UNIT-2. 17/09/18.

Fourier Series.

* A fun^c $f(x)$ is said to be even if $f(-x)=f(x)$

eg: (i) $f(x)=\cos x$

$f(-x)=\cos(-x)$

$=\cos x=f(x)$

(ii) $f(x)=x^2$

$f(-x)=(-x)^2$

$f(-x)=f(x)$.

* A function $f(x)$ is said to be odd if $f(-x)=-f(x)$

eg:- (i) $f(x)=\sin x$

$f(-x)=\sin(-x)$

$f(-x)=-\sin x$ $-f(x)$.

(ii) $f(x)=x^3$

$f(-x)=(-x)^3=-x^3=-f(x)$

* A function $f(x)$ is said to be periodic with period 'p.'

if $(x+p)=f(x)$

eq^n = $\sin x=\sin(2\pi+x)$

cosx/ sin x is Periodic function with Period 2π

* $$\int_{-l}^{l} f(x)dx = \begin{cases} 0, & \text{if } f(x) \text{ is odd} \\ 2\int_0^l f(x)dx & \text{if } f(x) \text{ is even.} \end{cases}$$

* Bernaulli's rules $=\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 \cdots$

where u', u'', u''' --- are successive $diff^n$

v_1, v_2, v_3 ------- integration.

* $\sin n\pi=0$

$\cos n\pi=(-1)^n$.

* consider a real value function which obeys the following condition known as Dirichlet's condition

(i) $f(x)$ is single value and is defined in interval $(a, a+2l)$

(ii) $f(x)$ is Periodic with period $2l$ i.e. $f(x+2l) = f(x)$

(iii) $f(x)$ is continous or it has finite no. of discontinuity in $(a, a+2l)$

(iv) $f(x)$ has at the most finite no. of maxima or minima in $(a, a+2l)$

Also let

$$a_0 = \frac{1}{l}\int_a^{a+2l} f(x)\,dx \quad — (1)$$

$$a_n = \frac{1}{l}\int_a^{a+2l} f(x)\cos\frac{n\pi}{l}x\,dx, \quad n = 1, 2 --- \quad — (2)$$

$$b_n = \frac{1}{l}\int_a^{a+2l} f(x)\sin\frac{n\pi}{l}x\,dx, \quad n = 1, 2 --- \quad \to (3)$$

Then the fourier series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi}{l}x + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi}{l}x \longrightarrow (4)$$

converges to $f(x)$ if $f(x)$ is continous at x

i.e. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi}{l}x + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi}{l}x$ — (5)

eq^n (5) in fourier series of $f(x)$ in $(a, a+2l)$

where $a_0, a_1, a_2 ----- b_1, b_2 ---$

are fourier co-efficients defined by Euler's formula (1) (2) & (3)

case (i) fourier series in $(0, 2\pi)$

$\left(a = 0 \quad a+2l = 2\pi \quad l = \pi\right)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n\sin nx$$

where $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx. \quad n = 1, 2, 3.$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx \quad n = 1, 2, 3 ----$$

Que obtain fourier series expansion for the function $f(x) = x$ in $(0, 2\pi)$

Sol^n fourier coeff^n are $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$

$$= \frac{1}{\pi}\int_0^{2\pi} x\,dx = \frac{1}{\pi}\left.\frac{x^2}{2}\right|_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[4\pi^2 - 0\right]$$

$$= 2\pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\cos nx\,dx$$

$$= \frac{1}{\pi}\left[x\cancel{\left(\frac{\sin nx}{n}\right)}^{\to 0} - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{\cos 2n\pi - 1}{n^2}\right] \qquad \cos 2n\pi = (-1)^{2n} = 1$$

$$= \frac{1}{\pi}\left[\frac{1-1}{n^2}\right] = 0, \quad n = 1, 2 ----$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\sin nx\,dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - (1)\cancel{\left(-\frac{\sin nx}{n^2}\right)}^{0}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}(-)\left[2\pi\left(\frac{\cos n 2\pi x}{n}\right)\right.$$

$$= +\frac{1}{\pi}\left[-2\pi\left(\frac{1}{n}\right)\right]$$

$$= -\frac{2}{n}, \quad n = 1, 2 ---$$

Fourier Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{2\pi}{\cancel{2}} + 0 + \sum_{n=1}^{\infty} -\frac{2}{n} \sin nx$$

$$x = \pi - 2 \sum_{n=1}^{\infty} \frac{\sin nx}{n}$$

Que: Obtained fourier series of $f(x) = x^2$ in $(0, 2\pi)$ and. deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

Soln: fourier co-efficient are:

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\,dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} x^2\,dx$$

$$= \frac{1}{\pi} \cdot \frac{x^3}{3} \Big|_0^{2\pi} = \frac{8\pi^2}{3}.$$

doubt

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx\,dx.$$

$$= \frac{1}{\pi} \int_0^{4\pi} x^2 \cos nx\,dx$$

$$= \frac{1}{\pi} \left[x^2 \left(\cancel{\frac{\sin nx}{n}}\right)^{0} - (2x)\left(-\frac{\cos nx}{n^2}\right) + (2)\left(\cancel{-\frac{\sin nx}{n^3}}\right)^{0} \right]_0^{2\pi}$$

$$\frac{1}{\pi} \left[2x \left(\frac{\cos nx}{n^2}\right) \right]_0^{2\pi}$$

$$= \frac{1}{\cancel{\pi}} \left[\frac{4\cancel{\pi}}{n^2} \right] = \frac{4}{n^2} \quad n = 1, 2 \dots$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} x^2 \sin nx\,dx$$

$$= \frac{1}{\pi} \left[x^2 \left(-\frac{\cos nx}{n}\right) - (2x)\left(\cancel{-\frac{\sin nx}{n^2}}\right)^{0} + (2)\left(\frac{\cos nx}{n^2}\right) \right]$$

$$\frac{1}{\cancel{\pi}} \left[-\frac{4\pi^{\cancel{2}} \cos 2n\pi}{n} \right]$$

$$= -\frac{4\pi}{n} \quad n = 1, 2 \dots$$

$$x^2 = \frac{\cancel{8}^{4}\pi^2}{3 \times \cancel{2}} + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos nx - \sum_{n=1}^{\infty} -\frac{4\pi}{n} \sin nx$$

at $x = 0$

$$0 = \frac{4\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\sum \frac{1}{n^2} = -\frac{\pi^2}{3} \rightarrow (1)$$

Put $x = 2\pi$: $4\pi^2 = \frac{4\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\pi^2 - \frac{\pi^2}{3} = \sum \frac{1}{n^2}$$

$$\frac{2\pi^2}{3} = \sum \frac{1}{n^2} \rightarrow (2)$$

Adding (1) + (2)

$$-\frac{\pi^2}{3} + \frac{2\pi^2}{3} = 2 \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\frac{\pi^2}{3 \times 2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$

Note:- $(0, 2\pi)$

If $f(2\pi - x) = f(x)$

then $f(2\pi - x) = +f(x)$

then $f(x)$ is even.

If $f(2\pi - x) = -f(x)$

then $f(x)$ is odd.

Note:-2.

odd × even :- odd

even × even :- even.

odd × odd = even.

Note:-3: ~~If $f(x)$ is even~~

$\frac{1}{\pi}$

Note:- 3.

If $f(x)$ is even

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$
$$= \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$
$$= \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx \quad x = 1, 2, \ldots$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$
$$= 0, \; n = 1, 2, \ldots$$

Note 4:- If $f(x)$ is odd

$$a_0 = 0, \; a_n = 0 \quad n = 1, 2 \ldots$$
$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx \quad n = 1, 2, 3 \ldots$$

Que.

Obtain fourier series of $f(x) = x(2\pi - x)$ in $(0, 2\pi)$

Sol^n If $(2\pi - x) = (2\pi - x)(2\pi - (2\pi - x))$
$$= (2\pi - x)x = f(x)$$

$f(x)$ is even in $(0, 2\pi)$

fourier co-efficient are.

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$
$$= \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$
$$= \frac{2}{\pi}\int_0^{\pi} x(2\pi - x)\,dx$$
$$= \frac{2}{\pi}\left[\frac{2\pi x^2}{2} - \frac{x^3}{3}\right]_0^{\pi} = \frac{4\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$
$$= \frac{2}{\pi}\int_0^{\pi} (2\pi x - x^2)\cos nx\,dx$$
$$= \frac{2}{\pi}\left[\cancel{(2\pi x - x^2)\left(\frac{\sin nx}{n}\right)} - (2\pi - 2x)\left(\frac{-\cos nx}{n^2}\right) \cancel{+ (-2)\left(\frac{-\sin nx}{n^3}\right)}\right]_0^{\pi}$$
$$= \frac{2}{\pi}\left[0 - \frac{2\pi\cos n\pi}{n^2}\right]$$
$$= \frac{-4}{n^2} \quad n = 1, 2 \ldots$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$
$$= \frac{1}{\pi}$$

f. ser $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n\sin nx$
$$= \frac{2\pi^2}{3} - 4\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}$$

Que (2)

$f(x) = \frac{\pi - x}{2}$ in $(0, 2\pi)$

and deduce that $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} \cdots = \pi/4$

Sol^n $f(x) = \frac{\pi - x}{2}$

$$f(2\pi - x) = \left(\frac{\pi - (2\pi - x)}{2}\right) = \frac{-\pi + x}{2} = -\left[\frac{\pi - x}{2}\right]$$
$$= -f(x)$$

