~~TAYLOR SERIES~~

UNIT-2

FOURIER THEOREM

If $f(t)$ is a periodic function which satisfies certain conditions, then it can be written as a sum of a number of sine functions of different amplitudes, frequency/periods and phases.

$$f(t) = A_0 + A_1 \sin(\omega t + \phi_1) + A_2 \sin(2\omega t + \phi_2) + A_3 \sin(3\omega t + \phi_3) \cdots + A_n \sin(n\omega t + \phi_n).$$

$$A_n \sin(n\omega t) = a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

The above eqn is called the nth harmonic.

$a_1 \cos\omega t + b_1 \sin\omega t \longrightarrow 1^{st}$ harmonic or fundamental harmonic or fundamental mode.

$A_0 = \frac{a}{2}$, $\omega = \frac{2\pi}{T}$, $T = 2L$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{L}\right) + \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi t}{L}\right)$$

a_0, a_n, b_n are called Fourier coefficients.

DIRICHLET THEOREM/CONDN

If $f(x)$ is defined in an interval $(C, C+2L)$ or $[C, C+2L]$

(i) is finite valued and periodic with $T = 2L$

(ii) has finite no. of maxima & minima.

(iii) has finite no. of finite discontinuities.

then $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right.$

converges to $f(x)$ at all points of continuity and to the avg. value of right hand limit and left hand limit at pts. of discontinuity.

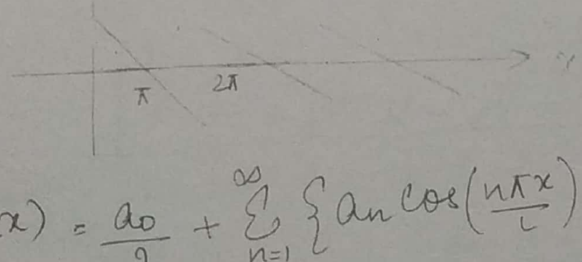
$$a_0 = \frac{1}{L} \int_c^{c+2L} f(x)\,dx$$

$$a_n = \frac{1}{L} \int_c^{c+2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$b_n = \frac{1}{L} \int_c^{c+2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx.$$

The above three equations are known as Euler formulae.

→ obtain the fourier series of $f(x) = \frac{\pi - x}{2}$ which is a ~~for~~ periodic function in $(0, 2\pi)$, and hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \cdots\cdots = \frac{\pi}{4}$.



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right\}$$

$(c, c+2L) = (0, 2\pi) \Rightarrow c = 0$ & $L = \pi$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos(nx) + b_n \sin(nx) \right\}$$

$$a_0 = \frac{1}{L} \int_c^{c+2L} f(x) = \frac{1}{\pi} \int_0^{2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos(nx)\,dx, \quad b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin(nx)\,dx$$

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$\cos(n(2\pi - x)) = \cos(2n\pi - nx) = \cos nx \longrightarrow$ even fn

$\sin(n(2\pi - x)) = -\sin nx. \quad \rightarrow$ odd fn

$$\int_0^{2a} F(x)\,dx = \begin{cases} 2\int_0^{a} F(x)\,dx & F(2a-x) = F \\ 0 & F(2a-x) = -F \end{cases}$$

Now, $f(2\pi - x) = \dfrac{\pi - (2\pi - x)}{2} = \dfrac{-\pi + x}{2} = -\left(\dfrac{\pi - x}{2}\right)$

$f(2\pi - x) = -f(x) \rightarrow$ odd function.

$\therefore f(x)$ is odd like fn, $a_0 = 0$

$\because$ odd like fn $\times$ even like fn = odd like fn ; $a_n = 0$

~~b_n ~~ ~~even.~~

b_n will be an even like fn.

$$b_n = \frac{1}{\pi}\, 2\int_0^{\pi} f(x)\sin(nx)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi - x}{2}\right)\sin(nx)\,dx$$

Bernoulli's rule:

$$\int uv = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots$$

$$b_n = \frac{1}{\pi}\left[\left(\frac{\pi - x}{2}\right)\right.$$

$$b_n = \frac{1}{\pi}\left[(\pi - x)\left\{-\frac{\cos(nx)}{n}\right\} - (-1)\left\{-\frac{\sin(nx)}{n^2}\right\}\right]_0^{\pi}$$

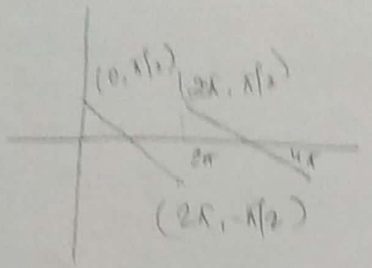
$$= \frac{1}{\pi}\left[0\left\{-\frac{\cos n\pi}{n}\right\} - \pi\left\{-\frac{\cos 0}{n}\right\} + \frac{1}{n^2}\left\{\cancel{\sin n\pi - \sin(0)}\right\}\right]$$

$$= \frac{1}{\pi} \times \frac{\pi}{n} = \frac{1}{n}$$

$$f(x) = 0 + 0 + \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{n} \sin(nx)$$

$f(0) = \frac{\pi}{2}$ & $f(2\pi) = -\frac{\pi}{2}$



→ End pts are discontinuous

$x = \frac{\pi}{2}$ [point of continuity].

$$f\left(\frac{\pi}{2}\right) = \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right)$$

$$\frac{\pi - \pi/2}{2} = \frac{1}{1}\sin\left(\frac{\pi}{2}\right) + \frac{1}{2}\sin\left(\frac{2\pi}{2}\right) + \frac{1}{3}\sin\left(\frac{3\pi}{2}\right) + \frac{1}{4}\sin\left(\frac{4\pi}{2}\right) + \frac{1}{5}\sin\left(\frac{5\pi}{2}\right) + $$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} + \cdots$$

Q→ Periodic fn $f(x) = x^2$ in $(-\pi, \pi)$ → And hence deduce

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$

$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} = \frac{\pi^2}{12}$

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right\}$$

$(c, c+2L) = (-\pi, \pi)$

$c = -\pi$ & $L = \pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos(nx) + b_n \sin(nx) \right\}$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)\,dx \qquad b_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \sin(nx)\,dx.$$

$f(x) \rightarrow$ even like f^n

$a_n \rightarrow$ even like f^n

$b_n \rightarrow$ odd like f^n

$a_0 \rightarrow$ even like f^n

$F(-x) = -F(x) \rightarrow$ odd

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} x^2\,dx = \frac{x^3}{3}\Big|_0^{\pi} \times \frac{2}{\pi}$$

$$a_0 = \frac{2\pi^2}{3}$$

$(b_n = 0) \rightarrow$ odd like f^n

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2\cos(nx)\,dx$$

$$\cancel{= \frac{2}{\pi}\left[x^2\left\{+\frac{\sin(nx)}{n}\right\} - 2x\left\{-\frac{\sin nx}{n^2}\right.\right.}$$

$$= \frac{2}{\pi}\left(x^2\left\{\frac{\sin(nx)}{n}\right\} - 2x\left\{-\frac{\cos(nx)}{n^2}\right\} + 2\left\{-\frac{\sin(nx)}{n^3}\right\}\right)$$

$$\cancel{= \frac{2}{\pi}\left[0 + 2\pi\left(\frac{\cos \pi n}{n^2}\right)\right]}$$

$$\cancel{= \frac{2}{\pi} \times 2\pi \times -\frac{1}{n^2}}$$

$$\cancel{= -\frac{4}{n^2}}$$

$$= \frac{2}{\pi} \times \frac{2}{n^2}\left[\pi\cos n\pi - 0\right]$$

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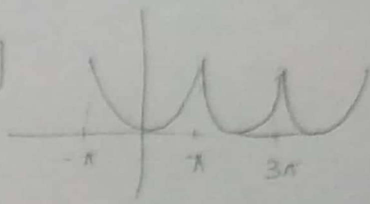
$\pi \qquad n^2 \quad n(-1) \qquad \cos(n\pi) = \begin{cases} 1 & n \to \text{even} \\ -1 & n \to \text{odd} \end{cases} = (-1)^n$

$$f(x) = \frac{1}{2} \times \frac{2\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos(nx)$$

$$f(x) = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n \cos(nx)}{n^2}$$

Now, $f(-\pi) = \pi^2 = f(\pi)$

$\Rightarrow f(x)$ is continuous in $[-\pi, \pi]$



* when $x = 0$

$$f(0) = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n \cos(0)}{n^2}$$

$$0 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}.$$

$$\frac{\pi^2}{3} = -4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

$$\frac{\pi^2}{12} = \frac{(-1)^2}{1} + \frac{(-1)^3}{2^2} + \frac{(-1)^4}{3^2} + \cdots$$

$$\frac{\pi^2}{2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

* ~~w~~ Now $(-1)^n (-1)^n = (-1)^{2n} = 1$

$\hookrightarrow \cos(n\pi) = 1^n$

$x = \pi$

$$f(\pi) = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)}{n^2}$$

$$\pi^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{n^2}$$

$$\cancel{\frac{2\pi^2}{3}} + x$$

$$\frac{2\pi^2}{3} = 4\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \quad \text{--- (2)}$$

(1) + (2)

$$\frac{\pi^2}{6} + \frac{\pi^2}{12} = 2\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} \cdots\right)$$

$$\frac{3\pi^2}{12} = 2\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$$

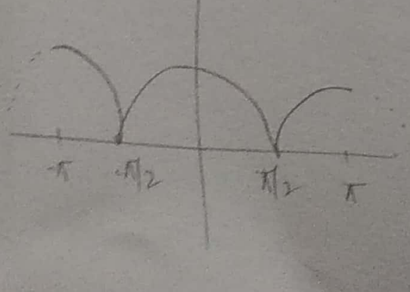
$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

Q→ Obtain FS of periodic fn $f(x) = |\cos x|$ in $(-\pi, \pi)$

Limits of integration

Do we get a fourier series for all values of n.

$$f(x) = |\cos x| = \begin{cases} -\cos x & -\pi < x < -\pi/2 \\ \cos x & -\pi/2 < x < \pi/2 \\ -\cos x & \pi/2 < x < \pi \end{cases}$$



$f(-x) = |\cos(-x)| = |\cos x| \rightarrow$ its an even fn.

