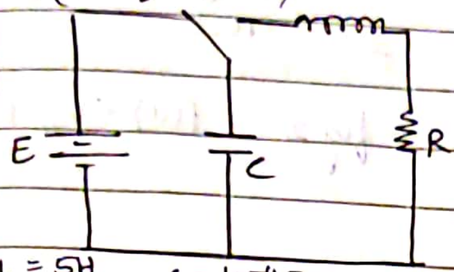
Numerical Methods

UNIT - 3

$Q(0) = q_0$

Requirement $Q(0.05) - 0.01\%$



$L = 5H$ $\quad$ $C = 10^{-4}F$

$$L\frac{di}{dt} + Ri + \frac{q}{c} = 0$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$$

$$q = e^{\alpha t}\left\{c_1 \cos\beta t + C_2 \sin\beta t\right\}$$

$$= q_0 e^{-Rt/2t} \cos\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\, t$$

$$0.01 q_t = q_0 e^{-0.005R} \cos\left\{\left(\sqrt{2000 - 0.01R^2}\right) 0.05\right\}$$

$$f(R) = e^{-0.005R} \cos\left\{0.05\sqrt{2000 - 0.01R^2}\right\} - 0.01$$

15 Oct '19

Transcendental functions

Functions which can't be expressed as polynomials of finite degree.

eg:- e^x, $\sin(x)$, $\cos(x)$, ... $\log(x)$.
$\sin^{-1}(x)$, $\cos^{-1}(x)$ ....

Transcendental equations

Eq^{ns} with transcendental functions.

Roots of a function $y = f(x)$

roots $= \{x / f(x) = 0\}$.

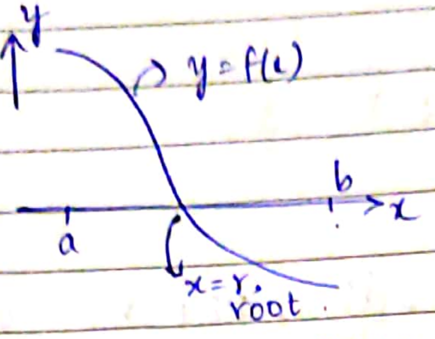


fig 1: $f(a) > 0$. $f(b) < 0$

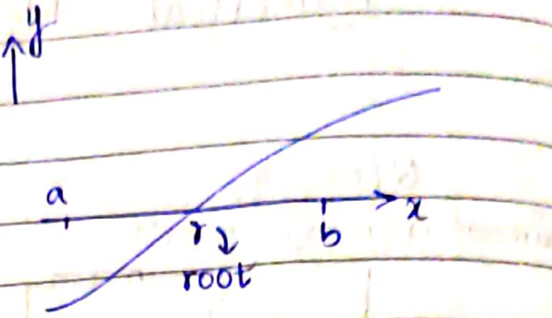


fig 2 : $f(a) < 0$, $f(b) > 0$

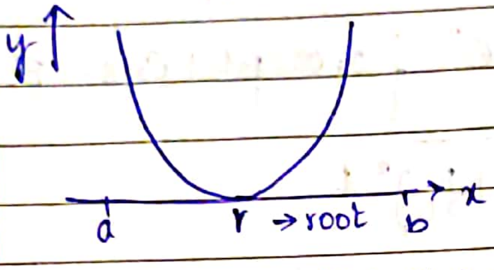
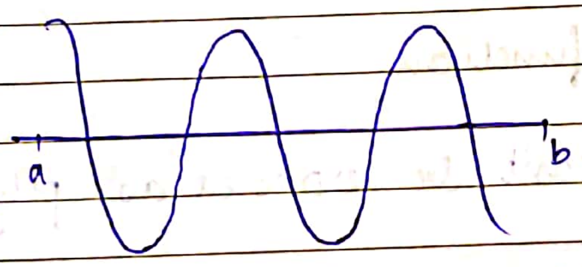


fig 3 : $f(a), f(b) > 0$

Intermediate value theorem



If $f(x)$ is continuous in $[a, b]$ with $f(a)$ & $f(b)$ of opposite signs then exist at least one root in (a, b)

Newton - Raphson Method

Taylor's Theorem — $f(x) + f(a) + (x-a)f'(a) +$ higher order terms

$$f(x) = f(x_0) + hf'(x_0)$$

suppose $h = x - x_0$ is very small.

i.e $x \sim x_0$

If x is a root then $f(x) = 0$

$$x - x_0 = \frac{-f(x_0)}{f'(x_0)}$$

$$x = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ → Newton-Raphson iterative formula.

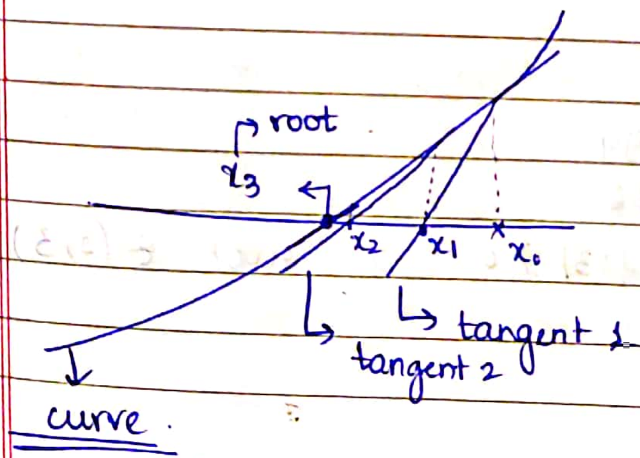
Q. Find the real root of the polynomial near $x = 0.5$. $[x^3 - 3x + 1 = 0]$.

$f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$ ⇒

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - (f(x_n)/f'(x_n))$
0	0.5	-0.375	-2.25	0.3333
1	0.3333	0.03712	-2.666	0.$\cancel{6}$3472
2	0.$\cancel{3}$3472	2.542×10^{-4}	-2.638	0.3472.



Q2) Find the root of $3x = \cos x + 1$ using Newton-Raphson method.

$3x = \cos x + 1$

$f(x) = \cos x + 1 - 3x$. → Imp: calculator should be in radian mode

$f(0) = 2$.