$f(x)$ is odd in $(0, 2\pi)$

fourier Coefficient are

$a_0 = 0, \ a_n = 0, \ n = 1, 2$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx \, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \frac{\pi - x}{2}\sin nx \, dx$$

$$= \frac{1}{\pi}\left[(\pi-x)\left(\frac{-\cos nx}{n}\right) - (-1)\cancel{\left(\frac{-\sin nx}{n^2}\right)}\right]$$

$$= \frac{1}{\pi}\left[0 - \left(\frac{-\pi\cos 0}{n}\right)\right]$$

$$\Rightarrow \frac{\pi}{n\pi} = \frac{1}{n}, \ n = 1, 2 \ldots$$

F.S in

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\frac{\pi - x}{2} = \sum_{n=1}^{\infty} \frac{\sin nx}{n}$$

Put $x = \pi/2$

$$\Rightarrow \frac{\pi - \pi/2}{2} = \sum_{n=1}^{\infty} \frac{\sin n\pi/2}{n}$$

$$\pi/4 = \frac{\sin \pi/2}{1} + \frac{\sin 2\cdot\frac{\pi}{2}}{2} + \frac{\sin \frac{3\pi}{2}}{3} + \frac{\sin\frac{4\pi}{2}}{4} + \frac{\sin\frac{5\pi}{2}}{5}$$

$$\pi/4 = \frac{1}{1} + 0 - \frac{1}{3} + 0 + \frac{1}{5} \ldots$$

Que. $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in $(0, 2\pi)$

and deduce that $\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots \frac{\pi^2}{12}$$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \ldots \frac{\pi^2}{6}$$

$$f(x) = \left(\frac{\pi - x}{2}\right)^2$$

$$f(2\pi - x) = \left(\frac{\pi - (2\pi - x)}{2}\right)^2 = \left(\frac{-\pi + x}{2}\right)^2 = \left(\frac{\pi - x}{2}\right)^2 = f(x)$$

$f(x)$ is even

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \frac{(\pi - x)^2}{4}\,dx$$

$$\frac{1}{2\pi}\left.\frac{(\pi - x)^3}{-3}\right|_0^{\pi} = \frac{\pi^2}{6}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx \, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \frac{(\pi - x)^2}{4}\cos nx \, dx$$

$$: \frac{1}{2\pi}\left[(\pi-x)^2\cancel{\left(\frac{\sin nx}{n}\right)} - [-2(\pi - x)]\left[\frac{-\cos nx}{n^2}\right] + (2)\left[\frac{-\sin nx}{n^3}\right]\right]_0^{\pi}$$

$$= \frac{1}{2\pi}\left[0 + \frac{2\pi}{n^2}\right] = \frac{1}{n^2} \quad n = 1, 2 \ldots$$

$b_n = 0, \ n = 1, 2 \ldots$

F.S.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\frac{(\pi - x)^2}{4} = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2} \rightarrow (1)$$

Qus. $f(x) = \begin{cases} x = f_1(x) & 0 \le x \le \pi \\ 2\pi - x : f_2(x) & \pi \le x \le 2\pi \end{cases}$ | for even find a_0, a_n

and deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

Sol^n In $(0,\pi)$ $f_1(x) = x$

$f_1(2\pi - x) = 2\pi - x$ $\quad (\pi, 2\pi)$

$= f_2(x)$

In $(\pi, 2\pi)$ $f_2(x) = 2\pi - x$

$f_2(2\pi - x) = 2\pi - (2\pi - x)$

$= x = f_1(x)$

$f(x) =$ even.

fourier coefficient.

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\,dx$$

$$\frac{2}{\pi}\frac{x^2}{2}\Big|_0^{\pi} = \frac{2\pi^2}{2\pi} = \pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$= \frac{2}{\pi}\Big[x\cancel{\left(\frac{\sin nx}{n}\right)}^{\to 0} - (1)\left(-\frac{\cos nx}{n^2}\right)\Big]_0^{\pi}$$

$$= \frac{2}{\pi}\,\frac{\cos n\pi - \cos 0}{n^2}$$

$$= \frac{2}{\pi}\,\frac{(-1)^n - 1}{n^2} \quad n = 1, 2 \cdots$$

$b_n = 0, \; n = 1, 2 \cdots$

$$f.s = f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi}\,\frac{(-1)^n - 1}{n^2}\cos nx + 0$$

Put $x = 0$

$$f(0) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2}$$

in $(0,\pi)$ $f(x) = x$

$f(0) = 0$

$$0 = \frac{\pi}{2} + \frac{2}{\pi}\Big[\frac{(-1)-1}{1^2} + \frac{(-1)^2-1}{2^2} + \frac{(-1)^3-1}{3^2} + \frac{(-1)^4-1}{4^2} + \cdots$$

$$\frac{\pi}{2} = -\frac{2}{\pi}\Big[\left(\frac{-1-1}{1^2}\right) + \left(\frac{1-1}{2^2}\right) + \left(\frac{-1-1}{3^2}\right) + \left(\frac{1-1}{4^2}\right) \cdots$$

$$\frac{\pi^2}{4} = -\Big[\frac{-2}{1^2} + 0 + \frac{(-2)}{3^2} + 0 - - -$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} - - \quad \text{Proof}$$

Qus obtained fourier series of $f(x) = \begin{cases} \pi - x : f_1 & 0 \le x \le \pi \\ x - \pi : f_2 & \pi \le x \le 2\pi \end{cases}$

Sol^n In $(0-\pi)$

$f_1(x) = \pi - x$

$f_1(2\pi - x) = \pi - (2\pi - x)$

$f_1(2\pi - x) = x - \pi = f_2(x)$

In $(\pi, 2\pi)$

$f_2(x) = x - \pi$

$f_2(2\pi - x) = 2\pi - x - \pi$

$= \pi - x = f_1(x)$

$\therefore$ $f(x)$ is even in $(0, 2\pi)$

fourier coefficient

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} (\pi - x)\,dx$$

$$= \frac{2}{\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{\pi}$$

$$= \pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx.$$

$$= \frac{2}{\pi}\int_0^{\pi} (\pi - x)\cos nx\,dx$$

$$= \frac{2}{\pi}\left[\cancel{(\pi - x)\left(\frac{\sin nx}{n}\right)}^{0} - (-1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\frac{(\cos n\pi - \cos(0))}{n^2}\right]$$

$$= \frac{2}{\pi}\left[\frac{1 - (-1)^n}{n^2}\right]$$

$b_n = 0,\ n = 1, 2, \ldots$

f.s is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$f(x) = \frac{\pi}{2} + \frac{2}{\pi}\left[\frac{1-(-1)^n}{n^2}\right]\cos nx$$

Que. An alternating current after passing through a rectifier has the form

$$I(x) = \begin{cases} I_0 \sin x & 0 \le x \le \pi \\ 0 & \pi \le x \le 2\pi \end{cases}$$ where I_0 is

the maximum current. Express 'I' as a fourier series.

Sol^n. $I(x)$ is defined in $(0, 2\pi)$

fourier coefficient are: $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$

$$= \frac{1}{\pi}\left[\int_0^{\pi} I_0 \sin x\,dx + \int_{\pi}^{2\pi} 0\,dx\right]$$

$$= \frac{I_0}{\pi}(-\cos x)_0^{\pi}$$

$$= \frac{I_0}{\pi}(+1 - (-1))$$

$$= \frac{2I_0}{\pi}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx.$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} I_0 \sin x \cos nx\,dx + \int_{\pi}^{2\pi} 0 \cdot \cos nx\,dx\right]$$

$$= \frac{I_0}{2\pi}\int_0^{\pi}\left[\sin(n+1)x - \sin(n-1)x\right]dx. \quad \text{—(1)}$$

$$= \frac{I_0}{2\pi}\left[-\frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right]_0^{\pi}$$

$$= \frac{I_0}{2\pi}\left[-\frac{(-1)^{n+1} + 1}{n+1} + \frac{(-1)^{n-1} - 1}{n-1}\right]$$

$$(-1)^{n+1} = (-1)^{n-1} = (-1)^n$$

$$= \frac{I_0}{2\pi}\left[\frac{1 + (-1)^n}{n+1} + \frac{(-1)^n - 1}{n-1}\right]$$

$$\frac{I_0}{2\pi}\left\{\frac{(n-1)[(-1)^n + 1] - (n+1)[(-1)^n + 1]}{(n+1)(n-1)}\right.$$

$$= \frac{I_0}{2\pi} \frac{((-1)^n+1)(-2)}{(n+1)(n-1)} \quad n = 2, 3, ---$$

At $n=1$ in (1)

$$a_1 = \frac{I_0}{2\pi} \int_0^{\pi} \sin 2x \, dx$$

$$= \frac{I_0}{2\pi} \left[-\frac{\cos 2x}{2} \right]_0^{\pi} = 0$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} I_0 \sin x \sin nx \, dx + \int_0^{2\pi} 0 \cdot \sin nx \, dx \right]$$

$$\frac{I_0}{\pi \times 2} \int_0^{\pi} \cos(n-1)x - \cos(n-1)x] \, dx \quad -(2)$$

$$\frac{I_0}{2\pi} \left[\frac{\sin(n-1)x}{n-1} - \frac{\sin(n+1)x}{n+1} \right]_0^{\pi}$$

$$= 0, \quad n = 2, 3 \; -----$$

at $n=1$ in (2)

$$b_1 = \frac{I_0}{2\pi} \int_0^{\pi} (1 - \cos 2x) \, dx$$

$$\frac{I_0}{2\pi} \left[x - \frac{\sin 2x}{2} \right]_0^{\pi}$$

$$= \frac{I_0}{2}$$

fourier series.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$I_0(x) = \frac{I_0}{\pi} + \left(\frac{I_0}{\pi} \frac{-((-1)^n+1)}{(n+1)(n-1)} \right) \cos nx + \frac{I_0}{2} \sin x =$$