$\Rightarrow b_n = 0$.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{1}{L}\int_c^{c+2L} f(x)\,dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2}\cos x + \int_{\pi/2}^{\pi} -\cos x\right]$$

$$= \frac{2}{\pi}\left[\sin x\Big|_0^{\pi/2} + (-\sin x)\Big|_{\pi/2}^{\pi}\right]$$

$$a_0 = \frac{4}{\pi}$$

$$a_n = \frac{1}{L}\int_c^{c+2L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx.$$

$$= \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)\,dx$$

$$\frac{2}{\pi}\left[\int_0^{\pi/2}\cos x\cos(nx)\,dx + \int_{\pi/2}^{\pi} -\cos x\cos(nx)\,dx\right]$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2}\frac{1}{2}\{\cos(nx+x) + \cos(nx-x)\} - \int_{\pi/2}^{\pi}\frac{1}{2}\{\cos(nx+x) + \cos(nx-x)\}\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/2}\cos((n+1)x) + \cos((n-1)x) - \int_{\pi/2}^{\pi}\cos((n+1)x) + \cos((n-1)x)\right]$$

$$= \frac{1}{\pi}\left\{\left[\frac{\sin((n+1)x)}{n+1} + \frac{\sin((n-1)x)}{n-1}\right]_0^{\pi/2} - \left[\frac{\sin((n+1)x)}{n+1} + \frac{\sin((n-1)x)}{n-1}\right]_{\pi/2}^{\pi}\right.$$

$$= \frac{1}{\pi}\left[\frac{\sin((n+1)\pi/2) - 0}{n+1} + \frac{\sin((n-1)\pi/2) - 0}{n-1}\right.$$

$$- \left[\frac{\cancelto{0}{\sin((n+1)\pi)} - \sin((n+1)\pi/2)}{n+1} + \frac{\cancelto{0}{\sin((n-1)\pi)} - \sin((n-1)\pi/2)}{n-1}\right]$$

$$= \frac{1}{\pi}\left[\frac{\sin((n+1)\pi/2)}{n+1} + \frac{\sin((n-1)\pi/2)}{n-1} + \frac{\sin((n+1)\pi/2)}{n+1} + \frac{\sin((n-1)\pi/2)}{n-1}\right]$$

$$= \frac{1}{\pi}\left[\frac{\cos\left(\frac{n\pi}{2}\right)}{n+1} - \frac{\cos\left(\frac{n\pi}{2}\right)}{n-1} + \frac{\cos\left(\frac{n\pi}{2}\right)}{n+1} - \frac{\cos\left(\frac{n\pi}{2}\right)}{n-1}\right]$$

$$\cancel{\frac{\cos n\pi}{2}}$$

$$= \frac{2}{\pi}\cos\left(\frac{n\pi}{2}\right)\left[\frac{1}{n+1} - \frac{1}{n-1}\right] \text{ when } n \neq 1$$

Evaluating for $n = 1$:

$$a_1 = \frac{2}{\pi}\left\{\int_0^{\pi/2} \frac{1}{2}(\cos 2x + 1)\,dx - \int_{\pi/2}^{\pi} \frac{1}{2}(\cos 2x + 1)\,dx\right\}$$

$$= \frac{1}{\pi}\left\{\left[\frac{\sin 2x}{2} + x\right]_0^{\pi/2} - \left[\frac{\sin 2x}{2} + x\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left[\frac{\pi}{2} - \left(\pi - \frac{\pi}{2}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{2} + \frac{\pi}{2} - \pi\right] = 0$$

$\cancel{f(x) = \frac{1}{2} \times \frac{4}{\pi}}$

$$f(x) = \frac{a_0}{2} + a_1 \cos x + \sum_{n=2}^{\infty} a_n \cos(nx) + 0$$

$$= \frac{4}{\pi} \times \frac{1}{2} + \sum_{n=2}^{\infty} \left\{ \frac{2}{\pi} \cos\left(\frac{n\pi}{2}\right) \left[\frac{1}{n+1} - \frac{1}{n-1} \right] \cos(nx) \right.$$

$$= \frac{2}{\pi} + \sum_{n=2}^{\infty} \frac{2}{\pi} \cos\left(\frac{n\pi}{2}\right) \left[\frac{1}{n+1} - \frac{1}{n-1} \right] \cos(nx)$$

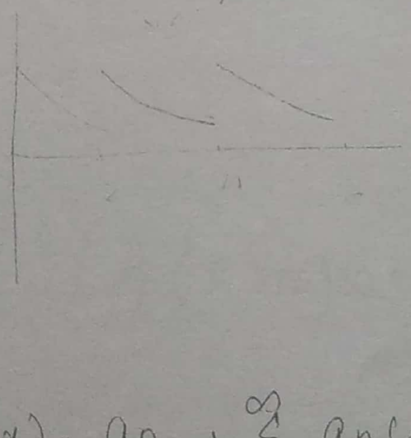
Q→ Obtain FS of the periodic fn $f(x) = e^{-x}$ in $(0,2)$

$(c, c+2L) = (0,2)$

$c = 0 \quad L = 1$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos(nx) + b_n \sin(nx) \right\}$$

$$a_0 = \frac{1}{L} \int_c^{c+2L} f(x)\,dx = \int_0^2 e^{-x}\,dx = -e^{-x}\Big|_0^2 = 1 - e^{-2}$$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

$(c, c+2L) = (0, 2)$

$c = 0 \quad L = 1$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x) + \sum_{n=1}^{\infty} b_n \sin(n\pi x)$$

$$a_0 = \frac{1}{L} \int_c^{c+2L} f(x)\,dx = \int_0^2 e^{-x}\,dx = 1 - e^{-2} = \frac{e^2 - 1}{e^2} = \int_0^2 f(x)\,dx$$

$$a_1 = \int_0^2 f(x) \cos(n\pi x)\,dx \qquad b_1 = \int_0^2 f(x) \sin(n\pi x)\,dx$$

$$a_n = \int_0^2 e^{-x} \cos(n\pi x)\, dx$$

$$\int e^{ax} \cos bx = \frac{e^{ax}}{a^2+b^2} \{ a\cos bx + b \sin bx \}$$

$$\int e^{ax} \sin bx = \frac{e^{ax}}{a^2+b^2} \{ a\sin(bx) - b\cos(bx) \}$$

$$a_n = \left[\frac{e^{-x}}{1+n^2\pi^2} \{ (-1)\cos(n\pi x) + n\pi \sin(n\pi x) \} \right]_0^2$$

$$= \cancel{\frac{1}{1+n^2\pi^2} [(-1)\cos(}$$

$$= \frac{1}{1+n^2\pi^2} \left[e^{-2} \{ -\cos(n2\pi) + n\pi \sin(n2\pi) \} - \{ -\cos(0) + n\pi \sin(0) \} \right]$$

$$= \frac{1}{1+n^2\pi^2} \left[e^{-2} \{ -1 \} + 1 \right]$$

$$= \frac{1}{1+n^2\pi^2} \left[1 - e^{-2} \right]$$

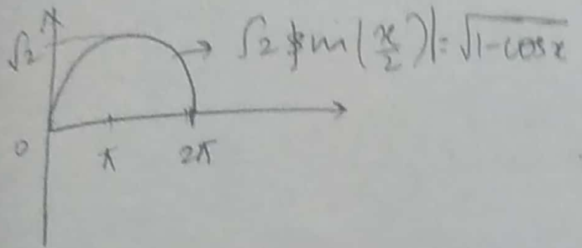
$$b_n = \left[\frac{e^{-x}}{1+n^2\pi^2} \{ (-1)\sin(n\pi x) \mp n\pi \cos(n\pi x) \} \right]_0^2$$