$f(1) = -1.45969$.

$f(0) > 0$ & $f(1) < 0$

root $\in (0,1)$

$f'(x) = -\sin x - 3$

Let $x_0 = 0.5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - f(x_n)/f'(x_n)$
0	0.5	0.37758	−3.47942	0.60851
1	0.60851	-4.9936×10^{-3}	−3.57163	0.60710
2	0.6071	5.8845×10^{-6}	−3.57048	0.60710

$\therefore x = 0.6071$.

17th oct '19.

Approximate root of $x \log_{10} x = 1.2$ correct to 4 decimal places using N-R method.

$f(x) = x \log_{10} x - 1.2$.

$f(1) = -1.2$

$f(2) = -0.59794$

$f(3) = 0.23136$

$f(2) < 0$ $f(3) > 0$ ⇒ root $\in (2,3)$

let $x_0 = 2.5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\cancel{f'(x) = \log_{10} x} \cdot \quad \frac{d}{dx} \log_e x = \frac{1}{x}$$

$$\therefore f(x) = \frac{x \log_e x}{\log_e 10} - 1.2$$

$$f'(x) = \frac{\log_e x + x(1/x)}{\log_e 10} = \frac{\log_e x}{\log_e 10} + \frac{1}{\log_e 10}$$

$$f'(x) = \underbrace{\log_{10} x}_{\to \text{ log button}} + 0.43429$$

$$x_{n+1} = x_n - \frac{x_n \log_{10} x_n - 1.2}{\log_{10} x_n + 0.43429}$$

$$x_{n+1} = \frac{0.43429 x_n + 1.2}{\log_{10} x_n + 0.43429}$$

n	x_n
0	2.5
1	2.74650
2	2.74064
3	2.74064

Approximate root is 2.74064 correct to 4 decimal places

$(x_0, y_0), (x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$

$$\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = y'(x_0)$$

$$f(x_0+h) - f(x_0) = \Delta f(x_0)$$

$x_0 + h = x_1$ $\quad y_1 - y_0 = \Delta y_0$

$x_2 = x_1 + h$ $\quad y_2 - y_1 = \Delta y_1$ $\Bigg\}$ 1st forward difference

$x_3 = x_2 + h$ $\quad y_3 - y_2 = \Delta y_2$

// This is used when x is in AP //
& method ?

$$\Delta y_1 - \Delta y_0 = \Delta^2 y_0$$
$$\Delta y_2 - \Delta y_1 = \Delta^2 y_1$$
} 2^{nd} forward difference

Foward difference table

Tab 1

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0			
		Δy_0		
x_1	y_1		$\Delta^2 y_0$	
		Δy_1		$\Delta^3 y_0$
x_2	y_2		$\Delta^2 y_1$	
		Δy_2		
x_3	y_3			

Backward difference table

Tab 2.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
x_0	y_0			
x_1	y_1	$y_1 - y_0 = \nabla y_1$		
x_2	y_2	$y_2 - y_1 = \nabla y_2$	$\nabla y_2 - \nabla y_1 = \nabla^2 y_2$	
x_3	y_3	$y_3 - y_2 = \nabla y_3$	$\nabla y_3 - \nabla y_2 = \nabla^2 y_3$	$\nabla^2 y_3 - \nabla^2 y_2 = \nabla^3 y_3$

Values in Tab 1 equal to Value in Tab 2.

Q.

x	10	15	20	25	30	35
y	19.97	21.51	22.47	23.52	24.65	25.89

$\Delta f(20)$ $\quad \Delta^3 f(10)$ $\quad \nabla^4 f(30)$ $\quad \nabla^3 f(35)$

Also predict $f(40)$

x
10
15

x	y	Δy or ∇y	$\Delta^2 y$ or $\nabla^2 y$	$\nabla^3 y$ or $\Delta^3 y$	$\nabla^4 y$ or $\Delta^4 y$	$\Delta^5 y$ or $\nabla^5 y$
$\boxed{10}$	19.97					
		1.54				
15	21.51		-0.58			
		0.96		0.67		
$\boxed{20}$	22.47		0.09		-0.68	
		1.05		-0.01		0.72
25	23.52		0.08		0.04	
		1.13		0.03		0.72
$\boxed{30}$	24.65		0.11		$a_1 = 0.76$	
		1.24		$a_2 = 0.79$		
$\boxed{35}$	25.89		$a_3 = 0.90$			
		$a_4 = 2.14$				
40	28.03					

$\Delta f(20) = 1.05$

$\Delta^3 f(10) = 0.67$

$\nabla^4 f(30) = -0.68$

$\nabla^3 f(35) = 0.03$

Assume $\Delta^5 y$ = constant

[i.e Assume $y = f(x)$ is approximated with polynomial of degree 5]

$a_1 - 0.04 = 0.72$

$a_2 - 0.03 = 0.76$

18th oct

Interpolation

Approximating the value of y for some intermediate value of x using $(x_0, y_0), (x_1, y_1), (x_2, y_2) \cdots\cdots (x_n, y_n)$.

$x_0, x_1, x_2 \cdots\cdots x_n \rightarrow$ equidistant points $\Rightarrow x_i$ are in AP.

$\Delta f(x) = f(x+h) - f(x) = Ef(x) - f(x) = Ef(x) =$
$(1+\Delta)f(x)$

$Ef(x) = f(x+h)$

$Ef(x+h) = f(x+2h) \Rightarrow f(x+2h) = E\{Ef(x)\} - E^2 f(x)$

$E^n f(x) = f(x+nh) \quad n = 0, 1, 2, 3 \cdots$

$x = x_0 + ph \qquad p \in (-1, 0) \cup (0, 1)$

$f(x) = f(x_0 + ph) = E^p f(x_0)$

$f(x) = (1+\Delta)^p f(x_0) = \left(1 + p\Delta + \frac{p(p-1)}{2!}\Delta^2 + \frac{p(p-1)(p-2)}{3!}\Delta^3 \cdots\right.$

Newton's Forward interpolation formula.
or
Newton - Gregory foward interpolation formula.