Que find f.s for $f(x) = x \sin x \quad 0 \le x \le 2\pi$

26/09/18

Case (2) F.S in $(0, 2l)$

Fourier co-efficient

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi}{l}x\,dx$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi}{l}x\,dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi}{l}x + \sum_{n=1}^{\infty} \sin\frac{n\pi}{l}x$$

Note:- $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^{a} f(x)\,dx & \text{if } f(2a-x) = f(x) \text{ (even)} \\ 0 & \text{if } f(2a-x) = -f(x) \text{ odd} \end{cases}$

$\Longrightarrow$

* Note (2) $f(2l-x) = f(x)$ $f(x)$ is even $\Rightarrow b_n = 0$

$f(2l-x) = -f(x)$ $f(x)$ is odd $\Rightarrow a_0 = a_n = 0$

$\Longrightarrow$

Que:-

Obtain f.s of the function

$f(x) = e^{-x}$ in $0 < x < 2$

$(0, 2l) \Rightarrow (0, 2)$

$2l = 2$ $l = 1$

Fourier co-eff are.

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$= \frac{1}{1}\int_0^{2} e^{-x}dx = \frac{e^{-x}}{-1}\Big|_0^{2}$$

$$= -(e^{-2} - 1)$$

$$= 1 - e^{-2}$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi}{l}x\,dx$$

$$= \frac{1}{1}\int_0^{2} e^{-x}\cos n\pi x\,dx$$

Note: $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

$$= \frac{e^{-x}}{n^2\pi^2+1}\left(-\cos n\pi x + n\pi\sin n\pi x\right)\Big|_0^{2}$$

$$= \frac{e^{-2}}{n^2\pi^2+1}\left(-\cos 2n\pi + n\pi\sin 2\pi n\right) - (-1)\Big]$$

$$= \frac{1}{n^2\pi^2+1}(1 - e^{-2}) \qquad n = 1, 2 ----$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi}{l}x\,dx$$

$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

$$= \frac{1}{1}\int_0^{2} e^{-x}\sin n\pi x\,dx$$

$$= \frac{e^{-1}}{1+n^2\pi^2}\left[-\cancel{\sin n\pi x} - n\pi\cos n\pi x\right]_0^{2}$$

$$= \frac{e^{-2}}{1+n^2\pi^2}\left[(-n\pi\cos 2n\pi) - (-n\pi)\right]$$

$$= \frac{n\pi}{n^2\pi^2+1}(1 - e^{-2}) \qquad n = 1, 2, -----$$

f.s of $f(x)$ is

$$e^{-x} = \frac{1-e^{-2}}{2} + 1-e^{-2}\sum_{n=1}^{\infty}\frac{\cos n\pi x}{1+n^2\pi^2} + \pi(1-e^{-2})\sum_{n=1}^{\infty}\frac{n\pi\sin n\pi x}{n^2\pi^2+1}$$

Que Express $f(x) = \begin{cases} 2-x & \text{in } 0 \le x \le 4 \\ x-6 & \text{in } 4 \le x \le 8 \end{cases}$ as f. series

and deduce that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

Soln. let $f_1 = 2-x \quad f_2 = x-6$

In $(0,4)$. $f(x) = 2-x$

$f_1(8-x) = 2-(8-x)$

$= x-6 = f_2(x)$

In $(4,8)$ $f_2(4,8)$ $f_2(x) = x-6$

$f_2(8-x) = 8-x-6$

$= 2-x = f_1(x)$

$f(x)$ is an even func

f.s are.

$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$

$= \frac{2}{l}\int_0^{l} f(x)\,dx$ $f(x)$ is even $\left[(0,2l) \Rightarrow (0,8)\right.$ $l \Rightarrow 4$

$= \frac{2}{4}\int_0^{4} (2-x)\,dx$

$= \frac{1}{2}\left[2x - \frac{x^2}{2}\right]_0^4$

$= 0$

$a_n = \frac{1}{l}\int_0^{2l} f(x) \cos\frac{n\pi}{l}x\,dx$

$= \frac{2}{4}\int_0^4 f(x) \cos\frac{n\pi}{4}x\,dx$

$= \frac{1}{2}\int_0^4 (2-x)\cancel{\left(\frac{\sin n\pi/4\, x}{n\pi/4}\right)}^{0} - (-1)\left(\frac{-\cos n\pi/4\, x}{(n\pi/4)^2}\right)\Big|_0^4$

$= -\frac{16}{2}\left[\frac{\cos n\pi - 1}{n^2\pi^2}\right]$

$= \frac{8}{\pi^2}\left[\frac{1-(-1)^n}{n^2}\right]$, $n = 1, 2 \ldots$

$b_n = \frac{1}{l}\int_0^{2l} f(x) \sin\frac{n\pi}{l}x\,dx$

$= 0 \;\because f(x)\sin\frac{n\pi}{l}x$ is odd.

$n = 1, 2 \ldots$

fourier series:

$f(x) = \frac{8}{\pi^2}\sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^2} \rightarrow (1)$

Put $x=0$ in (1) $\left[\begin{array}{l} f(x) = 2-x \\ f(0) = 2 \end{array}\right.$

$\frac{\pi^2}{8} f(0) = \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^2}$

$\frac{\cancel{2}\pi^2}{\cancel{8}4} = \frac{1-(-1)^1}{1^2} + \frac{1-(-1)^2}{2^2} + \frac{1-(-1)^3}{3^2} + \frac{1-(-1)^4}{4^2} + \cdots$

$= \frac{2}{1^2} + \frac{1-1}{2^2} + \frac{2}{3^2} + \frac{1-1}{4^2} + \cdots$

$\frac{\pi^2}{4} = 2\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$

$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

Que obtain fourier series of the function

$f(x) = \begin{cases} x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$

Soln Here $f(x)$ is defined in $(0,2)$

$\left[(0,2l) \Rightarrow (0,2)\right]$

$l = 1$

fourier coefficient.

~~$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$~~

~~$\frac{2}{l}\int_0^{l} f(x)\,dx$~~

~~$\frac{2}{1}[x]dx\Big|_0^1 = \frac{x \cdot x^2}{2}\Big|_0^1$~~

~~$a_0 = 1$~~

$= \int_0^1 f(x)dx + \int_1^2 f(x)dx$

$\Rightarrow \int_0^1 x\,dx + \int_1^2 0 \cdot dx$

$\frac{x^2}{2}\Big|_0^1 \Rightarrow 1/2$

$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi}{l}x\,dx$

$= \frac{1}{1}\int_0^2 f(x)\cos n\pi x\,dx$

$= \int_0^1 x\cos n\pi x\,dx + \int_1^2 (0)\cos n\pi x\,dx$

$= x\cancel{\left(\frac{\sin n\pi x}{n\pi}\right)}^{0} - (1)\left(\frac{-\cos n\pi x}{(n\pi)^2}\right)\Big|_0^1 + 0$

$= \cancel{\frac{\cos n\pi x}{(n\pi)^2}} \quad \frac{\cos n\pi - \cos 0}{(n\pi)^2} = \frac{(-1)^n - 1}{n^2\pi^2} \quad n = 1, 2 \ldots$

$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin n\frac{\pi}{l}x\,dx$

$= \frac{1}{1}\int_0^2 f(x)\sin \pi x\,dx$

$= \int_0^1 x\sin\pi x\,dx + \int_1^2 (0)\sin n\pi x\,dx$

$= x\left(\frac{-\cos n\pi x}{n\pi}\right) - (1)\cancel{\left(\frac{-\sin n\pi x}{n^2\pi^2}\right)}^{0}\Big|_0^1$

$= \frac{-\cos n\pi}{n\pi} \quad \frac{-(-1)^n}{n\pi}$

$= \frac{(-1)^{n}}{n\pi} \Rightarrow \frac{(-1)^{n+1}}{n\pi}$

f. s is.