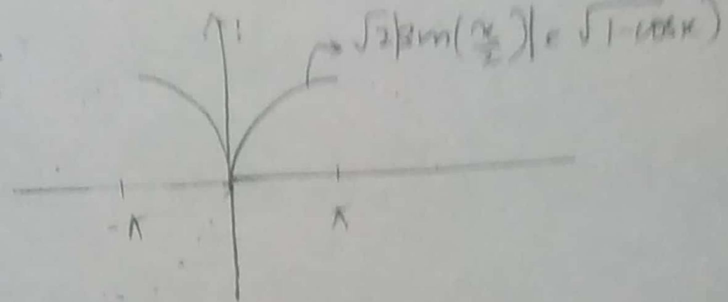
$$b_n = \frac{1}{1+n^2\pi^2} \left[e^{-2} \{ -\sin(n2\pi) - n\pi \cos(n\pi 2) \} - \{ -\sin(0) - n\pi\cos(0) \} \right]$$

$$b_n = \frac{1}{n^2\pi^2+1} [1 - e^{-2}]$$

$$f(x) = \frac{e^2 - 1}{2e^2} + \sum_{n=1}^{\infty} \frac{1}{1+n^2\pi^2} \left[1 - e^{-2}\right]$$

Q→ $f(x) = \sqrt{1-\cos x}$ $(0, 2\pi)$ & $f(x) = \sqrt{1-\cos x}$ $(-\pi, \pi)$





↓

Its an even fn.

$a_0 = \frac{2}{\pi} \int_0^{\pi} \sqrt{2} \sin\left(\frac{x}{2}\right) dx$

$a_n = \frac{2}{\pi} \int_0^{\pi} \sqrt{2} \sin\left(\frac{x}{2}\right) \cos(nx)\, dx$

$b_n = 0$

↓

Its an even fn

$\because \sqrt{1-\cos x} = \sqrt{2}\left|\sin\left(\frac{x}{2}\right)\right|$

$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx = \frac{2}{\pi} \int_0^{\pi} \sqrt{2} \sin\left(\frac{x}{2}\right) dx$

$a_n = \frac{2}{\pi} \int_0^{\pi} \sqrt{2} \sin\left(\frac{x}{2}\right) \cos(nx)\, dx$

$b_n = 0$

Q → Obtain fourier series of periodic fn $f(x) = x\cos\left(\frac{\pi x}{l}\right)$ in the interval $(-l, l)$

Ans: $f(x) \to$ odd fn.

$(c, c+2L) = (-l, l)$

$c = -l$

$L = l$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{1}{L}\int_c^{c+2L} f(x)\,dx$$

$$a_0 = \frac{1}{l}\int_{-l}^{l} x\cos\left(\frac{\pi x}{l}\right)dx = 0 \quad \because \text{its an odd fn}$$

$a_n = 0 \quad \because$ its an odd fn

$$b_n = \frac{2}{l}\int_0^l x\cos\left(\frac{\pi x}{l}\right)\sin\left(\frac{n\pi x}{l}\right)dx.$$

[we need to find b_n $n \neq 1$ & b_1

$$b_n = \frac{1}{l}\int_0^l x\left\{\sin\left((n+1)\frac{\pi x}{l}\right) + \sin\left((n-1)\frac{\pi x}{l}\right)\right\}dx$$

$$b_n = \frac{1}{l}\left[x\left\{\frac{-\cos\left((n+1)\frac{\pi x}{l}\right)}{\frac{(n+1)\pi}{l}} - \frac{\cos\left((n-1)\frac{\pi x}{l}\right)}{(n-1)\pi/l}\right\} - 1\left\{-\frac{\sin\left((n+1)\frac{\pi x}{l}\right)}{(n+1)^2\pi^2/l^2} - \frac{\sin\left((n-1)\pi x/l\right)}{(n-1)^2\pi^2/l^2}\right\}\right]_0^l$$

$$= \frac{1}{l}\left[l\left\{\frac{-\cos((n+1)\pi)}{(n+1)\pi/l} - \frac{\cos((n-1)\pi)}{(n-1)\pi/l}\right\} - \left\{-\frac{\sin((n+1)\pi)}{(n+1)^2\pi^2/l^2} - \frac{\sin((n-1)\pi)}{(n-1)^2\pi^2/l^2}\right\}\right]$$

$-0\left\{ = \quad ? \quad \cdot \quad 0 \quad ? \quad \right]$

$$= \frac{1}{l}\left[l \left\{ -\cos \right. \right.$$

$$= \frac{-\cos((n+1)\pi)}{(n+1)\pi/l} - \frac{\cos((n-1)\pi)}{(n-1)\pi/l}$$

$$= -\frac{l}{\pi}\left[\frac{(-1)^{n+1}}{n+1} + \frac{(-1)^{n-1}}{n-1} \right]$$

$$= -\frac{l}{\pi}(-1)^n \left[\frac{(-1)^1}{n+1} + \frac{(-1)^{-1}}{n-1} \right]$$

$$= -\frac{l}{\pi}(-1)^n \left[\frac{1}{n+1} + \frac{1}{n-1} \right]$$

$$= -\frac{l}{\pi}(-1)^n \left[\frac{2n}{n^2-1} \right] \quad \text{(check)}$$

$$b_n = \frac{2ln(-1)^{n+1}}{\pi(n^2-1)}$$

$$b_1 = \frac{1}{l}\int_0^l x \sin\left(\frac{2\pi x}{l}\right) dx$$

$$= \frac{1}{l}\left[x \left\{ \frac{-\cos(2\pi x/l)}{2\pi/l} \right\} - (1)\left\{ -\frac{\sin(2\pi x/l)}{(2\pi/l)^2} \right\} \right]_0^l$$

$$= \frac{1}{l}\left[\cancel{-l} \; \frac{-l}{2\pi}\left\{ l\cos(2\pi) - 0\cos 0 \right\} \right]$$

$$= \cancel{\frac{1}{l}\left[\right.} = -\frac{l}{2\pi}$$

$$f(x) = 0 + 0 + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$= b_1 \sin\left(\frac{\pi x}{l}\right) + \sum_{n=2}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{-l}{2\pi}\sin\left(\frac{\pi x}{l}\right) + \sum^{\infty} \frac{2n(-1)^{n+1}}{} \sin\left(\frac{n\pi x}{}\right)$$

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Q. $f(x) = x - x^2 \quad (-\pi, \pi)$ & hence deduce $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

Ans. $f(x)$ is neither even nor odd.

$(C, C+2L) \qquad C = -\pi \qquad L = \pi$

$$a_0 = \frac{1}{L}\int_{C}^{C+2L} f(x)\,dx$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} x - x^2\,dx.$$

$$= \frac{1}{\pi}\Big[\underbrace{\int_{-\pi}^{\pi} x\,dx}_{\text{odd}} - \underbrace{\int_{-\pi}^{\pi} x^2\,dx}_{\text{even}}\Big]$$

$$-\frac{2}{\pi}\int_{0}^{\pi} x^2\,dx$$

$$= -\frac{2}{\pi}\,\frac{x^3}{3}\Big|_0^{\pi} = \frac{-2\pi^2}{3}.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x - x^2)\cos(nx)\,dx$$

$$= 0 - \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos(nx)\,dx$$

$$= \frac{-2}{\pi}\left[x^2\left(\frac{\sin(nx)}{n}\right) - 2x\left(-\frac{\cos(nx)}{n^2}\right) + \left(-\frac{\sin(nx)}{n^3}\right)\right]_0^{\pi}$$

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$$= \frac{2}{\pi} \times \frac{\pi}{n^2} \times 2 =$$

$$= -\frac{2\times 2}{\pi}\left[\frac{-\pi\,(\cancel{1}-(-1)^n)}{n^2}\right.$$

$$= -\frac{4}{\pi} \times \pi\,\frac{(-1)^n}{n^2}.$$

$$a_n = -\frac{4(-1)^n}{n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x - x^2)\sin(nx)\,dx$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x\sin(nx)\,dx$$

$$b_n = \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\cancel{\sin nx}^{\,0}}{n^2}\right)\right]_0^{\pi}$$

$$b_n = -\frac{2(-1)^n}{\pi}$$

$$f(x) = -\frac{2\pi^2}{3}\times\frac{1}{2} + \sum_{n=1}^{\infty}\left\{-\frac{4(-1)^n}{n^2}\cos(nx) - \frac{2}{n}(-1)^n\sin(nx)\right\}$$

$$f(0) = -\frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left(-\frac{4(-1)^n}{n^2}\right) = 0$$

$$4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{3}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}.$$

$x = \pi$

$f(\pi) = -\frac{\pi^2}{3} - 4\sum_{n=1}^{\infty}$

complete

Q→ $f(x) = x\sin x$ $(0, 2\pi)$

(a) $\frac{1}{1\cdot3} - \frac{1}{3\cdot5} + \frac{1}{5\cdot7} - \frac{1}{7\cdot9} + \cdots = \frac{\pi-2}{4}$

(b) $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}$.