$$f(x) = f(x_0) + p\Delta f(x_0) + \frac{p(p-1)}{2!}\Delta^2 f(x_0) + \frac{p(p-1)(p-2)}{3!}\Delta^3 f(x_0) + \cdots$$

Q. Interpolate the pressure of the steam at 142°C from the following table:

Temp	140	150	160	170	180
Pressure	3.685	4.854	6.302	8.076	10.225

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T	P	ΔP	$\Delta^2 P$	$\Delta^3 P$	$\Delta^4 P$
140	$\boxed{3.685}$				
		1.169			
150	4.854		0.279		
		1.448		0.047	
160	6.302		0.326		0.002
		1.774		0.049	
170	8.076		0.375		
		2.149			
180	10.225				

$$f(x) = f(x_0) + p\Delta f(x_0) + \frac{p(p-1)\Delta^2 f(x_0)}{2!} + \dots$$

$$P = P_0 + p(\Delta P_0) + \frac{p(p-1)\Delta^2 P_0}{2!} + \dots$$

$x = 142 \qquad x_0 = 140$

$f(x_0)$ $\quad x = x_0 + ph \Rightarrow 142 = 140 + p(10) \Rightarrow p = 0.2$

$$P(142^\circ) = 3.685 + 0.2(1.169) + \frac{0.2(-0.8)(0.279)}{2!}$$

$$+ \frac{(0.2)(-0.8)(-1.8)(0.047)}{3!} + \frac{(0.2)(-0.8)(-1.8)(-2.8)(0.002)}{4!}$$

$\simeq 3.8986$

for backward

$\nabla f(x+h) = f(x+h) - f(x)$

$\nabla f(x) = f(x) - f(x-h) = f(x) - E^{-1}f(x)$

$E^{-1}f(x) = (1-\nabla)f(x) \Rightarrow Ef(x) = (1-\nabla)^{-1}f(x)$

$E^{+p}f(x) = (1-\nabla)^{-p}f(x)$

$f(x_n + ph) = (1-\nabla)^{-p}f(x_n)$

$$= \left\{ 1 + p\nabla + \frac{p(p+1)\nabla^2}{2!} + \frac{p(p+1)(p+2)}{3!}\nabla^3 + \dots \right\} f(x_n)$$

$$f(x) = f(x_n) + p\nabla f(x_n) + \frac{p(p+1)}{2!}\nabla^2 f(x_n) + \frac{p(p+1)(p+2)}{3!}\nabla^3 f(x_n) + \dots$$

Q. Given $f(40) = 184$, $f(50) = 204$, $f(60) = 226$, $f(70) = 250$, $f(80) = 276$, $f(90) = 304$, find $f(38)$ and $f(85)$ using suitable interpolation formula.

x	y	Δy or ∇y	$\Delta^2 y$ or $\nabla^2 y$	$\Delta^3 y$ or $\nabla^3 y$	$\Delta^4 y$ or $\nabla^4 y$
40	184				
		20			
50	204		2		
		22			
60	226		2		
		24			
70	250		2		
		26			
80	276		2		
		28			
90	304				

$f(38)$ = Newton's forward interpolation

$$f(x) = f(x_0) + p\Delta f(x_0) + \frac{p(p-1)}{2!}\Delta^2 f(x_0)$$

$x = x_0 + ph \quad x = 38 \quad x_0 = 40 \quad h = 10 \Rightarrow p = -0.2$

$$f(38) = 184 + (-0.2)(20) + \frac{(-0.2)(-1.2)(2)}{2!} = 180.24$$

$f(85) \to$ Newton's Backward interpolation

$$p(x) = f(x_n) + p\nabla f(x_n) + \frac{p(p+1)}{2!}\nabla^2 f(x_n) + \dots$$

$x = x_n + ph$ $x_n = 90$

$x = 85$, $h = 10$ $\Rightarrow$ $p = -0.5$

$$f(85) = 304 + (-0.5)(28) + \frac{(-0.5)(0.5)\times 2}{2!} = 289.75$$

Q. Fit a polynomial of degree 3 which takes the following values

x	3	4	5	6
y	6	24	60	120

using Newton's interpolation.

x	y	Δy or ∇y	$\Delta^2 y$ or $\nabla^2 y$	$\Delta^3 y$ or $\nabla^3 y$
3	6			
		18		
4	24		18	
		36		6
5	60		24	
		60		
6	120			

$x = x_0 + ph$ $x_0 = 3$ $h = 1$

$x = 3 + p \Rightarrow p = x - 3$

$$f(x) = 6 + (x-3)(18) + \frac{(x-3)(x-4)(18)}{2!} + \frac{(x-3)(x-4)(x-5)(6)}{3!}$$

$$= x^3 + (-3)x^2 + (2)(x) + 0$$

$$= x^3 - 3x^2 + 2x$$

21 oct '19

Estimate the no. of students who have scored between 70-80 marks from the data.

Marks	0-20	20-40	40-60	60-80	80-100
No. of students	41	62	65	50	17

x	No. of students	Δy	$\Delta^2 y$	$\Delta^3 y$
20	41			
		62		
40	103		3	
		65		-18
60	168		-15	
		50		-18
80	218		-33	
		17		
100	235			

$f(70) = ?$

$x = 70$, $x = x_n + ph$ $h = 20$

$x_n = 80$ $p = -0.5$

$$f(70) = 218 + (-0.5)(50) + \frac{(-0.5)(0.5)(-15)}{2!}$$

$$+ \frac{(-0.5)(0.5)(1.5)(-18)}{3!}$$

$= 196 =$ No. of students scoring < 70 marks

No of students scoring 70 - 80 marks $= 218 - 196 = 22$

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$x_0 \quad x_1$

$y_0 \quad y_1 \qquad \dfrac{y - y_0}{x - x_0} = \dfrac{y_1 - y_0}{x_1 - x_0}$

$$y = (\quad)y_0 + (\quad)y_1$$

$$y = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x - x_0) + y_0$$

$$y = \left(\frac{x - x_1}{x_0 - x_1}\right)y_0 + \left(\frac{x - x_0}{x_1 - x_0}\right)y_1$$

$$\begin{aligned} y = {} & A_0(x-x_1)(x-x_2) \cdots (x-x_n) \\ & + A_1(x-x_0)(x-x_2) \cdots (x-x_n) \\ & + A_i(x-x_0)(x-x_1) \cdots (x-x_{i-1})(x-x_{i+1}) \cdots (x-x_n) \\ & \vdots \\ & + A_n(x-x_0)(x-x_1) \cdots (x-x_{n-1}) \end{aligned}$$