$f(x) = \frac{1}{4} - \sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2\pi^2}\cos n\pi x + \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n\pi}\sin n\pi x$

Qu find f.s of

$f(x) = \frac{\pi - x}{2}$ in $(0, 2)$

Case III fourier series in $(-\pi, \pi)$

$(-\pi, \pi) \Rightarrow (a, a+2l)$

$a = -\pi \quad l = \pi$

fourier co-efficient are

$a_0 = \frac{1}{l}\int_a^{a+2l} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$

$a_n = \frac{1}{l}\int_a^{a+2l} f(x)\cos\frac{n\pi}{l}x\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \quad n = 1, 2, \ldots$

$b_n = \frac{1}{l}\int_a^{a+2l} f(x)\sin\frac{n\pi}{l}x\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \quad n = 1, 2 \ldots$

f. series of $f(x)$ in $(-\pi, \pi)$ is

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n\sin nx$

Note (i) To check even or odd in $(-\pi, \pi)$

If $f(-x) = -f(x)$ then $f(x)$ is odd

$a_0 = 0$

$a_n = 0 \quad n = 1, 2 \ldots$

$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx, \; n = 1, 2 \ldots$

f. series of $f(x)$ in $(-\pi, \pi)$ is

$f(x) = \sum_{n=1}^{\infty} b_n\sin nx$

* If $f(-x) = f(x)$ then $f(x)$ is even

$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$

$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx \quad n = 1, 2 \ldots$

$b_n = 0$

f. series of $f(x)$ in $(-\pi, \pi)$ is

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx$

Qu. Obtain f. series of the function $f(x)=|x|$ in $(-\pi,\pi)$ & deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}---$

Sol^n. given fun^c $f(x)=|x|$

$f(-x)=|-x|=|x|=f(x)$

$\therefore f(x)$ is even function

- fourier coefficient are

$$f(x)=|x| = \begin{cases} x & x\geq 0 \\ -x & x<0 \end{cases}$$

$$a_0=\frac{2}{\pi}\int_0^{\pi} f(x)dx$$

$$\frac{2}{\pi}\int_0^{\pi} x\,dx$$

$$=\frac{2}{\pi}\frac{x^2}{2}\Big|_0^{\pi}=\frac{\cancel{2}}{\pi}\cdot\frac{\pi^{\cancel{2}}}{\cancel{2}}=\pi$$

$$a_n=\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$=\frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$\frac{2}{\pi}\left[x\cancel{\frac{(\sin nx)}{n}}^{0}-(1)\frac{(-\cos nx)}{n^2}\right]_0^{\pi}$$

$$\frac{2}{\pi}\left(\frac{\cos nx}{n^2}\right)\Big|_0^{\pi}=\frac{2}{\pi}\left[\frac{\cos n\pi-\cos 0}{n^2}\right]$$

$$=\frac{2}{\pi}\left[\frac{(-1)^n-1}{n^2}\right)\quad n=1,2$$

$b_n=0 \quad n=1,2 ---$

f.s of $f(x)$ in $(-\pi,\pi)$ is

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx$$

$$|x|=\pi/2+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2}\cos nx \quad -①$$

Put $x=0$ in eq^n ①

$$0=\pi/2+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2}$$

$$(-\pi/2)(\pi/2)=\frac{-1-1}{1^2}+\frac{1-1}{2^2}+\frac{-1-1}{3^2}+\frac{1-1}{4^2}---$$

$$\frac{-\pi^2}{4}=-2\left[\frac{1}{1^2}+\frac{1}{3^2}-----\right.$$

$$\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}-----$$

Qu. Obtain f. series of $f(x)=\sin mx$ where m is neither '0' not an integer

Sol^n $f(x)=\sin mx$

$f(-x)=\sin m(-x)=-\sin mx$

$f(x)=-f(x)$

$\because f(x)$ is odd in $(-\pi,\pi)$

$a_0=0 \quad a_n=0$

$$b_n=\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$\frac{2}{\pi}\int_0^{\pi}\sin mx\cdot\sin nx$$

$$\frac{\cancel{2}}{\pi}\cdot\frac{1}{\cancel{2}}\int_0^{\pi}\left[\cos(m-n)x-\cos(m+n)x\right]$$

$$=\frac{1}{\pi}\left[\frac{\sin(m-n)x}{m-n}-\frac{\sin(m+n)x}{m+n}\right]_0^{\pi}$$

$$\frac{1}{\pi}\left[\frac{\sin(m-n)\pi}{m-n}-\frac{\sin(m+n)\pi}{m+n}\right]$$

$$\frac{1}{\pi}\left[\frac{\sin m\pi\cos n\pi-\cancel{\cos m\pi\cdot\sin n\pi}^{0}}{m-n}\right.$$

$$\left.-\frac{\sin m\pi\cos n\pi+\cancel{\cos m\pi\sin n\pi}^{0}}{m+n}\right]$$

$$\frac{(-1)^n\sin m\pi}{\pi}\left[\frac{\cancel{m}+n-\cancel{m}+n}{m^2-n^2}\right]$$

$$\frac{2n(-1)^n\sin m\pi}{\pi(m^2-n^2)},\ n=1,2---$$

f. series $f(x)=\sum_{n=1}^{\infty}b_n\sin nx$

$$=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{n(-1)^n\sin m\pi\sin nx}{m^2-n^2}$$

Qu Obtain fourier series of function $f(x)=x\sin x$ in $(-\pi,\pi)$ & deduce that $\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+---\frac{\pi-2}{4}$

Sol^n $f(x)=x\sin x$

$f(-x)=(-x)\sin(-x)$

$=(-x)(-\sin x)=x\sin x=f(x)$

$\therefore f(x)$ is even

$$a_0=\frac{2}{\pi}\int_0^{\pi}x\sin x\,dx$$

$$\frac{2}{\pi}\left[x(-\cos x)-(1)(-\sin x)\right]_0^{\pi}$$

$\cos\pi=-1$

$$\frac{2}{\pi}\left[\pi\left[(-\cos\pi)+\cos 0\right]\right.$$

$$=\frac{2}{\pi}\left[\pi(+\cos\pi)\right)$$

$=2$

$$a_n=\frac{2}{\pi}\int_0^{\pi}f(x)\cos nx\,dx$$

$$\frac{2}{\pi}\int_0^{\pi}x\sin x\cos nx\,dx$$

$$=\frac{2}{\pi}\cdot\frac{1}{2}\int_0 x\left[\sin(n+1)x-\sin(n-1)x\right]dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right) - (1)\left(\cancel{-\frac{\sin(n+1)x}{(n+1)^2}} + \cancel{\frac{\sin(n-1)x}{(n-1)^2}}\right)\right]_0^\pi$$

$$= \frac{1}{\pi}\left[\pi\left(-\frac{\cos(n+1)\pi}{n+1}\right) + \frac{\cos(n-1)\pi}{n-1}\right]$$

$$= \frac{-(-1)^{n+1}}{n+1} + \frac{(-1)^{n-1}}{n-1}$$

$$= (-1)^n\left[\frac{1}{n+1} - \frac{1}{n-1}\right] = (-1)^n\left[\frac{n-1-n-1}{(n+1)(n-1)}\right]$$

$$= (-1)^n\left[\frac{-2}{(n+1)(n-1)}\right] \quad n = 2, 3 \ldots$$

At $n = 1$ in (1)

$$a_1 = \frac{1}{\pi}\int_0^\pi x \sin 2x \, dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos 2x}{2}\right) - (1)\left(-\frac{\sin 2x}{4}\right)\right]_0^\pi$$

$$= \frac{1}{\pi}\pi\left(-\frac{1}{2}\right) = -\frac{1}{2}$$

Fourier series $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

$$x \sin x = 1 + a_1 \cos x + \sum_{n=2}^{\infty} a_n \cos nx$$

$$= 1 - \frac{1}{2}\cos x \sum_{n=2}^{\infty} \frac{2(-1)^n}{(n+1)(n-1)} \quad -(2)$$

Put $x = \pi/2$ in (2)

$$\frac{\pi}{2} = 1 - 0 - \sum_{n=2}^{\infty} \frac{2(-1)^n \cos n\pi/2}{(n+1)(n-1)}$$

$$\frac{\pi}{2} - 1 = 2\left[\frac{-1\cos\pi}{1.3} - \frac{\cos 3\pi/2}{4.2} + \frac{\cos 4\pi}{3.5} \ldots\right.$$

$$\frac{\pi - 2}{4} = \frac{1}{1.3} - \frac{1}{3.5} \ldots$$

Que. $f(x) = |\cos x|$ in $(-\pi, \pi)$ $\quad \cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

Sol^n $f(-x) = |\cos(-x)|$

$$= |\cos(x)| = f(x)$$

$f(x)$ is even in $(-\pi, \pi)$

$$f.s = \quad a_0 = \frac{2}{\pi}\int_0^\pi f(x)dx = \frac{2}{\pi}\int_0^\pi |\cos x| dx$$

$$\left\{\because |\cos x| = \begin{cases} \cos x & 0 < x < \pi/2 \\ -\cos x & \pi/2 < x < \pi \end{cases}\right.$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \cos x\, dx + \int_{\pi/2}^{\pi} (-\cos x) dx\right]$$

$$= \frac{2}{\pi}\left[\sin x\Big|_0^{\pi/2} - \sin x\Big|_{\pi/2}^{\pi}\right]$$

$$= \frac{2}{\pi}\left[(1-0) - (0-1)\right] = \frac{4}{\pi}$$

$$a_n = \frac{2}{\pi}\int_0^\pi f(x) \cos nx\, dx$$

$$= \frac{2}{\pi}\int_0^\pi |\cos x| \cos nx\, dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \cos x \cos nx\, dx + \int_{\pi/2}^{\pi} (-\cos x)(\cos nx) dx\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{2}\int_0^{\pi/2} \cos(n+1)x + \cos(n-1)x\right]dx$$

$$-\frac{1}{2}\int_{\pi/2}^{\pi}\left[\cos(n+1)x + \cos(n-1)x\right]dx\Bigg] \quad -(1)$$

$$a_n = \frac{1}{\pi}\left\{\left[\frac{\sin(n+1)x}{n+1} + \frac{\sin(n-1)x}{n-1}\right]_0^{\pi/2}\right.$$