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$

$(c, c+2L) = (0, 2\pi) \Rightarrow c = 0,\ L = \pi$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \{a_n \cos(nx) + b_n \sin(nx)\}$

$a_0 = \frac{1}{L}\int_{c}^{c+2L} f(x) = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$

$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)\,dx \quad ; \quad b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin(nx)\,dx$.

$a_0 = \frac{1}{\pi}\int_0^{2\pi} x\sin x\,dx = \frac{1}{\pi}\left[x\{-\cos x\} - (1)(-\sin x)\right]_0^{2\pi}$

$= +\frac{1}{\pi} \times -2\pi$

$a_0 = \cancel{2} - 2$

$a_n = \frac{1}{\pi}\int_0^{2\pi} x\sin x\cos(nx)\,dx = \frac{1}{\pi}\int_0^{2\pi} x\cos(nx)\sin x\,dx$

$a_n = \frac{1}{\pi}\int_0^{2\pi} \frac{x}{2}\{\sin(nx+x) - \sin(nx-x)\}\,dx$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\{\sin((n+1)x) - \sin((n-1)x)\}\,dx$$

$$\frac{1}{2\pi}\left[x\left\{\frac{-\cos((n+1)x)}{n+1} + \frac{\cos((n-1)x)}{n-1}\right\} - \left\{\frac{-\sin((n+1)x)}{(n+1)^2} + \frac{\sin((n-1)x)}{(n-1)^2}\right\}\right]_0^{2\pi}$$

$n \neq 1$; a_1 has to be found separately

$$a_n = \frac{1}{2\pi}\left[2\pi\left\{\frac{-\cos((n+1)2\pi)}{n+1} + \frac{\cos((n-1)2\pi)}{n-1}\right\} - 0\left\{\frac{-1}{n+1} + \frac{1}{n-1}\right\} - \{0 \cdot\}\right]$$

$$a_n = \frac{1}{n-1} - \frac{1}{n+1} = \frac{2}{(n-1)(n+1)}$$

$$a_1 = \frac{1}{2\pi}\int_0^{2\pi} x\sin(2x)\,dx$$

$$a_1 = \frac{1}{2\pi}\left[x\left\{\frac{-\cos(2x)}{2}\right\} - (1)\left\{\frac{-\sin(2x)}{2^2}\right\}\right]_0^{2\pi}$$

$$a_1 = \frac{1}{2\pi}\left[\frac{-1}{2}\left\{2\pi\cos 2\pi - 0\cos 0\right\} + \frac{1}{4}\{0-0\}\right]$$

$$a_1 = \frac{-1}{2}.$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x\sin x\sin(nx)\,dx = \frac{1}{\pi}\int_0^{2\pi} x\sin(nx)\sin x\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\frac{-x}{2}\{\cos(nx+x) - \cos(nx-x)\}\,dx = \frac{1}{2\pi}\int_0^{2\pi} x\{\cos((n-1)x) - \cos((n+1)x)\}$$

$$= \frac{1}{\pi}\left[x\left\{\frac{\sin((n-1)x)}{n-1} - \frac{\sin((n+1)x)}{n+1}\right\} - 1\left\{\frac{-\cos((n-1)x)}{(n-1)^2} + \frac{\cos((n+1)x)}{(n+1)^2}\right\}\right]$$

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$\frac{1}{\pi}$ ~~γ~~

$b_n = 0 \qquad n \neq 1$

$b_1 = \frac{1}{2\pi}\int_0^{2\pi} x \sin x \sin x$

$b_1 = \frac{1}{2\pi}\int_0^{2\pi} x(1-\cos 2x)\,dx$

$= \frac{1}{2\pi}\int_0^{2\pi} x - x\cos 2x\,dx$

$= \frac{1}{2\pi}\left[\frac{x^2}{2} - x\left(\frac{\sin 2x}{2}\right) + (1)\left(-\frac{\cos 2x}{2}\right)\right]_0^{2\pi}$

$= \frac{1}{2\pi}\left[\frac{(2\pi)^2}{2}\right]$

$= \frac{1}{2\pi} \times \frac{4\pi^2}{2}$

$b_1 = \pi$

$f(x) = \frac{a_0}{2} + a_1\cos x + \sum_{n=2}^{\infty} a_n\cos(nx) + b_1\sin x + \sum_{n=2}^{\infty} b_n\sin(nx)$

$f(x) = \frac{1}{2}x - 2 + \frac{-1}{2}\cos x + \sum_{n=2}^{\infty} \frac{2}{n^2-1}\cos(nx) + \pi\sin x + 0$

$= -1 - \frac{1}{2}\cos x + \sum_{n=2}^{\infty}\frac{2}{n^2-1}\cos(nx) + \pi\sin x.$

* Put $x = \pi/2$

$\frac{\pi}{2}(1) = -1 - 0 + \sum_{n=2}^{\infty}\frac{2}{(n-1)(n+1)}\cos\left(\frac{n\pi}{2}\right) + \pi(1)$

$\pi - \frac{\pi}{2} + 1 = -2\left\{\frac{-1}{1\cdot3} + 0 + \frac{1}{3\cdot5} + 0 - \frac{1}{5\cdot7} + 0 + \frac{1}{7\cdot9} - \right\}$

$\dots \frac{}{5\cdot 7} + \frac{1}{7\cdot 9} + \dots$

$x = 0$

$$0 = -1 - \frac{1}{2} + 2\sum_{n=2}^{\infty} \frac{1}{n^2-1}(1) + 0$$

$$\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{1}{2}\left(\frac{3}{2}\right) = \frac{3}{4}$$

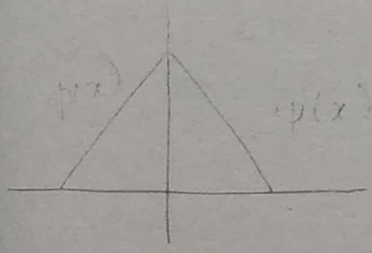
H.W.
Q→ $f(x) = x\sin x$ in $(-\pi, \pi)$
& hence deduce that $\cancel{\frac{1}{1\cdot 3} + \frac{1}{5\cdot 3}}$

$$\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \frac{1}{7\cdot 9} \dots = \frac{\pi - 2}{4}.$$

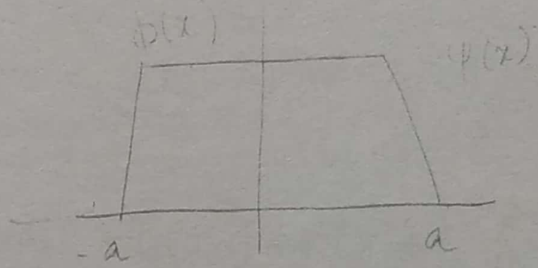
Q→ $\cancel{f(x) = |\cos x|}$

Q→ $f(x) = \begin{cases} \phi(x) & -a < x < 0 \\ \psi(x) & 0 < x < a. \end{cases}$

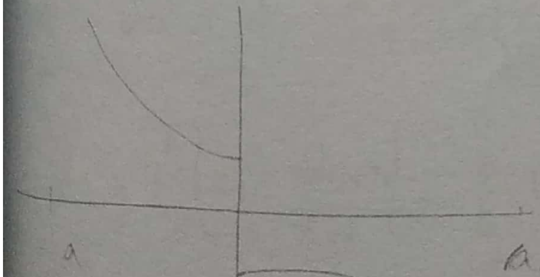
Condn for $f(x)$ to be odd or even



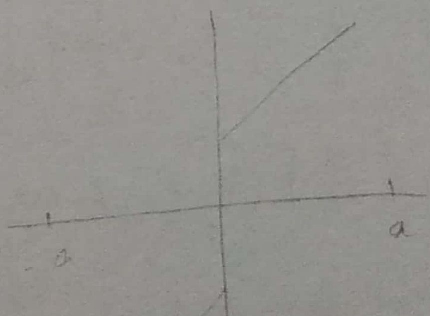
even



even



odd



odd.