Lagrange's Interpolation.

$$y = \frac{(x-x_1)(x-x_2)(x-x_3) \cdots (x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3) \cdots (x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3) \cdots (x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3) \cdots (x_1-x_n)} y_1$$

$+ \cdots$

Q Fit an interpolating polynomial of the form $x = f(y)$ and hence find $x(5)$ and $y(5)$

x	2	10	7
y	1	3	4
	y_0	y_1	y_2

$x \notin AP$, $y \notin AP$

$\therefore$ Lagrange's Interpolation is used

$$x = \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} x_0 + \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)} x_1 + \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)} x_2$$

$$x = \frac{(y-3)(y-4)(2)}{(1-3)(1-4)} + \frac{(y-1)(y-4)(10)}{(3-1)(3-4)} + \frac{(y-1)(y-4)}{(4-1)(4-3)} 17$$

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$$x = \frac{y^2-7y+12}{(-2)(-3)}(2) + \frac{y^2-5y+4}{(2)(-1)}(10) + \frac{y^2-4y+3}{(3)(1)}(17)$$

$$= \frac{1}{3}(y^2-7y+12) - 5(y^2-5y+4) + \frac{17}{3}(y^2-4y+3)$$

$$x = y^2 + 0y + 1$$

$$x = y^2 + 1$$

$$x(5) = (5)^2 + 1 = 26$$

$$y(5) = 5 = y^2 + 1 \Rightarrow y^2 = 4 \Rightarrow y = 2$$

[$y = -2$ can be ignored]

Q Find the distance moved by particle & acc^n at the end of ~~acceleration~~ 4sec if the time v/s velocity values are given accordingly.

	t_0	t_1	t_2	t_3
t	0	1	3	4
v	21	15	12	10
	v_0	v_1	v_2	v_3

$$v = \frac{(t-t_1)(t-t_2)(t-t_3)}{(t_0-t_1)(t_0-t_2)(t_0-t_3)}v_0 + \frac{(t-t_0)(t-t_2)(t-t_3)}{(t_1-t_0)(t_1-t_2)(t_1-t_3)}v_1$$

$$+ \frac{(t-t_0)(t-t_1)(t-t_3)}{(t_2-t_0)(t_2-t_1)(t_2-t_3)}v_2 + \frac{(t-t_0)(t-t_1)(t-t_2)}{(t_3-t_0)(t_3-t_1)(t_3-t_2)}v_3$$

distance, $s = \int_0^t v\,dt$ [i.e $\frac{ds}{dt} = v$, $ds = v\,dt$, $s = \int_{t_0}^{t_n} v\,dt$, $t_0 = 0$, $t_n = 4$]

acceleration $= a = \left.\frac{dv}{dt}\right|_{t=4}$

$$v = \frac{(t-1)(t-3)(t-4)}{(-1)(-3)(-4)}(21) + \cancel{\frac{(t-0)(t-3)(t-4)}{(\;)}}$$

$(t-1)(t-3)(t-4) \Rightarrow (t^2-3t-t+3)(t-4) = t^3-4t^2-3t^2+12t - \ldots + 4t + 3t - 12$

$t(t-3)(t-4) = (t^2-3t)(t-4) = t^3-4t^2-3t^2+12t \qquad = t^3 - 8t^2 + 19t - 12$

$t(t-1)(t-4) = (t^2-t)(t-4) = t^3 -$

$$+ \frac{(t-0)(t-3)(t-4)}{(1-0)(1-3)(1-4)}(15) + \frac{(t-0)(t-1)(t-4)}{(3-0)(3-1)(3-4)}(12)$$

$$+ \frac{(t-0)(t-1)(t-3)}{(4-0)(4-1)(4-3)}(10)$$

$$\Rightarrow \cancel{x = \frac{5}{12}t^3 + \frac{19t^2}{6}}$$

$$v = \frac{7}{4}(t^2-4t+3)(t-4) + \frac{5}{2}(t^3-7t^2+12t) - 2(t^3-5t^2+4t)$$
$$+ \frac{5}{6}(t^3-4t^2+3t)$$

$$v = \frac{7}{4}(t^3-8t^2+19t-12) + \frac{5}{2}(t^3-7t^2+12t) - 2(t^3-5t^2+4t)$$
$$+ \frac{5}{6}(t^3-4t^2+3t)$$

$$v = -\frac{5t^3}{12} + \frac{19}{6}t^2 - \frac{35}{4}t + 21$$

$$s = \int_0^4 v\,dt = \left. -\frac{5}{12}\frac{t^4}{4} + \frac{19}{6}\frac{t^3}{3} - \frac{35t^2}{4\times 2} + 21t \right]_0^4$$

$$= \frac{-5(4^4)}{48} + \frac{19(4^3)}{18} - \frac{35}{8}(4)^2 + 21(4)$$

$$= 54.88$$

$$a = \left.\frac{dv}{dt}\right|_{t=4} = \left[-\frac{5t^2}{4} + \frac{19}{3}t - \frac{35}{4}\right]_{t=4}$$

$$= -\frac{5(4)^2}{4} + \frac{19}{3}(4) - \frac{35}{4} = -3.4167$$

Q. Find a root of the eq $f(x) = 0$ given that $f(30) = -30$, $f(34) = -13$, $f(38) = 3$, $f(42) = 18$

Find x where $y = 0$ → Apply Lagrange's interpolation inversely

x	30	34	38	42
y	-30	-13	3	18

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} x_1$$

$$+ \frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)} x_3$$

$y = 0$

$$x = \frac{(13)(-3)(-18)}{(-17)(-33)(-48)}(30) + \frac{(30)(-3)(-18)}{(17)(-16)(-31)}(34)$$