$$\left. - \left[\frac{\sin(n+1)x}{n+1} + \frac{\sin(n-1)x}{n-1}\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\left[\frac{\sin(n+1)\pi/2}{n+1} + \frac{\sin(n-1)\pi/2}{n-1}\right) - 0\right]$$

$$-\left[0 - \frac{\sin(n+1)\pi/2}{n+1} - \frac{\sin(n-1)\pi/2}{n-1}\right]$$

$$= \frac{2}{\pi}\left[\frac{(\sin n\pi/2 \cos \pi/2 + \sin \pi/2 \cos n\pi/2)}{n+1} + \frac{\sin n\pi/2 \cos \pi/2 - \cos n\pi/2 \sin \pi/2}{n-1}\right]$$

$$= \frac{2}{\pi}\cos n\pi/2 \left[\frac{1}{n+1} - \frac{1}{n-1}\right]$$

$$= \frac{2}{\pi}\cos n\pi/2 \left[\frac{n-1-n-1}{(n+1)(n-1)}\right]$$

$$= \frac{-4\cos n\pi/2}{\pi(n+1)(n-1)} \quad n = 2, 3 \text{-----}$$

at $n=1$ in (1)

$$a_1 = \frac{1}{\pi}\left[\int_0^{\pi/2}(\cos 2x + 1)\,dx - \int_{\pi/2}^{\pi}(\cos 2x + 1)\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{\sin 2x}{2} + x\right)_0^{\pi/2} - \left(\frac{\sin 2x}{2} + x\right)_{\pi/2}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[(0 + \pi/2) - (0 + \pi - \pi/2)\right]$$

$$= \frac{1}{\pi}\left[\pi/2 - \pi + \pi/2\right] = 0$$

$b_n = 0, \; n = 1, 2, 3 \text{---}$

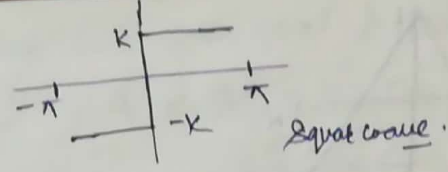
f.s of $f(x)$ in $(-\pi, \pi)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$= \frac{4}{2\pi} + a_1 \cos x + \sum_{n=2}^{\infty} a_n \cos nx.$$

$$|\cos x| = \frac{2}{\pi} - \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{\cos n\pi/2}{n^2-1}\cos nx.$$

Ques: Obtain fourier series of rectangular or square wave function $f(x) = \begin{cases} -K & \text{if } -\pi < x < 0 \\ K & \text{if } 0 < x < \pi \end{cases}$



Squar wave.

Put $f_1(x) = -k \quad f_2(x) = k$

In $(-\pi, 0)$ $\quad f_1(x) = -k$

$$f_1(-x) = -k = -f_2(x)$$

In $(0, \pi)$

$$f_2(x) = k$$
$$f_2(-x) = k$$
$$= -f_1(x)$$

$f(x)$ is odd function in $(-\pi, \pi)$

$a_0 = 0, \; a_n = 0, \; n = 1, 2 \text{----}$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} k \sin nx\,dx$$

$$= \frac{2k}{\pi}\left[\frac{-\cos nx}{n}\right]_0^{\pi}$$

$$= \frac{2k}{\pi n}\left[-(-1)^n + 1\right] \quad n = 1, 2, \text{---}$$

f.s of squale wave

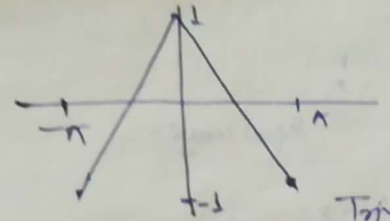
$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{2\pi}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n}\sin nx.$$

Ques. Obtain f. series for triangular wave form of

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi < x < 0 \\ 1 - \frac{2x}{\pi} & 0 < x < \pi \end{cases}$$

Sol^n



Trigulat wave.

Put $f_1(x) = 1 + \frac{2x}{\pi}$ $\quad f_2(x) = 1 - \frac{2x}{\pi}$

In $(-\pi, 0)$ $f_1(x) = 1 + \frac{2x}{\pi}$

$f_1(-x) = 1 - \frac{2x}{\pi} = f_2(x)$

In $(0, \pi)$ $f_2(x) = 1 - \frac{2x}{\pi}$

$f_2(-x) = 1 + \frac{2x}{\pi} = f_1(x)$

$f(x)$ is even function in $(-\pi, \pi)$

$$a_0 = \frac{2}{\pi}\int_0^\pi f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^\pi \left(1 - \frac{2x}{\pi}\right)dx$$

$$= \frac{2}{\pi}\left[x - \frac{\cancel{2}}{\pi}\frac{x^2}{\cancel{2}}\right]_0^\pi$$

$$= \frac{2}{\pi}\left[x - \frac{x^2}{\pi}\right] = 0$$

$$a_n = \frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx$$

$$\frac{2}{\pi}\int_0^\pi \left(1 - \frac{2x}{\pi}\right)\cos nx\,dx$$

$$= \frac{2}{\pi}\left[\cancel{\left(1 - \frac{2x}{\pi}\right)\left(\frac{\sin nx}{n}\right)}^{0} - \left(-\frac{2}{\pi}\right)\left(-\frac{\cos nx}{n^2}\right)\right]_0^\pi$$

$$= \frac{4}{\pi^2}\left[\frac{-(-1)^n + 1}{n^2}\right], n = 1, 2, \ldots$$

$$b_n = 0$$

F. Series $= \frac{4}{\pi^2 n^2}\sum_{n=1}^{\infty}\left[-(-1)^n + 1\right]$

$$f(x) = \frac{4}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{-(-1)^n + 1}{n^2}\right]\cos nx$$

Qus. Obtain F.S of the func & deduce the value of $\frac{1}{1^2} + \frac{1}{3^2} + \ldots$

$$f(x) = \begin{cases} -x + 1 & -\pi < x < 0 \\ x + 1 & 0 < x < \pi \end{cases}$$

Put $f_1(x) = -x + 1$ $\quad f_2(x) = x + 1$

$f_1(-x) = x + 1 = f_2(x)$ $\quad f_2(-x) = -x + 1 = f_1(x)$

It is an even function.

$$a_0 = \frac{2}{\pi}\int_0^\pi f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^\pi (x+1)\,dx$$

$$= \frac{2}{\pi}\left[\frac{x^2}{2} + x\right]_0^\pi = \frac{2}{\pi}\left[\frac{\pi^2}{2} + \pi\right]$$

$$= \frac{\cancel{2}}{\pi}\left[\frac{\pi^2 + 2\pi}{\cancel{2}}\right]$$

$$= \frac{\cancel{\pi}[\pi + 2]}{\cancel{\pi}}$$

$$a_0 = [\pi + 2]$$

$$a_n = \frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\left[(x+1)\cos nx\right]_0^\pi$$

$$= \frac{2}{\pi}\left[\cancel{(x+1)\left(\frac{\sin nx}{n}\right)}^{0} - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^\pi$$

$$= \frac{2}{\pi}\left[\frac{\cos nx}{n^2}\right]_0^\pi$$

$$= \frac{2}{\pi}\left[\frac{(-1)^n - 1}{n^2}\right], n = 1, 2, \ldots$$

Note:- If $f(x)$ is discontinous at x then

$$f(x) = \frac{f(x^+)+f(x^-)}{2}$$

$$= \frac{R.H.L + L.H.L}{2}$$

$$R.H.L = \lim_{h\to 0} f(x+h)$$

$$L.H.L = \lim_{h\to 0} f(x-h)$$

Problem:- obtain fourier series of the function

$$f(x) = \begin{cases} -\pi & -\pi<x<0 \\ x & 0<x<\pi \end{cases}$$ and hence

deduce that the sum of the reciprocal square of odd integer is equal to $\frac{\pi^2}{8}$

Soln. $f(x)$ is defined in $(-\pi,\pi)$ and it is neither odd nor even fourier coefficient are

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[-\int_{-\pi}^{0}\pi\,dx + \int_0^{\pi} x\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi x\Big|_{-\pi}^{0} + \frac{x^2}{2}\Big|_0^{\pi}\right]$$

$$a_0 = -\pi/2.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[-\int_{-\pi}^{0}\pi\cos nx + \int_0^{\pi} x\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi\cancel{\frac{\sin nx}{n}}\Big|_{-\pi}^{0} + \left(\cancel{x\frac{\sin nx}{n}} - \frac{(-\cos nx)}{n^2}\right)\Big|_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\cos nx}{n^2}\right]_0^{\pi} \Rightarrow \frac{1}{\pi}\left[\frac{(-1)^n-1}{n^2}\right] \quad n=1,2\ldots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[-\int_{-\pi}^{0}\pi\sin x + \int_0^{\pi} x\sin x\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi\cos nx}{n}\Big|_{-\pi}^{0} + \left[x\left(\frac{-\cos nx}{n}\right) - 1\cancel{\left(\frac{-\sin nx}{n^2}\right)}\right]_0^{\pi}\right]$$

$$\frac{1}{\pi}\left[\frac{\pi\cos 0 - \pi\cos n\pi}{n} - \frac{\pi\cos n\pi}{n}\right]$$

$$= \frac{\cancel{\pi}}{\cancel{\pi}}\left[\frac{1-2(-1)^n}{n}\right] \quad n=1,2\ldots$$