$x>0 \Rightarrow -x<0$

$f(-x) = \phi(-x) = \psi(x) \Rightarrow f(x)$ is even

$\phi(-x) = -\psi(x) \Rightarrow f(x) = \text{odd}$

$\phi(2\pi - x) = \psi(x)$ - even like

$\phi(2\pi - x) = -\psi(x)$ odd

If it has more sub-intervals

Q→ obtain fourier series of $f(x) = \begin{cases} x - \frac{\pi}{2} & -\pi < x < 0 \\ x + \frac{\pi}{2} & 0 < x < \pi \end{cases}$

$\phi(x) = x - \frac{\pi}{2} \quad \psi(x) = x + \frac{\pi}{2}$

$\phi(-x) = -x - \frac{\pi}{2} = -(x + \frac{\pi}{2}) - \psi(x)$

$f(x)$ is an odd fn

$a_0 = a_n = 0$

$b_n = \frac{1}{l} \int^{c+2l} f(x) \sin\left(\frac{n\pi x}{l}\right) \quad c = -\pi \quad l = \pi$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx \qquad \left.\begin{matrix} f(x) \rightarrow \text{odd} \\ \sin(nx) \rightarrow \text{odd} \end{matrix}\right\} \text{even}$

$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin(nx) dx$

$b_n = \frac{2}{\pi} \int_0^{\pi} \left(x + \frac{\pi}{2}\right) \sin(nx) dx$

$= \frac{2}{\pi} \left[\left(x + \frac{\pi}{2}\right)\left(-\frac{\cos nx}{n}\right) - 1 \left\{ -\frac{\sin nx}{n^2} \right\} \right]_0^{\pi}$

$= \frac{2}{\pi} \left[-\frac{1}{n} \left\{ \frac{3\pi}{2} \cos n\pi - \frac{\pi}{2} \cos 0 \right\} + \frac{1}{n^2} (0 - 0) \right]$

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$= \frac{-2}{n\pi} \times \frac{\pi}{2} (3(-1)^n - 1)$

$= \frac{1}{n} \{1 - 3(-1)^n\}$

$f(x) = 0 + 0 + \sum_{n=1}^{\infty} \frac{1}{n} \{1 - 3(-1)^n\} \sin(nx)$

Q→ Find fourier series of triangular wave represented by

$$f(x) = \begin{cases} x & 0 < x < \pi \\ 2\pi - x & \pi < x < 2\pi \end{cases}$$

sum of reciprocal squares of odd integers $= \frac{\pi^2}{8}$.

$\phi(x) = x \qquad \psi(x) = 2\pi - x$

$\phi(2\pi - x) = 2\pi - x = \psi(x) \Rightarrow f(x)$ is an even like fn

$\Rightarrow b_n = 0$ in the FS.

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \{ a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \}$

$a_0 = \frac{1}{L} \int_c^{c+2L} f(x)\,dx = \frac{1}{\pi} \int_0^{2\pi} f(x)\,dx$.

$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos(nx)\,dx$

$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\,dx = \frac{2}{\pi} \int_0^{\pi} x\,dx$

$a_0 = \frac{2}{\pi} \times \left[\frac{x^2}{2}\right]_0^{\pi} = \pi$

$a_n = \frac{1}{\pi} \times 2 \int_0^{\pi} f(x) \cos(nx)\,dx = \frac{2}{\pi} \int_0^{\pi} x \cos(nx)\,dx$

$$a_n = \frac{2}{\pi}\left[x\frac{\sin(nx)}{n} - (1)\left\{\frac{-\cos(nx)}{n^2}\right\}\right]_0^{\pi}$$

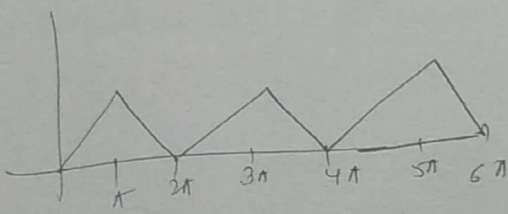
$$= \frac{2}{\pi}\left[\frac{1}{n}\{\pi(0) - 0\} + \frac{1}{n^2}\{\cos(n\pi) - \cos 0\}\right]$$

$$a_n = \frac{2}{n^2\pi}\{(-1)^n - 1\}$$

$$f(x) = \frac{1}{2}\times\pi + \sum_{n=1}^{\infty}\frac{2}{n^2\pi}\{(-1)^n - 1\}\cos nx + 0$$

~~$x = 0$~~

$x = 0 \cdot \{f(0) = 0$ $\quad f(2\pi) = 2\pi - 2\pi = 0$



$x = 0$ is a pt. of continuity

$$f(0) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{2}{n^2\pi}\{(-1)^n - 1\}$$

$$0 = \frac{\pi}{2} + \frac{2}{\pi}\left\{\frac{-2}{1^2} + 0 + \frac{-2}{3^2} + 0 + \frac{-2}{5^2} + \cdots\right\}$$

$$\frac{\pi}{2} = \frac{4}{\pi}\left\{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right\}$$

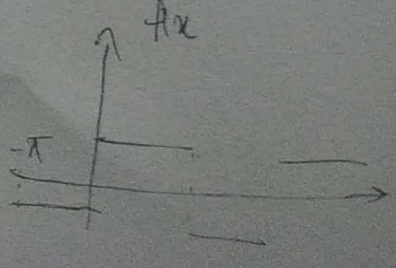
$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

Q →

$$f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 0 & x = 0 \\ 1 & 0 < x < \pi \end{cases}$$

PT $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}$

$f(x)$ is an odd fn.



$a_0 = 0 = a_n$

$$f(x) = 0 + 0 + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \sin(nx)\,dx$$

$$= \frac{2}{\pi}\left[-\frac{\cos(nx)}{n}\right]_0^{\pi}$$

$$b_n = \frac{2}{n\pi}\left[1-(-1)^n\right]$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left\{1-(-1)^n\right\}\sin(nx)$$

$$x=\frac{\pi}{2} \Rightarrow 1 = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(+1 - (-1)^n\right)\sin\left(\frac{n\pi}{x}\right)$$

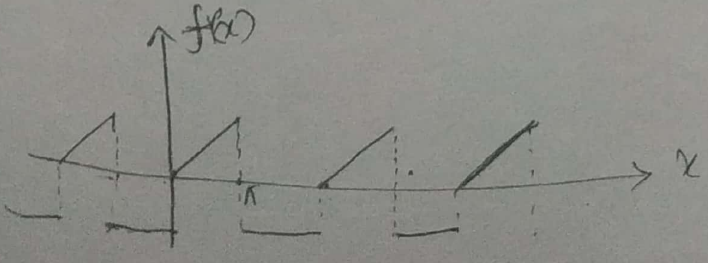
$$\frac{\pi}{2} = \sum_{n=1} \frac{1-(-1)^n}{n}\sin\left(\frac{n\pi}{2}\right) = \frac{2}{1} + 0 + \frac{2}{3}(-1) + 0 + \frac{2}{5}(1)$$

$$+ 0 + \frac{2}{7}(-1) + \ldots$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

Q→ FS of $f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$ & hence deduce that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)$$

$L = \pi$

$$a_0 = \frac{1}{L}\int_c^{c+2L} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$$\cancel{a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}}$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\,dx + \int_0^{\pi} x\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi x\Big|_{-\pi}^{0} + \frac{x^2}{2}\Big|_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\pi^2 + \frac{\pi^2}{2}\right]$$

$$a_0 = -\frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\cos(nx)\,dx + \int_0^{\pi} x\cos(nx)\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi\left[\frac{\sin(nx)}{n}\right]_{-\pi}^{0} + \left[x\left(\frac{\sin(nx)}{n}\right) - \left(-\frac{\cos(nx)}{n^2}\right)\right]_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n^2}\left(\cos(nx)\right)\right]_0^{\pi}$$

$$= \frac{1}{n^2\pi}\left[(-1)^n - 1\right]$$

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$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\sin(nx)\,dx + \int_{0}^{\pi} x\sin(nx)\,dx\right]$$

$$= \frac{1}{\pi}\left\{\left[-\pi\left(\frac{-\cos(nx)}{n}\right)\right]_{-\pi}^{0} + \left[x\left(\frac{-\cos(nx)}{n}\right) - \left(\frac{-\sin(nx)}{n^2}\right)\right]_{0}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left(\frac{\pi}{n}\left[\cos(nx)\right]_{-\pi}^{0} + \left[\frac{\pi(-1)^n}{n}\right]\right.$$

$$b_n = \frac{1}{n}(1-(-1)^n) \cancel{\mp} \frac{(-1)^n}{n}.$$

$$b_n = \frac{1}{n}\left[1 - (-1)^n - (-1)^n\right]$$

$$b_n = \frac{1}{n}\left[1 - 2(-1)^n\right]$$

$$f(x) = \frac{1}{2}\left(-\frac{\pi}{2}\right) + \sum_{n=1}^{\infty} \frac{\{(-1)^n - 1\}\cos(nx)}{n^2\pi}$$

$$+ \sum_{n=1}^{\infty} \frac{1-2(-1)^n \sin(nx)}{n}$$

$x = 0$ is a pt. of discontinuity.