$$+ \frac{(30)(13)(-18)}{(33)(16)(-15)}(38) + \frac{(30)(13)(-3)}{(48)(31)(15)}(42)$$

$$= -0.7820 + 6.5322 + 33.68 - 2.2016$$

$$= 37.229$$

22 Oct '19

Q Interpolate the voltage at $i = 1.15A$

	i_0	i_1	i_2	i_3	i_4
i =	0.25	0.75	1.25	1.5	2
V =	−0.45	−0.6	0.7	1.88	6

$$v = \frac{(i-i_1)(i-i_2)(i-i_3)(i-i_4)}{(i_0-i_1)(i_0-i_2)(i_0-i_3)(i_0-i_4)} v_0 + \dots$$

$$v = \frac{(0.4)(-0.1)(-0.35)(-0.85)(-0.45)}{-(0.5)(-1)(-1.25)(-1.75)} + \frac{(0.9)(-0.1)(-0.35)(-0.85)(-0.6)}{(0.5)(-0.5)(-0.75)(-1.25)}$$

$$+ \frac{(0.9)(0.4)(-0.35)(-0.85)(0.7)}{(1)(0.5)(-0.25)(-0.75)} + \frac{(0.9)(0.4)(-0.1)(-0.85)(1.88)}{(1.25)(0.75)(0.25)(-0.5)}$$

$$+ \frac{(0.9)(0.4)(-0.1)(-0.35)(6)}{(1.75)(1.25)(0.75)(0.5)}$$

$$v = + 0.001424 - 0.068544 + 0.799 - 0.4904 + 0.09216$$

$$v = 0.337$$

Q $y(3) = 6$, $y(5) = 24$, $y(7) = 58$, $y(9) = 108$, $y(11) = 174$

Find x when $y = 100$.

x	3	5	7	9	11
y	6	24	58	108	174

Applying Lagrange's interpolation inversely.

$$x = \frac{(94)(76)(42)(-8)(-74)(3)}{(-18)(-52)(-102)(-168)} + \frac{(94)(42)(-8)(-74)(5)}{(18)(-34)(-84)(-150)}$$

$$+ \frac{(94)(76)(-8)(-74)(7)}{(52)(34)(-50)(-116)} + \frac{(94)(76)(42)(-74)(9)}{(102)(84)(50)(-66)}$$

$$+ \frac{(94)(76)(42)(-8)(11)}{\dots}$$

$= +0.3534 - 1.51546 + 2.8870 + 7.0675 - 1.5054$

8.656

Find
Q. Voltage drop across capacitor $= 10^{-5}F$ when $t = t_n$, $D = ?$

$i(t) = (60-t)^2 + (60-t)\sin(\sqrt{t})$.

$$V = \frac{1}{C}\int_0^{t_n} i(t)\,dt$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^{T}[i(t)]^2\,dt}$$

Scanned by CamScanner

$$\int_{x_0}^{x_n} f(x)dx = \int_0^n f(x_0+ph)\,h\,dp \qquad \left[\because x = x_0+ph \Rightarrow dx = h\,dp\right]$$

$$= h\int_0^n \left[f(x_0) + \frac{p(p-1)\Delta^2 f(x_0)}{2!} + \dots\right]dp$$

$n=1$ $x_1 \Rightarrow$ Trapezoidal rule [Not in syllabus]

$$\int_{x_0}^{x_1} f(x)dx = h\left[\frac{f(x_0)+f(x_1)}{2}\right] = \frac{h}{2}(y_0+y_1)$$

$$\int_{x_0}^{x_2} f(x) = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx$$

$$= \frac{h}{2}(y_0+y_1) + \frac{h}{2}(y_1+y_2) = \frac{h}{2}[\underbrace{y_0}_{\text{1st}} + \underbrace{y_2}_{\text{last}} + \underbrace{2y_1}_{\text{remaining}}]$$

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[y_0+y_n+2(y_1+y_2+y_3 \dots y_{n-1})\right]$$

$n =$ multiples of 2

$$\int_{x_0}^{x_n} y\,dx = \int_{x_0}^{x_2} y\,dx + \int_{x_2}^{x_4} y\,dx + \dots$$

$$= \frac{h}{3}[y_0+4y_1+y_2] + \frac{h}{3}[y_2+4y_3+y_4] + \dots$$

$$= \frac{h}{3}\left[y_0+y_n+4(y_1+y_3+y_5+\dots) + 2(y_2+y_4+\dots y_{n-2})\right]$$

$n =$ multiples of 3 simpson's $\frac{3}{8}^{th}$ rule

$$\int_{x_0}^{x_n} y\,dx = \int_{x_0}^{x_3} y\,dx + \int_{x_3}^{x_6} y\,dx + \int_{x_6}^{x_9} y\,dx + \dots$$

$$= \frac{3h}{8}\left[y_0 + 3y_1 + 3y_2 + y_3\right] + \frac{3h}{8}\left[y_3 + 3y_4 + 3y_5 + y_6\right].$$

$$= \frac{3h}{8}\left[y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5 + \dots) + 2(y_3 + y_6 + y_9 + \dots y_{n-3})\right]$$

$n = 6$ Weddle's rule

$$\int_{x_0}^{x_1} y\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$$

$n = 2k$ 1 4 1 (last term) → simpson's 1/3 rule → $\frac{h}{3}[\quad]$

2 → n = multiples of 2.

$y_0\ y_1\ y_2$

$n = 3k$ 1 3 3 1 (last term) Simpson's $\frac{3}{8}$th rule

y_0 others 2 → n = multiples of 3

$\frac{3h}{8}[\quad]$.