$\therefore$ fourier series:-

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos n + \sum_{n=1}^{\infty} b_n\sin nx$$

$$f(x) = -\frac{\pi}{4} + \sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2\pi}\cos nx + \sum_{n=1}^{\infty}\frac{1-2(-1)^n}{n}\sin nx \quad \text{—(1)}$$

$f(x)$ is discontinous at $x=0$

$$f(0) = \frac{f(0^+)+f(0^-)}{2}$$

$$= \frac{0+(-\pi)}{2} = -\pi/2.$$

Put $x=0$ in eqn (1)

$$f(0) = -\pi/4 + \sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2\pi}$$

$$-\pi/2 = -\pi/4 + \sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2\pi}$$

$$\left[-\pi/2 + \pi/4\right] = \frac{1}{\pi}\left[\frac{-1-1}{1^2} + \frac{1-1}{2^2} + \frac{-1-1}{3^2} + \cdots\right]$$

$$\frac{-\cancel{2}\pi^2}{8} = -\cancel{2}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2}\cdots\right]$$

$$\frac{\pi^2}{8} = \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2}\cdots\right]$$

Qus:- obtain fourier series of the function

$$f(x) = \begin{cases} -\frac{(\pi+x)}{2} & -\pi<x<0 \\ \frac{\pi-x}{2} & 0\le x<\pi \end{cases}$$

$$f_1(x) = -\left(\frac{\pi+x}{2}\right) \quad f_2(x) = \left(\frac{\pi-x}{2}\right)$$

In this interval $(-\pi, 0)$ $f_1(x) = -\left(\frac{\pi+x}{2}\right)$

$$f_1(-x) = -\left(\frac{\pi-x}{2}\right) = -f_2(x)$$

In other interval $(0,\pi)$

$$f_2(x) = \frac{\pi-x}{2}$$

$$f_2(-x) = \frac{\pi-(-x)}{2} = \frac{\pi+x}{2} = -f_1(x)$$

$f(x)$ is odd, $a_0 = 0$, $a_n = 0$, $n = 1, 2, 3 \ldots$

$$b_n = \frac{2}{\pi}\int_0^\pi f(x)\sin nx\,dx$$

$$= \frac{2}{\pi}\int_0^\pi \left(\frac{\pi-x}{2}\right)\sin nx\,dx$$

It is even so take the limit +ve, so that's why limit is from $0-\pi$

$$\frac{2}{\pi}\left[\left(\frac{\pi-x}{2}\right)\left(\frac{-\cos nx}{n}\right) - \left(\frac{-1}{2}\right)\cancel{\left(\frac{-\sin nx}{n^2}\right)}^{0}\right]_0^\pi$$

$$\frac{2}{\pi}\left[0 - \frac{\pi}{2}\frac{\cos 0}{n}\right] = \frac{1}{n}, \; n = 1, 2, \ldots$$

fourier series is

$$f(x) = \sum_{n=1}^{\infty} \frac{\sin nx}{n}$$

Qus- obtain the fourier series of the function

$$f(x) = \begin{cases} -x+1 & -\pi\le x\le 0 \\ x+1 & 0\le x\le\pi \end{cases}$$

$$f_1(x) = (-x+1) \quad f_2(x) = (x+1)$$

In $(-\pi, 0)$

$$f_1(-x) = (x+1) = f_2(x)$$

In $(0,\pi)$ $f_2(-x) = (-x+1) = f_1(x)$ function is even.

Qu.

$$f(x) = \begin{cases} 0 & -\pi\le x\le 0 \\ x^2 & 0\le x\le\pi \end{cases}$$

given function is neither odd nor even.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_0^\pi x^2\,dx\right]$$

$$= \frac{1}{\pi}\frac{x^3}{3}\Big|_0^\pi = \frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} 0\cos nx\,dx + \int_0^\pi x^2\cos nx\,dx$$

$$= \frac{1}{\pi}\left[x^2\cancel{\left(\frac{\sin nx}{n}\right)}^{0} - 2x\left(\frac{-\cos nx}{n^2}\right) + 2\cancel{\left(\frac{-\sin nx}{n^3}\right)}^{0}\right]_0^\pi$$

$$= \frac{1}{\pi}\left[+2x\left(\frac{\cos nx}{n^2}\right)\right]_0^\pi$$

$$= \frac{1}{\cancel{\pi}}\left[2\cancel{\pi}\left(\frac{\cos n\pi}{n^2}\right)\right] = \frac{2(-1)^n}{n^2} \quad n = 1, 2 \ldots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$\frac{1}{\pi}\int_{-\pi}^{0} 0\sin nx\,dx + \int_0^\pi x^2\sin nx\,dx$$

$$\frac{1}{\pi}\left[x^2\left(\frac{-\cos nx}{n}\right) - 2x\cancel{\left(\frac{-\sin nx}{n^2}\right)}^{0} + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^\pi$$

$$\frac{1}{\pi}\left[x^2\left(\frac{-\cos nx}{n}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^\pi$$

$$\frac{1}{\pi}\left[\pi^2\left(\frac{-\cos n\pi}{n}\right) + \frac{2\cos n\pi}{n^3} - \frac{2\cos 0}{n^3}\right)$$

$$= \frac{\pi(-1)^n}{n} + \frac{2(-1)^n}{n^3} - \frac{2}{n^3}$$

$\frac{1}{\pi}\left[-\pi^2 \frac{\cos n\pi}{n} \pm \frac{2}{n^3}(\cos n\pi - \cos 0)\right]$

$= \frac{1}{\pi}\left[\frac{\pi^2(-1)^{n+1}}{n} + \frac{2}{n^3}[(-1)^n - 1]\right]$

$n = 1, 2 ---$

Case IV - fourier series in $(-l, l)$

fourier coefficient are $a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$

$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi}{l}x\,dx \quad n=1,2 ---$

$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi}{l}x\,dx \quad n=1,2 ---$

fourier series of $f(x)$ in $(-l, l)$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi}{l}x + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi}{l}x$

Note:- to check odd or even use condition

$f(-x) = f(x)$ for even

$\Rightarrow b_n = 0$

$f(-x) = -f(x)$ for odd

$\Rightarrow a_0 = 0$

$a_n = 0$

Ques. Obtain fourier series of $f(x) = \sin mx$ in $(-l, l)$

Soln. $f(x) = \sin mx$

$f(-x) = \sin m(-x)$

$= -\sin mx$

$a_0 = 0 \quad a_n = 0 \quad n = 1, 2 ---$

$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi}{l}x\,dx$

$= \frac{2}{l}\int_0^l \sin mx \sin\frac{n\pi}{l}x\,dx$

$= \frac{2}{l^2}\int_0^l \sin mx \sin n\pi x\,dx$

Ques. Obtain fourier series

$f(x) = \frac{l}{2} - x \quad (-l, 0)$

$\frac{l}{2} + x \quad (0, l)$

hence deduce value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} ---$

Soln. Let $f_1(x) = \frac{l}{2} - x \quad f_2(x) = \frac{l}{2} + x$

In $(-l, 0)$ $f_1(x) = \frac{l}{2} - x$

$f_1(-x) = \frac{l}{2} - (-x)$

$= \frac{l}{2} + x = f_2(x)$

In $(0, l)$ $f_2(x) = \frac{l}{2} + x$

$f_2(-x) = \frac{l}{2} + (-x) = \frac{l}{2} - x = f_1(x)$

So, it is an even function $(-l, l)$

$b_n = 0$

$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$

$= \frac{2}{l}\int_0^l \frac{l}{2} + x\,dx$

$\Rightarrow \frac{2}{l}\left(\frac{l}{2}x + \frac{x^2}{2}\right]_0^l$

$= \frac{2}{l}\left(\frac{l^2}{2} + \frac{l^2}{2}\right) \Rightarrow \frac{2}{l}\left(\frac{2l^2}{2}\right)$

$= 2l$

$a_n = \frac{2}{l}\int_0^l \left(\frac{l}{2} + x\right)\cos\frac{n\pi}{l}x\,dx$

$= \frac{2}{l}\left[\left(\frac{l}{2}+x\right)\cancel{\frac{\sin n\pi x}{n\pi/l}}^{\to 0} - (1)\left(\frac{-\cos n\pi x/l}{n^2\pi^2/l^2}\right)\right]_0^l$

$$\frac{2}{l}\left[\frac{\cos\frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2}\right]_0^l$$

$$\frac{2}{\cancel{l}}\cdot\frac{l^{\cancel{2}}}{(n\pi)^2}\left[\cos n\pi - \cos 0\right]$$

$$= \frac{2l}{n^2\pi^2}\left[(-1)^n - 1\right] \quad n = 1, 2, \ldots$$

$b_n = 0, \; n = 1, 2, \ldots$

f.s of $f(x)$ is

$$f(x) = l + \frac{2l}{\pi^2}\sum \frac{(-1)^n - 1}{n^2}\cos\frac{n\pi}{l}x \quad \text{—} (1)$$