$f^+(0) = RHL = 0 \qquad f^-(0) = LHL = -\pi$

$$\frac{RHL + LHL}{2} = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2\pi}$$

$$\frac{0-\pi}{2} = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2\pi}$$

$$-\frac{\pi}{4} = \frac{1}{\pi}\left\{\frac{-2}{1^2} + 0 + \frac{-2}{3^2} + 0 + \frac{-2}{5^2} + \cdots\right\}$$

$$-\frac{\pi^2}{8} = \sum \frac{1}{1^2}.$$

Q→ $f(x) = \begin{cases} 0 & -2 < x < -1 \rightarrow \phi_2 \\ 1+x & -1 < x < 0 \rightarrow \phi_1 \\ 1-x & 0 < x < 1 \rightarrow \psi_1 \\ 0 & 1 < x < 2 \rightarrow \psi_2 \end{cases}$

$\phi_2(-x) = \psi_2(x)$

$\phi_1 = 1+x \qquad \phi_1(-x) = 1-x = \psi_1(x)$

$f(x) \rightarrow$ even fn $\Rightarrow b_n = 0$

Complete it

Q→ An alternating current after passing th' the half wave rectifier has the form,

$$I = \begin{cases} I_0 \sin x & 0 < x < \pi \\ 0 & \pi < x < 2\pi \end{cases} \qquad 2\pi \text{ is the pd.}$$

Express I as fourier series. Also evaluate the value of (a) $\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \frac{1}{7\cdot 9} + \ldots$

(b) $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \frac{1}{7\cdot 9} + \ldots$

Ans $\quad L = \pi \qquad (C, C+2L) = (0, 2\pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right\}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos(nx) + b_n \sin(nx) \right\}$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_0^{\pi} I_0 \sin x\,dx + \int_{\pi}^{2\pi} 0\,dx \right]$$

~~$a_0 = 0$~~

$$a_0 = \frac{I_0}{\pi}\left[-\cos x\right]_0^{\pi} = \frac{2I_0}{\pi}$$

$$\cancel{a_n = \frac{1}{\pi}\left\{\int_0^{2\pi} I_0 \sin x \cos(nx)\, dx + \int_\pi^{2\pi} 0 \times \cos(nx)\, dx\right\}}$$

$a_n = 0$

$$\cancel{b_n = \frac{1}{\pi}\left\{\int_0^{\pi} I_0 \sin x \sin(nx)\, dx + \int_\pi^{2\pi} 0 \times \sin nx\, dx\right\}}$$

$$\cancel{= \frac{I_0}{2\pi}\left\{\int_0^{\pi} (\sin(n+1\right.}$$

$$b_n = \frac{I_0}{2\pi}\left\{\int_0^{\pi} \cos((n-1)x) - \cos((n+1)x)\right\}$$

$$\cancel{b_n = \frac{I_0}{2\pi}\left\{\frac{\sin((n-1)x)}{n-1} - \frac{\sin((n+1)x)}{n+1}\right\}}$$

$$a_n = \frac{1}{\pi}\left\{\int_0^{\pi} I_0 \sin x \cos(nx)\, dx + \int_\pi^{2\pi} 0 \times \cos(nx)\, dx\right)$$

$$= \frac{I_0}{2\pi}\left\{\int_0^{\pi} \sin((n+1)x) - \sin((n-1)x)\right\}$$

$$= \frac{I_0}{2\pi}\left\{-\frac{\cos((n+1)x)}{(n+1)} + \frac{\cos((n-1)x)}{n-1}\right\}_0^{\pi}$$

$$= \frac{I_0}{2\pi}\left[\frac{\cos((n-1)x)}{n-1} - \frac{\cos((n+1)x)}{n+1}\right]_0^{\pi}$$

$$= \frac{I_0}{2\pi}\left[\frac{(-1)^{n-1}}{n-1} - \frac{1}{n-1} - \frac{(-1)^{n+1}}{n+1} + \frac{1}{n+1}\right] \quad n \neq 1$$

$$(-1)^{n-1} = (-1)^{n+1} = -(-1)^n$$

$$a_n = \frac{I_0}{2\pi}\left\{-(-1)^n - 1\right\}\left\{\frac{1}{n-1} - \frac{1}{n+1}\right\} = \frac{-I_0\left\{(-1)^n + 1\right\}}{\pi(n^2-1)}$$

$a_1 = \frac{I_0}{2\pi}\int_0^\pi \sin 2x\,dx = \frac{I_0}{\pi}\{-\cos 2x\}\Big|_0^\pi = 0$

$b_n = 0 \quad n \neq 1$

$b_1 = \frac{1}{\pi}\int_0^\pi \frac{1}{2}(1-\cos 2x)\,dx.$

$b_1 = \frac{1}{2\pi}\left[x - \frac{\sin 2x}{2}\right]_0^\pi$

$b_1 = \frac{1}{2\pi}[\pi - 0] = \frac{1}{2}$

$f(x) = \frac{I_0}{\pi} + \sum_{n=2}^{\infty} \frac{-I_0\{(-1)^n+1\}}{\pi(n^2-1)}\cos nx + \frac{1}{2}\sin x.$

★ $x = 0$

$0 = \frac{I_0}{\pi} - \frac{I_0}{\pi}\left\{\frac{2}{1\cdot 3} + 0 + \frac{2}{3\cdot 5} + 0 + \frac{2}{5\cdot 7} + 0 + \cdots\right\}$

$\frac{I_0}{\pi} = \frac{2I_0}{\pi}\left\{\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots\right\}$

$\frac{1}{2} = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots$

★ $x = \pi$, we won't get the reqd. expression

★ $x = \frac{\pi}{2}$

$I_0 = \frac{I_0}{\pi} - \frac{I_0}{\pi}\sum_{n=2}^{\infty}\frac{(-1)^n+1}{(n-1)(n+1)}\cos\left(\frac{n\pi}{2}\right) + \frac{I_0}{2}$

$\frac{I_0}{2} - \frac{I_0}{\pi} = -\frac{I_0}{\pi}\sum_{n=2}^{\infty}\left\{\frac{2}{1\cdot 3}(-1) + 0 + \frac{2}{3\cdot 5}(1) + \frac{2}{5\cdot 7}(-1) + 0 + \cdots\right.$

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$$\frac{\pi-2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots$$

Q→ Obtain the fourier series of the periodic fns.

(a) $f(x) = \begin{cases} 1+\frac{4x}{3} & -\frac{3}{2} \le x \le 0 \\ 1-\frac{4x}{3} & 0 \le x \le \frac{3}{2} \end{cases}$ even

(b) $f(x) = \begin{cases} 2-x & 0<x<4 \\ x-6 & 4<x<8 \end{cases}$ even

(c) $f(x) = \begin{cases} -\cos x & -\pi<x<0 \\ \cos x & 0<x<\pi \end{cases}$ odd $a_0 = 0 = a_n$

(d) full wave rectifier $f(x) = \sin x$ $0<x<\pi$

$f(x) = \sin x$ $0<x<\pi$

$\Rightarrow f(x) = |\sin x|$ $0<x<2\pi$

---

* Square wave

$f(x) = \begin{cases} -k & -\pi<x<0 \\ k & 0<x<\pi \end{cases}$

or

$f(x) = \begin{cases} -1 & -1<x<0 \\ 0 & x=0 \\ 1 & 0<x<1 \end{cases}$

* Triangular wave

$f(x) = \begin{cases} x & 0<x<\pi \\ 2\pi-x & \pi<x<2\pi \end{cases}$

$f(x) = \begin{cases} 1+\frac{4x}{3} & -\frac{3}{2}<x<0 \\ 1-\frac{4x}{3} & \frac{3}{2}<x<0 \end{cases}$

* Saw tooth wave

$f(x) = x \quad 0 < x < 2\pi$ or $0 < x < 2l$

or $-\pi < x < \pi$ or $-l < x < l$

* Half wave rectifier

$f(x) = \begin{cases} 10\sin x & (0, \pi) \\ 0 & (\pi, 2\pi) \end{cases}$

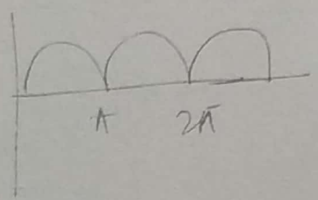
* full wave rectifier

Problem

(d) full wave rectifier

$f(x) = \sin x \quad 0 < x < \pi$

change it to $f(x) = |\sin x| \quad 0 < x < 2\pi$.