15|6|51

$n = 6k$ Weddle's rule → $\frac{3h}{10}[\quad]$

1 5 1 6 1 5 1

Q) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by simpon's rule taking 4 equal strips

$\frac{(1-0)}{4} = 0.25$

$y = \frac{1}{1+x^2}$.

x	0	0.25	0.5	0.75	1
y	1	16/17	4/5	16/25	0.5
	y_0	y_1	y_2	y_3	y_4

$n = 4 \Rightarrow$ Simpson's $\frac{4}{3}$ rule

1 4 1 (n = last)

2 → even

n = odd

$y_0\ y_1\ y_2$

$$I = \frac{h}{3}\left[y_0 + y_4 + 4(y_1 + y_3) + 2(y_2)\right]$$

$$= \frac{(0.25)}{3}\left[1 + 0.5 + 4\left(\frac{16}{17} + \frac{16}{25}\right) + 2\left(\frac{4}{5}\right)\right]$$

$=$

1) Obtain Fourier series of

a) $f(x) = \sin(mx)$ $(-l, l)$ $m =$ neither an integer nor zero.

b) $f(x) = 2x - x^2$ $(0, 3)$ and hence deduce

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$$

c) $f(x) = \begin{cases} x & -1 < x < 0 \\ x+2 & 0 < x < 1 \end{cases}$

2) Obtain the complex Fourier of $f(x) = x$ $(0, 2\pi)$

3) Compute the Fourier series of $f(x)$ upto ~~f(x)~~ 2^{nd} harmonic given

x	0	2	4	6	8	10	12
y	9.0	18.2	24.4	27.8	27.5	22.0	9.0

4) Solve the integral eq^n $\int_0^\infty f(x)\cos(sx) = \begin{cases} 1-s & 0 \le s \le 1 \\ 0 & s > 1 \end{cases}$. Hence deduce that

$$\int_0^\infty \frac{1-\cos x}{x^2}dx = \int_0^\infty \frac{\sin^2 x}{x^2} = \frac{\pi}{2}.$$

5) Apply the Parseval's identity for fourier cosine transform and show that

$$\int_0^\infty \frac{\sin(ax)}{x(a^2+x^2)}\,dx = \frac{\pi(1-e^{-a^2})}{2a^2} \qquad a > 0$$

6) Apply parseval's identity for Fourier transform on $f(x) = \begin{cases} a - |x| & |x| \le a \\ 0 & |x| > a \end{cases}$ and show that

$$\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2} \quad \text{and} \quad \int_0^\infty \left(\frac{\sin t}{t}\right)^4 = \frac{\pi}{3}$$

$f(-x) = 2(-x) - (-x)^2$
$= -2x - x^2$

1) a) $f(x) = \sin(mx)$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{n\pi t}{l}\right) + b_n\sin\left(\frac{n\pi t}{l}\right)\right)$$

$$a_0 = \cancel{\frac{1}{l}\int_{-l}^{l} f(x)\,dx} = \cancel{\frac{1}{l}\int_{-l}^{l}\sin(mx)\,dx}$$

$$= \cancel{\frac{1}{l}\left[\frac{-\cos(mx)}{m}\right]_{-l}^{l}} =$$

a_0 and $a_n = 0$

$$b_n = \frac{1}{l}\int_{-l}^{l}\sin(mx)\sin\left(\frac{\pi x n}{l}\right)dx.$$

$$= \frac{1}{2l}\int_{-l}^{l}\left[\cos\left(mx - \frac{\pi x n}{l}\right) - \cos\left(mx + \frac{\pi x n}{l}\right)\right]dx.$$

$$= \frac{1}{2l}\int_{-l}^{l}\left[\cos\left(\overline{m - \frac{\pi n}{l}}\,x\right) - \cos\left(\overline{m + \frac{\pi n}{l}}\,x\right)\right]dx$$

$$= \frac{\cancel{2}}{\cancel{2}l}\left[\frac{\sin\left(\overline{m - \frac{\pi n}{l}}\,x\right)}{m - \frac{\pi n}{l}} - \frac{\sin\left(\overline{m + \frac{\pi n}{l}}\,x\right)}{m + \frac{\pi n}{l}}\right]_0^l$$

$$= \frac{1}{l}\left[\frac{\sin(ml - \pi n)}{m - \frac{\pi n}{l}} - \frac{\sin(ml + \pi n)}{m + \frac{\pi n}{l}}\right]$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$= \sum_{n=1}^{\infty}\frac{1}{l}\left[\frac{\sin(ml - \pi n)}{m - \frac{\pi n}{l}} - \frac{\sin(ml + \pi n)}{m + \frac{n\pi}{l}}\right]\sin\left(\frac{n\pi x}{l}\right)$$

$c + 2l \Rightarrow 3$

$0 + 2l = 3$

b) $f(x) = 2x - x^2$ in the interval $(0,3)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)\right)$$

$$a_0 = \frac{2}{3}\int_0^3 (2x - x^2)\,dx \Rightarrow \frac{2}{3}\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$= \frac{2}{3}\left[\frac{\cancel{2}(3^2)}{\cancel{2}} - \frac{(3^3)}{3}\right] = 0$$

$$a_n = \frac{2}{3}\int_0^3 (2x - x^2)\cos\left(\frac{n\pi x}{3/2}\right)dx$$

$$= \frac{2}{3}\left[\frac{(2x-x^2)\sin\left(\frac{2n\pi x}{3}\right)}{\frac{2n\pi}{3}} - (2-2x)\left[\frac{-\cos\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)^2}\right] + (-2)\left[\frac{-\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)^2}\right]\right]_0^3$$

$$= \frac{2}{3}\Big[\ \big($$

5) $\displaystyle\int_0^\infty \frac{\sin ax}{x(a^2+x^2)}\,dx = \frac{\pi(1-e^{-a^2})}{2a^2}, \quad a>0.$

Taking $f(x) = \begin{cases} 1, & 0<x<a \\ 0, & x>a \end{cases}$ and $g(x) = e^{-ax}$ we have

$$F_c[f(x)] = \int_0^\infty f(x)\cos(ux)\,dx = \int_0^a 1.\cos ux\,dx = \left[\frac{\sin ux}{u}\right]_0^a$$

$$= \cancel{\frac{\sin ax}{a}} \quad \frac{\sin ax}{u}$$

$$F_c[g(x)] = F_c(u) = \frac{a}{a^2+u^2}$$

Using Parseval's identity.

$$\frac{2}{\pi}\int_0^\infty F_c(u)\,G_c(u)\,du = \int_0^\infty f(x)\,g(x)\,dx$$

$$\frac{2}{\pi}\int_0^\infty \frac{\sin au}{u}\cdot\frac{a}{a^2+u^2}\,du = \int_0^a f(x)g(x)\,dx + \int_a^\infty f(x)g(x)\,dx$$

$$\Rightarrow \frac{2a}{\pi}\int_0^\infty \frac{\sin au}{u(a^2+u^2)}\,du = \left[\frac{e^{-ax}}{-a}\right]_0^a$$

7th Nov

Evaluate $\displaystyle\int_0^2 e^{x^2}dx$ by taking 10 equal sub intervals.