Put $x = 0$

$$f(0) = l/2$$

$$f(0) = l + \frac{2l}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2}$$

$$= l/2 = l + \frac{2l}{\pi^2}\left[\frac{-1-1}{1^2} + \frac{1-1}{2^2} + \frac{-1-1}{3^2} \ldots\right]$$

$$\Rightarrow l/2 - l = \frac{2l}{\pi^2}\left[-\frac{2}{1^2} - \frac{2}{3^2} \ldots\right]$$

$$\Rightarrow \left(\frac{-l}{2}\right)\times\frac{\pi^2}{2l} = -2\left[\frac{1}{1^2} + \frac{1}{3^2} \ldots\right]$$

$$= \frac{\cancel{-}\pi^2}{4} = \cancel{-}2\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \ldots\right]$$

$$= \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \ldots$$

Ques Obtain fourier series of fun^n

$$f(x) = \begin{cases} 1 + \frac{4x}{3} & -\frac{3}{2} < x < 0 \\ 1 - \frac{4x}{3} & 0 < x < 3/2 \end{cases}$$

hence deduce $\frac{1}{1^2} + \frac{1}{3^2} \ldots = \pi^2/8$.

$$f_1(x) = 1 + \frac{4x}{3}$$

$$f_1(-x) = 1 + \frac{4(-x)}{3} = f_2(x)$$

$$f_2(x) = 1 - \frac{4x}{3}$$

$$f_2(-x) = 1 - \frac{4(-x)}{3}$$

$$= 1 + \frac{4x}{3} \rightarrow f_1(x)$$

even function

$b_n = 0$.

$$a_0 = \frac{2}{l}\int_0^{3/2} f(x)\,dx$$

$$= \frac{2}{l}\int_0^{3/2}\left(1 - \frac{4x}{3}\right)dx$$

$$= \frac{2}{l}\left[x - \frac{4x^2}{6}\right]_0^{3/2}$$

$$= \frac{2}{l}\left[3/2 - \frac{4(9/4)}{6}\right] \Rightarrow \frac{2}{l}\left[3/2 - \frac{\cancel{9}\,3}{\cancel{6}\,2}\right)$$

$$= \frac{2}{l}(0) = a_0 = 0$$

$$a_n = \frac{1}{l}\int_0^{3/2} f(x)\cos\frac{n\pi}{l}x\,dx.$$

$$= \frac{1}{l}\int_0^{3/2}\left(1 - \frac{4x}{3}\right)\cos n\frac{\pi}{l}x\,dx$$

$$\cancel{= \frac{1}{l}\left[\left(1 - \frac{4x}{3}\right)\frac{\sin\frac{n\pi}{l}x}{n\pi/l} - \left(-\frac{4}{3}\right)\left(\frac{-\cos\frac{n\pi}{l}x}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^{3/2}}$$

$$= \frac{1}{l}\left[\frac{4}{3}\left(\frac{-\cos\frac{n\pi}{l}x}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^{3/2}$$

$$\frac{4}{3l}\,\frac{l^2}{n\pi^2}\left[-\cos\frac{n\pi}{l}x\right]_0^{3/2}$$

$$\cancel{\frac{4}{3l}\,\frac{l^2}{n\pi^2}\left[-\cos 2n\pi\right.}$$

$$= \frac{2}{3/2}\int_0^{3/2}\left(1 - \frac{4x}{3}\right)\cos\frac{n\pi x}{3/2}\,dx$$

$$= \frac{4}{3}\left[\left(1 - \frac{4x}{3}\right)\left(\frac{\sin\frac{2n\pi}{3}x}{2n\pi/3}\right) - \left(-\frac{4}{3}\right)\frac{\left(-\cos\frac{2n\pi}{3}x\right)}{(2n\pi/3)^2}\right]_0^{3/2}$$

$$= \frac{16}{9}\times\frac{9}{4n^2\pi^2}\left[(-1)^n + 1\right] = \frac{4}{n^2\pi^2}\left[1 - (-1)^n\right] \quad n = 1, 2, \ldots$$

$b_n = 0 \; n = 1, 2 \ldots$

Scanned by CamScanner

F.S. of $f(x)$ is

$$f(x)=\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2}\cos\frac{n\pi l x}{3} \quad -(1)$$

Put $x=0$ in (1)

$$f(0)=\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2}$$

$$\Rightarrow 1=\frac{4}{\pi^2}\left[\frac{1+1}{1^2}+\frac{1-1}{2^2}+\frac{1+1}{3^2}+\cdots\right]$$

$$\frac{\pi^2}{4}=2\left[\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right]$$

$$\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}\cdots$$

Que. Obtain fourier series of function

$$f(x)=\begin{cases} x & -1<x\le 0 \\ x+2 & 0<x<1\end{cases}$$

$f(x)$ is define in $(-1,1)$ the function is neither odd nor even. fourier co-efficient are

$$a_0=\frac{1}{1}\int_{-1}^{1}f(x)\,dx$$

$$=\int_{-1}^{0}x\,dx+\int_0^1(x+2)\,dx$$

$$=\frac{x^2}{2}\Big|_{-1}^{0}+\left(\frac{x^2}{2}+2x\right)\Big|_0^1=0$$

$$a_n=\frac{1}{1}\int_{-1}^{1}f(x)\cos\frac{n\pi}{l}x\,dx \qquad \{l=1\}$$

$$=\int_{-1}^{0}x\cos n\pi x\,d+\int_0^1(x+2)\cos n\pi x\,dx$$

$$=\left[x\frac{\cancel{\sin n\pi x}}{n\pi}-(1)\left(\frac{-\cos n\pi x}{(n\pi)^2}\right)\right]_{-1}^{0}$$

$$+\left[(x+2)\frac{\cancel{\sin n\pi x}}{n\pi}-(1)\left(\frac{-\cos n\pi x}{(n\pi)^2}\right)\right]_0^1$$

$$=\Rightarrow\frac{1-(-1)^n}{n^2\pi^2}+\frac{(-1)^n-1}{n^2\pi^2}=0$$

$n=1,2\cdots$

$$b_n=\frac{1}{l}\int_{-l}^{l}f(x)\sin\frac{n\pi}{l}x\,dx \qquad n=1,2\cdots$$

$$b_n=\frac{1}{1}\int_{-1}^{0}x\sin\frac{n\pi}{l}x+\int_0^1(x+2)\sin\frac{n\pi}{l}x$$

$$b_n=\left[x\left(\frac{-\cos n\pi x}{n\pi}\right)-(1)\left(\frac{\cancel{-\sin n\pi x}}{(n\pi)^2}\right)\right]_{-1}^{0}+\left[(x+2)\left(\frac{-\cos n\pi x}{n\pi}\right)-(1)\left(\frac{\cancel{-\sin n\pi x}}{(n\pi)^2}\right)\right]_0^1$$

$$b_n=-\left[\frac{\cos n\pi}{n\pi}-\left[\frac{3\cos n\pi}{n\pi}+\frac{2\cos 0}{n\pi}\right]\right]$$

$$=\frac{-4(-1)^n+2}{n\pi}, \quad n=1,2\cdots$$

fourier series:-

$f(x)$ in $(-1,1)$ in $f(x)=1+\sum_{n=1}^{\infty}\frac{2-4(-1)^n}{n\pi}\sin n\pi x$

Que.

Obtain fourier series of the function

$$f(t)=\begin{cases}0 & (-l,0)\\ E\sin\omega t & (0,l)\end{cases}$$

Sol^n. $f(t)$ is defined in $(-l,l)$ the function is neither odd or even.

$$a_0=\frac{1}{l}\int_{-l}^{l}f(x)\,dx$$

$$=\frac{1}{l}\int_{-l}^{0}0\,dx+\int_0^l E\sin\omega t$$

$$=\frac{1}{l}\left[E\frac{(-\cos\omega t)}{\omega}\right]_0^l$$

$$=\frac{1}{l}-\left(\frac{E\cos\omega t}{\omega}\right)_0^l=-\frac{E}{l\omega}((-1)^n-1)$$

$$=\frac{E}{l\omega}\left(1-(-1)^n\right)$$

$$a_n=\frac{1}{l}\int_{-1}^{1}f(x)\cos\frac{n\pi}{l}x$$

$$a_n=\frac{1}{l}\int_{-1}^{0}(0)\cos\frac{n\pi}{l}x+\int_0^1 E\sin\omega t\cos\frac{n\pi}{l}x\,dx$$

Harmonic Analysis:-

Property:- The average of the function $f(x)$ over the interval $[a,b]$ is denoted by $[[f(x)]]$ and is given by $[[f(x)]] = \frac{1}{b-a}\int_a^b f(x)\,dx$

The fourier series of $f(x)$ over $(a, a+2l)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{l}x + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{l}x$$

where

$$a_0 = \frac{1}{l}\int_a^{a+2l} f(x)\,dx = 2[[f(x)]]$$

$$a_n = \frac{1}{l}\int_a^{a+2l} f(x)\cos\frac{n\pi}{l}x\,dx = 2\left[\left[f(x)\cos\frac{n\pi}{l}x\right]\right] \quad n=1,2---$$

$$b_n = \frac{1}{l}\int_a^{a+2l} f(x)\sin\frac{n\pi}{l}x\,dx = 2\left[\left[f(x)\sin\frac{n\pi}{l}x\right]\right] \quad n=1,2---$$

Obtain fourier series based on above process is called H.A.

$\frac{a_0}{2}$ is called constant term

$a_1\cos\frac{\pi}{l}x + b_1\sin\frac{\pi}{l}x$ is called first harmonic term.

$a_2\cos\frac{2\pi}{l}x + b_2\sin\frac{2\pi}{l}x$ ——— second ———

and so on.

$\sqrt{a_1^2+b_1^2}$ is called amplitude of 1st harmonic

$\sqrt{a_2^2+b_2^2}$ is called amplitude of 2nd harmonic.