~~$T = 2\pi$ is~~ $T = \pi \rightarrow$ for full wave rectifier

$T = 2\pi \rightarrow$ for fourier series

In $(c, c+2L)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right\}$$

$$\left.\begin{aligned} e^{i\theta} &= \cos\theta + i\sin\theta \\ e^{-i\theta} &= \cos\theta - i\sin\theta \end{aligned}\right\} \Rightarrow \quad \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

~~$\sin\theta = e^{i\theta}$~~ $\quad \sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \left\{ \frac{\exp\left(\frac{in\pi x}{L}\right) + \exp\left(-\frac{in\pi x}{L}\right)}{2} \right\} + b_n \left\{ \frac{\exp\left(\frac{in\pi x}{L}\right) - \exp\left(-\frac{in\pi x}{L}\right)}{2i} \right\} \right]$$

$$= \frac{a_0}{2} + \sum_{n=1} \left\{ \left(\frac{a_n - ib_n}{2}\right) \exp\left(\frac{in\pi x}{L}\right) + \left(\frac{a_n + ib_n}{2}\right) \exp\left(-\frac{in\pi x}{L}\right) \right\}$$

$$f(x) = C_0 + \sum_{n=1}^{\infty} \left\{ C_n \exp\left(\frac{in\pi x}{L}\right) + \sum_{n} C_{-n} \exp\left(\frac{-in\pi x}{L}\right) \right\}$$

$$f(x) = \sum_{n=\infty}^{\infty} C_n \exp\left(\frac{in\pi x}{L}\right)$$

$$\cancel{C_n = \frac{a_n - ib_n}{2} = \frac{1}{2} \times \frac{1}{L} \int_c^{c+2L} f(x) \times \left\{ \cos\left(\frac{n\pi x}{L}\right) + i\sin\left(\frac{n\pi x}{L}\right) \right\} dx}$$

$$C_n = \frac{a_n - ib_n}{2} = \frac{1}{2} \times \frac{1}{L} \int_c^{c+2L} f(x) \left\{ \cos\left(\frac{n\pi x}{L}\right) - i\sin\left(\frac{n\pi x}{L}\right) \right\} dx$$

$$C_n = \frac{1}{2L} \int_c^{c+2L} f(x) \exp\left(-\frac{in\pi x}{L}\right) dx \qquad n = 1 \text{ to } \infty$$

$$C_{-n} = \frac{a_n + ib_n}{2} = \frac{1}{2} \times \frac{1}{L} \int_c^{c+2L} f(x) \left\{ \cos\left(\frac{n\pi x}{L}\right) + i\sin\left(\frac{n\pi x}{L}\right) \right\} dx$$

$$C_{-n} = \frac{1}{2L} \int_c^{c+2L} f(x) \exp\left(\frac{in\pi x}{L}\right) dx \qquad n = 1 \text{ to } \infty$$

$$C_0 = \frac{a_0}{2} = \frac{1}{2} \times \frac{1}{L} \int_c^{c+2L} f(x)\, dx$$

$$C_n = \frac{1}{2L} \int_c^{c+2L} f(x) \exp\left(-\frac{in\pi x}{L}\right) dx \qquad n = -\infty \text{ to } \infty$$

$$f(x) = \sum_{n=-\infty}^{\infty} C_n \exp\left(\frac{in\pi x}{L}\right); \quad C_n = \frac{1}{2L} \int_c^{c+2L} f(x) \exp\left(-\frac{in\pi x}{L}\right) d$$

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Q→ obtain complex form of fourier series $f(x) = e^{ax}$ in the interval $-l < x < l$.

Ans - $(c, c+2L) = (-l, l)$

$c = -l \quad L = l$

$$f(x) = \sum_{n=-\infty}^{\infty} C_n \exp\left(\frac{in\pi x}{L}\right)$$

$$C_n = \frac{1}{2L}\int_c^{c+2L} f(x) \exp\left(-\frac{in\pi x}{L}\right) dx$$

$$C_n = \frac{1}{2l}\int_{-l}^{l} f(x) \exp\left(-\frac{in\pi x}{l}\right) dx.$$

$$= \frac{1}{2l}\int_{-l}^{l} \exp(ax) \exp\left(-\frac{in\pi x}{l}\right) dx$$

$$= \frac{1}{2l}\int_{-l}^{l} \exp\left(ax - \frac{in\pi x}{l}\right) dx$$

$$= \frac{1}{2l}\int_{-l}^{l} \exp\left\{\left(a - \frac{in\pi}{l}\right)x\right\} dx$$

$$= \frac{1}{2l} \left. \frac{\exp\left\{\left(\frac{al - in\pi}{l}\right)x\right\}}{\frac{al - in\pi}{l}} \right|_{-l}^{l}$$

$$= \frac{1}{2(al - in\pi)}\left[\exp(al - in\pi) - \exp(-(al - in\pi))\right]$$

$$= \frac{1}{2(al - in\pi)}\left[e^{al} e^{-in\pi} - e^{-al} e^{in\pi}\right]$$

$e^{\pm in\pi} = \cos n\pi \pm i\sin(n\pi) = \cos n\pi = (-1)^n$

$= \frac{(-1)^n}{al - in\pi}\left\{\frac{e^{al} - e^{-al}}{2}\right\} = \frac{(-1)^n \sinh(al)}{al - in\pi}$

$f(x) = \sum_{n=-\infty}^{\infty} \frac{(-1)^n \sinh(al)}{(al - in\pi)} \exp\left(\frac{in\pi x}{l}\right)$

Q. Obtain complex form of fourier series.

$f(x) = x \quad -\pi < x < \pi$

Ans - $(c, c+2l) = (-\pi, \pi)$

$\Rightarrow c = -\pi \quad L = \pi$

$f(x) = \sum_{n=-\infty}^{\infty} c_n \exp\left(\frac{in\pi x}{l}\right)$

$c_n = \frac{1}{2L}\int_c^{c+2L} f(x) \exp\left(-\frac{in\pi x}{L}\right) dx$

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x \exp\left(-\frac{in\pi x}{L}\right) dx$ ILATE

$\cancel{= \frac{1}{2\pi}\left[\frac{x^2}{2}\exp\left(-\frac{in\pi x}{L}\right)\right.}$

$\cancel{= \frac{1}{2\pi}\left[x \exp\left(\frac{in\pi x}{L}\right) - \int e\right.}$

$\cancel{= \frac{1}{2\pi}\left[\frac{x \exp\left(-\frac{in\pi x}{L}\right)}{\left(-\frac{in\pi x}{L}\right)} - \int \frac{\exp\left(-\frac{in\pi x}{L}\right)}{\left(-\frac{in\pi x}{L}\right)}\right]_{-\pi}^{\pi}}$

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x \exp\left(-inx\right)$

(5) $\cancel{c_n = \frac{1}{2\pi}\left[x \frac{\exp(-inx)}{-inx} - \int \frac{\exp(-inx)}{-inx} \right]_{-\pi}^{\pi}}$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\left\{\cos(nx) - i\sin(nx)\right\}dx]$$

, on integrating goes to zero → odd function

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x\cos(nx) - xi\sin(nx)\,dx$$

→ even function

$$= \frac{-i}{2\pi}\left[x\left\{-\frac{\cos(nx)}{n}\right\} - \left\{-\frac{\sin(nx)}{n^2}\right\}\right]_{0}^{\pi} \quad n \neq 0$$

$$= \frac{-i}{2\pi}\left[-\frac{1}{n}\left[\pi(-1)^n\right]\right]$$

$$= \cancel{\frac{(-1)^n}{2\pi}} + \cancel{\frac{i(-1)^n}{n\pi}}$$

$$\frac{i(-1)^n}{n} \quad n \neq 0$$

$$C_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\,dx = 0.$$

$$f(x) = C_0 + \sum_{n=-\infty}^{\infty} C_n \exp(inx)$$

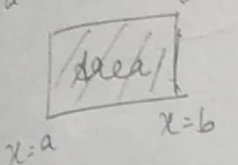
$$\cancel{= C_0 + \sum}$$

$$f(x) = \sum_{n=-\infty}^{\infty} \frac{i(-1)^n}{n}\exp(inx) \quad n \neq 0$$

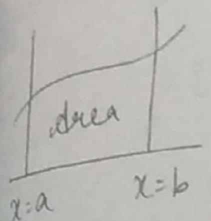
PRACTICAL HARMONIC ANALYSIS

$\int_a^b y\,dx =$ Area under $y = f(x)$ b|w $x = a$ & $x = b$

For a rectangle



$$\frac{\text{Area}}{b-a} = \text{height}$$



$$\frac{1}{b-a}\int_a^b y\,dx = \text{Average height} = \text{Average}(y)$$

$$f(x) = \frac{a_0}{2} + \underbrace{a_1\cos\left(\frac{\pi x}{L}\right) + b_1\sin\left(\frac{\pi x}{L}\right)}_{\text{1st harmonics or fundamental harmonic}} + \underbrace{a_2\cos\left(\frac{2\pi x}{L}\right) + b_2\sin\left(\frac{2\pi x}{L}\right)}_{\text{2nd harmonic}} + \ldots$$

$$\ldots + a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right) \rightarrow n^{th} \text{ harmonic}$$

$$a_0 = \frac{1}{L}\int_c^{c+2L} f(x)\,dx \qquad b-a = c+2L-c = 2L$$

$$= 2\left(\frac{1}{2L}\int_c^{c+2L} f(x)\,dx\right) = 2\,\text{avg}[f(x)]$$

$$a_n = \frac{1}{L}\int_c^{c+2L} f(x)\cos\left(\frac{n\pi x}{L}\right) = 2\left[\frac{1}{2L}\int_0^{c+2L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx\right)$$

$$= 2\,\text{Avg}\left[f(x)\cos\left(\frac{n\pi x}{L}\right)\right] = 2\,\text{Avg}[y\cos(n\theta)]$$

Similarly;

$$b_n = 2\,\text{Avg}\left[f(x)\sin\left(\frac{n\pi x}{L}\right)\right] = 2\,\text{Avg}[y\sin(n\theta)]$$

Express y as fourier series

θ	0	60	120	180	240	300	360
y							

$y = \frac{a_0}{2} + a_1\cos\theta + b_1\sin\theta + a_2\cos 2\theta + b_2\sin 2\theta$.