$x_0 = 0, \quad x_n = 2, \quad n = 10$

$x_n = x_0 + nh \Rightarrow 2 = 0 + 0h \Rightarrow h = 0.2$

x	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2.0
y	1	1.04	1.173	1.433	1.896	2.718	4.220	7.099	12.935	25.533	54.59

n = even no. [not divisible by 3 or 6]

Simpson's $\frac{1}{3}$rd rule. $\Rightarrow$

$x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow dx = \frac{dt}{2\sqrt{t}}$

$e^t \frac{dt}{2\sqrt{t}}$

$$I = \frac{h}{3}\left[y_0 + y_{10} + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)\right]$$

$$\Rightarrow \frac{0.2}{3}\left[1 + 54.59 + 4(1.04 + 1.433 + 2.718 + 7.099 + 25.533) + 2(1.173 + 1.896 + 4.220 + 12.935)\right]$$

$$\frac{0.2}{3}\left[1 + 54.59 + 151.292 + 40.448\right]$$

$\Rightarrow 16.4886$ //

Q) Evaluate $\int_0^6 \frac{\sin x}{x}\,dx$ using weddle's rule

Weddle's rule $\Rightarrow$ n = multiples of 6

let $n = 6$

$x_0 = 0 \qquad x_n = 6 \qquad x_n = x_0 + nh \Rightarrow 6 = 0 + 6h$

$h = 1$

x	0	1	2	3	4	5	6
$y = \frac{\sin x}{x}$	1	0.8414	0.454	0.047	−0.189	−0.191	−0.0465

$$I = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$$

$$I = \frac{3(1)}{10}\left[1 + 5(0.841) + 0.454 + 6(0.047) + (-0.189) + 5(-0.191) + (-0.0465)\right]$$

$I = 1.4251$

Q)

Q.

distance s, ft	0	10	20	30	40	50	60
velocity v, ft per sec	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft.

$$\frac{ds}{dt} = v \Rightarrow dt = \frac{ds}{v}$$

$$\int_0^T dt = \int_0^{60} \frac{ds}{v} \Rightarrow t\Big]_0^T = \int_0^{60} \frac{ds}{v}$$

$$T = \int_0^{60} \frac{1}{v}\, ds$$

1 3 3 ~~1~~ → 2 for y → last

y_0 y_1 y_2 y_3 other y_{3k}

$$I = \int_{x_0}^{x_n} y\,dx \Rightarrow y = \frac{1}{v}$$

$$I = \frac{3h}{8}\left[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3\right]$$

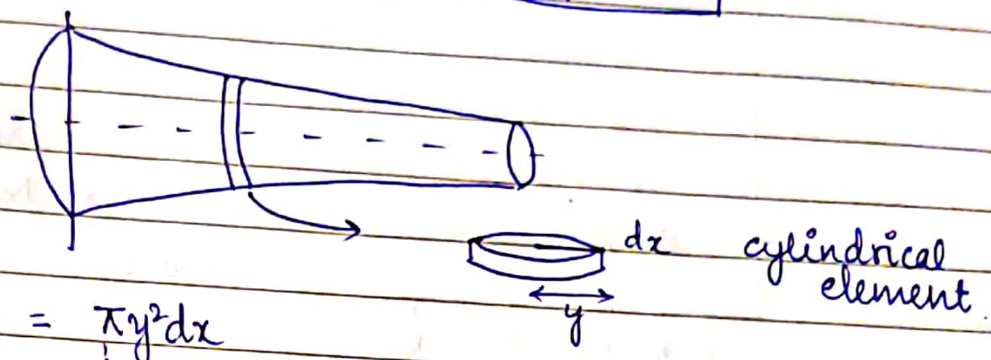
$$T = \frac{3h}{8}\left[\frac{1}{V_0} + \frac{1}{V_6} + 3\left(\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_4} + \frac{1}{V_5}\right) + 2\left(\frac{1}{V_3}\right)\right]$$

$$T = \frac{3 \times 10}{8}\left[\frac{1}{47} + \frac{1}{38} + 3\left(\frac{1}{58} + \frac{1}{64} + \frac{1}{61} + \frac{1}{52}\right) + 2\left(\frac{1}{65}\right)\right]$$

$$T = 1.0643$$

Q) Compute the volume of solid of Revolutions formed by rotating about x-axis. The area between x-axis, x=0, x=1 and the curve through the points given below.

x	0.00	0.25	0.50	0.75	1.00
y	1	0.9896	0.9589	0.9089	0.8415



$$dV = \pi y^2 dx$$

$$V = \pi \int_0^1 y^2 dx \qquad y^2 = Y \Rightarrow V = \pi \int_0^1 Y dx$$

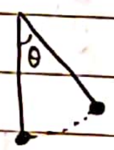
$n = 4 \Rightarrow$ simpson's $\frac{1}{3}$ rule as $3/n$ and $6/n$ but $2/n$

$$V = \pi \left[\frac{h}{3} \left(Y_0 + Y_4 + 4(Y_1 + Y_3) + 2Y_2 \right) \right]$$

5th Nov

$$\frac{dy}{dx} = f(x) \qquad \int_a^b y\,dx$$

$$\frac{dy}{dx} = f(x,y) \qquad y(x_0 = y_0)$$



1) $\frac{dy}{dx} = x + y^2$

$$\frac{d^2\theta}{dt^2} = -\omega^2 \sin\theta$$

2) $\frac{dy}{dx} = x + \sin y$

$\theta \to$ small $\Rightarrow \sin\theta \sim \theta$

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

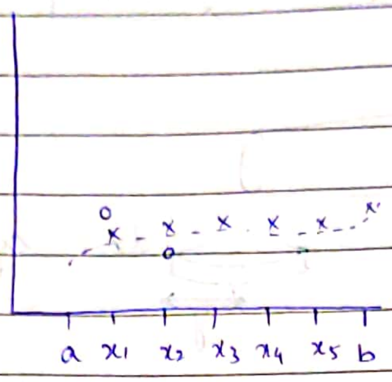
3) $\frac{dy}{dx} = xy + y^2 \rightarrow$ Bernoulli's

eq^n 3 is non linear but has analytical soln

If a soln of $\frac{dy}{dx} = f(x,y) \qquad y(x_0) = y_0$

i) $F(x,y) = c$ [i.e a curve]

Problem defined i.e $\frac{dy}{dx} = f(x,y) =$ slope of the ~~curv~~ curve.