Harmonic analysis in $(0, 2\pi)$
$(0, 360)$

$$a_0 = 2[[f(x)]]$$

$$a_n = 2[[f(x)\cos nx]] \quad n=1,2--$$

$$b_n = 2[[f(x)\sin nx]]$$

F.S of $f(x)$ in $(0,2\pi)$ is

$$f(x) = \frac{a_0}{2} + (a_1\cos x + b_1\sin x) + (a_2\cos 2x + b_2\sin 2x) + \; - - -$$

Que Express y as F.S upto 2nd harmonic given

x	0	60	120	180	240	300
y	4	3	2	4	5	6

Soln at $x = 360$
$y = 4$

y is defined in $(0, 2\pi)$

x	y	$y\cos x$	$y\cos 2x$	$y\sin x$	$y\sin 2x$
0	4	4	4	0	0
60	3	3/2	-1.5	2.6	2.6
120	2	-1	-1	1.7	-1.7
180	4	-4	4	0	0
240	5	-2.5	-2.5	-4.3	4.3
300	6	3	-3	-5.2	-5.2
Σ	24	1	0	-5.2	0

$$a_0 = 2[[f(x)]] = 2[[y]] = 2\left[\frac{24}{6}\right] = 2[4] = 8$$

$$a_n = 2[[f(x)\cos nx]]$$

at $n=1$ $a_1 = 2[[y\cos x]] = 2\left[\frac{1}{6}\right] = 0.333$

$n=2$ $a_2 = 2[[y\cos 2x]] = 2\left[\frac{0}{6}\right] = 0$

$$b_n = 2[[f(x)\sin nx]]$$

at $n=1$

$$b_1 = 2[[y\sin x]] = 2\left[\frac{-5.2}{6}\right] = -1.73$$

at $n=2$ $b_2 = 2[[y\sin 2x]] = 2\left[\frac{0}{6}\right] = 0$

$\therefore$ F.S upto 2nd harmonic is

$$y = 4 + (0.333\cos x - 1.73\sin x$$

Scanned by CamScanner

Qus. obtain fourier series for the flowing data neglecting terms higher than 1st harmonic

data.

x	0	60	120	180	240	300	360
y	7.9	7.2	3.6	0.5	0.9	6.8	7.9

soln y is defined in $(0, 2\pi)$

x	y	$y\cos x$	$y\sin x$
0	7.9	7.9	0
60	7.2	3.6	6.23
120	3.6	-1.8	3.11
180	0.5	-0.5	0
240	0.9	-0.45	-0.77
300	6.8	3.4	-5.88
Σ	26.9	12.15	2.69

$a_0 = 2[[f(x)] = 2[[y]] = 2\left[\frac{26.9}{6}\right] = 8.99$

$\Rightarrow a_n = 2[[f(x)\cos nx]]$

put $n=1$ $a_1 = 2[[y\cos x]] = 2\left[\frac{12.15}{6}\right] = 4.05$

$b_n = 2[[f(x)\sin nx]]$

put $n=1$ $b_1 = 2[[y\sin x]]$

$= 2\left[\frac{2.69}{6}\right] = 0.9$

f.s upto 1st harmonic is

$y = 4.46 + (4.05\cos x + 0.9\sin x)$

Q1 Obtain f.s upto 2nd harmonic & find the amplitude in 1st harmonic in data.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

x	y	$y\cos x$	$y\cos 2x$	$y\sin x$	$y\sin 2x$
0	1.0	1.0	1.0	0	0
$\pi/3$	1.4	0.7	-0.7	1.2	1.2
$2\pi/3$	1.9	-0.95	-0.95	1.64	-1.64
π	1.7	-1.7	1.7	0	0
$4\pi/3$	1.5	-0.75	-0.75	-1.3	1.3
$5\pi/3$	1.2	0.6	-0.6	-1.03	-1.03
Σ	8.7	-1.1	-0.3	~~0.05~~ +0.52	-0.17

$a_0 = 2[[f(x)] = 2[[y]] = 2\left[\frac{8.7}{6}\right] = 2.9$

$a_n = 2[[f(x)\cos nx]]$

put $n=1$ $a_1 = 2[[y\cos x]] = 2\left[\frac{-1.1}{6}\right] = 0.36$

$n_2 \cdot a_2 = 2[[y\cos 2x]] = 2\left[\frac{-0.3}{6}\right] = 0.1$

$b_n = 2[[f(x)\sin nx]]$

$n_1 = b_1 = 2[[y\sin x]]$

$= 2\left[\frac{0.52}{6}\right] = 0.17$

$b_2 = 2[[y\sin 2x]] = 2\left[\frac{-0.17}{6}\right] = -0.06$

f.s upto 2nd harmonic is

$y = 1.45 + (-0.36\cos x + 0.17\sin x)$

$+ (-0.1\cos 2x - 0.06\sin 2x)$

Amplitude of 1st harmonic is $= \sqrt{a_1^2 + b_1^2} = 0.39.$

Scanned by CamScanner

⇒ Harmonic analysis in $(0,2l)$

$a_0 = 2\left[\left[f(x)\right]\right]$

$a_n = 2\left[\left[f(x) \cos \frac{n\pi}{l} x\right]\right]$

$b_n = 2\left[\left[f(x) \sin \frac{n\pi}{l} x\right]\right]$

F. S of $f(x)$ in $(0,2l)$ is

$f(x) = \frac{a_0}{2} + \left(a_1 \cos\frac{\pi}{l}x + b_1 \sin\frac{\pi}{l}x\right) + \left(a_2 \cos\frac{2\pi}{l}x + b_2 \sin\frac{2\pi}{l}x\right) + \ldots$

Q Obtain the constant term & 1st Harmonic from the given data

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Solⁿ At $x = 6$, $y = 9$

y is determined in $(0,6) = (0,2l)$

$l \Rightarrow 3$

x	y	$y\cos\frac{\pi}{3}x$	$y \sin \frac{\pi}{3}x$
0	9	9	0
1	18	9	15.58
2	24	−12	20.7
3	28	−28	0
4	26	−13	−22.51
5	20	10	−17.32
Σ	125	−25	−3.5

$a_0 = 2\left[\left[f(x)\right]\right] = 2\left[\frac{125}{6}\right] = 41.67$

$a_n = 2\left[\left[f(x)\cos\frac{n\pi}{l}x\right]\right] = 2\left[\left[y\cos\frac{n\pi}{3}x\right]\right]$

At $n=1$ $a_1 = 2\left[\left[y\cos\frac{\pi}{3}x\right]\right] = 2\left[\frac{-25}{6}\right] = -8.33$

$b_n = 2\left[\left[y \sin n\frac{\pi}{3}x\right]\right]$

At $n=1$ $b_1 = 2\left[\left[y \sin\frac{\pi}{3}x\right]\right]$

$= 2\left[\frac{-3.5}{6}\right] = -1.166$

Constant term $= \frac{a_0}{2} = 20.84$

first harmonic term $= a_1 \cos\frac{\pi}{3}x + b_1 \sin\frac{\pi}{3}x$

$= -8.33 \cos\frac{\pi}{3}x - 1.166 \sin\frac{\pi}{3}x$.

Q4 The following data is variation of Periodic ~~time~~ current over a Period. ~~Direct current 0.7 amp in variable.~~

t sec	0	T/6	T/3	T/2	2T/3	5T/6
i amp	1.98	1.3	1.05	1.3	−0.88	−0.25

Show that the Direct Current of 0.7 amp in variable & find the Amplitude of 1st harmonic.

At $t = T$ $i = 1.98$

in amp is defined in $(0,T) \Rightarrow (0,2l)$

$l = T/2$

t	i	$i\cos\frac{2\pi}{T}t$	$i\sin\frac{2\pi}{T}t$
T/6	1.98	1.98	0
T/3	1.3	0.65	1.126
	1.05	−0.525	0.909
T/2	1.3	−1.3	0
2T/3	−0.88	0.44	0.762
5T/6	−0.25	−0.125	0.216
Σ	4.5	1.12	3.01

$1.3 \cos \frac{2\pi}{T} \times \frac{T}{3}$

3

$a_0 = 2\left[\left[f(x)\right]\right] = 2\left[\left[i\right]\right] = 2 \times \frac{4.5}{6} = 1.5$

$a_n = 2\left[\left[f(x)\cos\frac{n\pi}{l}x\right]\right] = 2\left[\left[i\cos\frac{2n\pi}{T}t\right]\right]$

At $n=1$ $a_1 = 2\left[\left[i\cos\frac{2\pi t}{T}\right]\right] = 2\left[\frac{1.12}{6}\right] = 0.31$

$b_n = 2\left[\left[i \sin\frac{2n\pi}{T}t\right]\right]$ At $n=1$ $b_1 = 2\left[\left[i\sin\frac{2\pi}{T}t\right]\right]$

$= 2\left[\frac{3.01}{6}\right] \doteq 1.004$

Direct current Part is $\frac{a_0}{2} = \frac{1.5}{2} = 0.75$

Amplitude of 1st harmonic

$\sqrt{a_1^2 + b_1^2} = 1.07$.

Q2 The following data gives variation of a Periodic Current over a Period

x	0	2	4	6	8	10	12
y	9.0	18.2	24.4	27.8	27.5	22.0	9.0

Find F.S upto 2nd harmonic