n y is defined in $(0,12) \Rightarrow (0,2l)$

$l \Rightarrow 6$

$$a_0 = 2\left[\left[f(x)\right]\right] =$$

$$a_n = 2\left[\left[f(x)\cos\frac{n\pi}{l}x\right]\right] = 2\left[\left[y\cos\frac{n\pi}{6}x\right]\right]$$

At $n = 1$. $a_1 = 2\left[\left[y\cos\frac{\pi}{6}x\right]\right] = 2\left[\frac{-24.65}{6}\right] = -8.216$

$n = 2$ $a_2 = 2\left[\left[y\cos\frac{\pi}{3}x\right]\right] = 2\left[\frac{-9.25}{6}\right] = -3.08$

$$b_n = 2\left[\left[y\sin\frac{n\pi}{6}x\right]\right]$$

At $n=1$ $b_1 = 2\left[\left[y\sin\frac{\pi}{6}x\right]\right]$

At $n=2$ $b_2 = 2\left[\left[y\sin\frac{\pi}{3}x\right]\right]$

x	y	$y\cos\frac{\pi}{3}x$	$y\sin\frac{\pi}{6}x$	$y\sin\frac{\pi}{3}x$
0	9.0	9.0		
2	18.2	9.1		
4	24.4	0		
6	27.8	−13.8		
8	27.5	−23.8		
10	22.0	−22		
Σ	128.9	−42.82		

x	y	$y\cos\frac{\pi}{6}x$	$y\cos\frac{\pi}{3}x$	$y\sin\frac{\pi}{6}x$	$y\sin\frac{\pi}{3}x$
0	9.0	9.0	9.0	0	0
2	18.2	9.1			
4	24.4	−12.2			
6	27.8	−27.8			
8	27.5	−13.75			
10	22.0	11			
Σ	128.9	−24.65	−9.25	−5.97	−0.6125

$$y = 64.45 - 8.216\cos\frac{\pi}{6}x - 3.08\cos\frac{\pi}{3}x - 1.99\sin\frac{\pi}{6}x$$

$$- 0.204\sin\frac{\pi}{3}x.$$