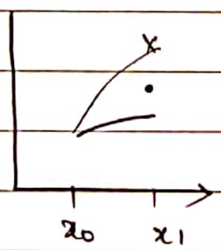
- - - - exact
× × × × Numerical
Euler's.

Suppose (x_0, y_0) (x_1, y_1) be approximated by a line $y = y_0 + hf(x_0, y_0)$ → Euler's method

Euler's Modified method.

$$y_1^{(0)} = y_0 + hf(x_0, y_0) \rightarrow \text{Approximation}$$

$$y_1^{(k+1)} = y_0 + h\{f(x_0, y_0) + f(x_1, y_1^{(k)})\}/2, \quad k = 0, 1, 2, 3 \ldots$$



× Euler's
• Euler's modified method
— exact soln.

Q) Solve $\frac{dy}{dx} = x + \sin(y)$ $y(0) = 1$, for $x = 0.2$ and 0.4 by Euler's modified method

$x_0 = 0$ $y_0 = 1$

$x_1 = 0.2$, $x_2 = 0.4$ ⇒ $h = 0.2$

Euler's modified method

$$y_1^{(0)} = y_0 + hf(x_0, y_0)$$

classmate

Date

Page

$$y_1^{(k+1)} = y_0 + h\left\{\frac{f(x_0,y_0) + f(x_1, y_1^{(k)})}{2}\right\} \quad , k=0,1$$

[for test, exam or as mention in Qn]

$$y_1^{(0)} = 1 + 0.2\{0 + \sin(1)\} = 1.16829$$

$$k=0, \quad y_1^{(1)} = 1 + \frac{0.2}{2}\{0 + \sin(1) + 0.2 + \sin(1.16829)\}$$

$$= 1.19615$$

$$k=1, \quad y_1^{(2)} = 1 + \frac{0.2}{2}\{0 + \sin(1) + 0.2 + \sin(1.19615)\}$$

$$= 1.19721$$

let $y_1 \sim 1.19721$

$$y_2^{(0)} = y_1 + hf(x_1, y_1)$$

$$y_2^{(k+1)} = y_1 + \frac{h}{2}\{f(x_1,y_1) + f(x_2, y_2^{(k)})\}$$

$$y_2^{(0)} = 1.19721 + \frac{0.2}{2}\{0.2 + \sin(1.19721)\}$$

$$= 1.42341$$

$$k=0, \quad y_2^{(1)} = 1.19721 + \frac{0.2}{2}\{f(x_1,y_1) + f(x_2, y_2^{(0)})\}$$

$+ \sin(1.19721) + 0.4$

$$= 1.19721 + \frac{0.2}{2}\{0.2 + \sin(1.42341)\} = 1.44923$$

$$k=1, \quad y_2^{(2)} = 1.19721 + \frac{0.2}{2}\{0.2 + \sin(1.19721) + 0.4 + \sin(1.44923)\}$$

$$= 1.44922$$

Solve $\frac{dy}{dx} = \sqrt{1+xy}$ $y(0)=1$, for $y(0.075)$ using 3 iterations of the Euler's modified method

$x_0 = 0$, $y_0 = 1$

let $x_1 = 0.075 \Rightarrow h = 0.075$

$y_1^{(0)} = y_0 + hf(x_0, y_0)$

$y_1^{(0)} = 1 + \frac{0.075}{2}\left[\sqrt{1+(0)(1)}\right] = 1.0375$

$y_1^{(k+1)} = y_0 + \frac{h}{2}\left\{f(x_0, y_0) + f(x_1, y_1^{(k)})\right\}$ $k = 0, 1, 2$

$x_0 = 0$; $\cancel{x}n$

$1331 \rightarrow y_n$ 151615 1

$y_0 \leftarrow$ $2(y_3 + y_6 y_9) + 5y_1 + y_2 + 6y_3 + 5y_4 + 5y_5 + y_6$

$F\left[e^{-a^2x^2}\right] = \int_{-\infty}^{\infty} \exp\left[-\left\{\left(ax - \frac{is}{2a}\right)^2\right\} + \frac{s^2}{4a^2}\right] dx$

$= \int_{-\infty} \exp \{$

Solve the integral eq^n $\int_0^\infty f(x)\cos(sx)dx = e^{-s}$

find $f(x)$

given $F_c[f(x)] = e^{-s}$

$f(x) = F_c^{-1}[e^{-s}]$

Ternai 141

2(even)

1331

$2 \rightarrow$ (multiples of 3)

HW Solve the integral eq $\int_0^\infty f(x)\sin(sx)dx = \begin{cases} 1 & 0<s<1 \\ 2 & 1<s<2 \\ 0 & s>2 \end{cases}$

MATLAB

1.
```
x0 = 0; xn = 1; tp = 5;
x = linspace (x0, xn, tp);
y = 1./(1+x.^2);
h = (xn - x0)/(tp - 1);
I = (h/3)*(y(1)+y(tp)+ 4*sum(y(2:2:tp-1))+
                2*sum(y(3:2:tp-2)))
```

indexing for Matrix starts from 1.

y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y
1	2	3	4	5	6	7	8	9

sum(y(2 : 2 : tp-1))

start, difference, last

sum(y(2) y(4) y(6) y(8))