$a_0 = 2\,\text{Avg}[y]$ $\quad a_n = 2\,\text{Avg}[y\cos(n\theta)]$ $\quad b_n = 2\,\text{Avg}[y\sin(n\theta)]$

θ	y	$\cos\theta$	$y\cos\theta$	$\sin\theta$	$y\sin\theta$	$\cos 2\theta$	$y\cos 2\theta$	$\sin 2\theta$	$y\sin 2\theta$
0	4	1	4	0	0	1	4	0	0
60	3	0.5	1.5	0.866	2.598	-0.5	-1.5	0.8660	2.598
20	2	-0.5	-1	0.866	1.732	-0.5	-1	-0.866	-1.732
80	4	-1	-4	0	0	1	4	0	0
40	5	-0.5	-2.5	-0.866	-4.33	-0.5	-2.5	[illegible]	[illegible]
00	6	+0.5	3	-0.866	-5.196	-0.5	-3	-0.866	-5.196
JM	24		1		-5.196		0		0

$a_0 = 2 \times \frac{24}{6} = 8$

$a_1 = 2 \times \frac{1}{6} = 0.333$

$b_1 = 2 \times \frac{-5.196}{6} = -1.732$ $\qquad a_2 = 0,\ b_2 = 0$

$y = 4 + 0.333\cos\theta - 1.732\sin\theta$.

$f(x) = x^2 \quad -l < x < l$

$$f(x) = \sum_{n=-\infty}^{\infty} C_n \exp\left(\frac{in\pi x}{L}\right)$$

$L = l$
$C = -l$

$$C_n = \frac{1}{2L}\int_{c}^{c+2L} f(x) \times \exp\left(-\frac{in\pi x}{L}\right)dx$$

$$= \frac{1}{2l}\int_{-l}^{l} x^2 \exp\left(-\frac{in\pi x}{l}\right)dx$$

$x^2\left(\cos\left(\frac{n\pi x}{l}\right) - i\sin\left(\frac{n\pi x}{l}\right)\right)$

$$= \frac{1}{2l}\int_{-l}^{l} x^2\left[\cos\left(-\frac{in}{}\right.\right.$$

$$= \frac{1}{2l}\int_{-l}^{l} x^2\left[\cos\left(\frac{n\pi x}{l}\right) - \cancel{i\sin\left(\frac{n\pi x}{l}\right)}\right]dx$$

$$= \frac{1}{2l} \times 2\int_{0}^{l} x^2 \cos\left(\frac{n\pi x}{l}\right)dx.$$

$$\cancel{\frac{1}{l}\left[x^2\left\{\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right\} - 2x\left\{\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right\} + 2\left\{\cos\left(\right.\right.\right.}$$

$$= \frac{1}{l}\left[x^2\left\{\frac{+\sin\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right\} - 2x\left\{\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right\} + 2\left\{\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\frac{n^3\pi^3}{l^3}}\right\}\right]$$

Q→ Obtain the fourier series neglecting the terms higher than first harmonic, given.

x	0	T/6	T/3	T/2	2T/3	5T/6	T
y	7.9	7.2	3.6	0.5	0.9	6.8	7.9

$$y = \frac{a_0}{2} + a_1\cos\left(\frac{m\pi x}{L}\right) + b_1\sin\left(\frac{\pi x}{L}\right) \quad (C, C+2L)$$

$$(C, C+2L) = (0, T) \Rightarrow C = 0 \quad 2L = T \Rightarrow L = T/2$$

$$y = \frac{a_0}{2} + a_1\cos\left(\frac{2\pi x}{T}\right) + b_1\sin\left(\frac{2\pi x}{T}\right) \quad \left[\because \theta = \frac{2\pi x}{T}\right.$$

~~$a_0 = 2a_0$.~~

$a_0 = 2\,\text{Avg}[y]$ $\quad a_1 = 2\,\text{Avg}(y\cos\theta)$ $\quad b_1 = 2\,\text{Avg}(y\sin\theta)$

x	$\theta = 2\pi x/T$	y	$\cos\theta$	$y\cos\theta$	$\sin\theta$	$y\sin\theta$
0	0	7.9	1	7.9	0	0
T/6	$\pi/3 = 60°$	7.2	0.5	3.6	0.8660	3.1176
T/3	$2\pi/3 = 120°$	3.6	-0.5	-1.8	0.8660	-1.5588
T/2	$\pi = 180°$	0.5	-1	-0.5	0	0
2T/3	$4\pi/3 = 240$	0.9	-0.5	~~-0.18~~ 0.45	-0.866	-0.3897
5T/6	$5\pi/3 = 300°$	6.8	0.5	~~13.6~~ 3.4	-0.866	2.9444
SUM		26.9		12.15		2.68428.

$$a_0 = 2\,\text{Avg}[y] = 2\left(\frac{26.9}{6}\right) = 8.96$$

$$a_1 = 2\,\text{Avg}(y\cos\theta) = 4.05$$

$$b_1 = 2\,\text{Avg}(y\sin\theta) = 0.8948$$

$$y = 4.48 + 4.05\cos\left(\frac{2\pi x}{T}\right) + 0.8948\sin\left(\frac{2\pi x}{T}\right)$$

Q: Express y as a fourier series upto third harmonic, given

x	0	1	2	3	4	5
y	4	8	15	7	6	2

$$y = \frac{a_0}{2} + a_1\cos\left(\frac{\pi x}{L}\right) + b_1\sin\left(\frac{\pi x}{L}\right) + a_2\cos\left(\frac{2\pi x}{L}\right) + b_2\sin\left(\frac{2\pi x}{L}\right) + a_3\cos\left(\frac{3\pi x}{L}\right) + b_3\sin\left(\frac{3\pi x}{L}\right)$$

$(c, c+2L) = (0, 6)$ [$\because$ y is periodic ~~to~~ $0 = 0$ & 2π should have the same value

$2L = 6$
$L = 3$

$$y = \frac{a_0}{2} + \sum_{n=1}^{3}\left[a_n\cos\left(\frac{n\pi x}{3}\right) + b_n\sin\left(\frac{n\pi x}{3}\right)\right]$$

$\theta = \pi x/3$	y	$y\cos\theta$	~~$y\cos^2\theta$~~	$\sin\theta$	$y\sin\theta$	$\cos 2\theta$	$y\cos 2\theta$
0	4	4	~~16~~	0	0	1	4
$\pi/3 = 60°$	8	4	~~32~~	0.866	6.928	−0.5	−4
$2\pi/3 = 180°$	15	−7.5	~~−112.5~~	0.866	12.99	−0.5	−7.5
180°	7	−7	~~−49~~	0	0	1	7
$4\pi/3 = 240°$	6	−3	~~−18~~	−0.866	−5.196	−0.5	~~8~~ −3
$5\pi/3 = 300°$	2	−1	~~−2~~	−0.866	−1.732	−0.5	−1
SUM	42	−10.5			12.99		−4.5

$\sin 2\theta$	$y\sin 2\theta$	$\cos 3\theta$	$y\cos 3\theta$	$\sin 3\theta$
0	0	1	4	0
0.866	6.928	−1	−8	0
−0.866	−12.99	1	15	0
0.	0	−1	−7	0
0.866	5.196	1	6	0
−0.866	−1.732	−1	−2	0
	−2.598		8	

$y = a_0 = 2 \times 7 = 14$

$a_1 = -3.5 \quad b_1 = 4.33$

$a_2 = -1.5 \quad b_1 = 0.866$

$a_3 = 2.67 \quad b_3 = 0.$

$$y = 14 - 3.5\cos\left(\frac{\pi x}{3}\right) + 4.33\sin\left(\frac{\pi x}{3}\right) - 1.5\cos\left(\frac{2\pi x}{3}\right) + 0.866\sin\left(\frac{2\pi x}{3}\right) + 2.67\cos(\pi)$$

$A_1 = \sqrt{a_1^2 + b_1^2} = 5.17$

$A_2 = \sqrt{a_2^2 + b_2^2} = 1.732$

Q → obtain const term & coeff of $\sin\theta$ & $\sin 2\theta$ in fourier series expansion of y from the given data

θ	0	60	120	180	240	300	360
y	0	9.2	14.4	17.8	17.3	11.7	0

Q → obtain FS upto 2nd harmonic given

θ	30	60	90	120	150	180	210	240	270	300	330	360
y	1.8	1.1	0.3	0.16	1.5	1.3	2.16	1.25	1.30	1.52	1.76	2

→ Amplitude of the nth harmonic, A_n.

$$A_n = \sqrt{a_n^2 + b_n^2}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{L} + \phi_n\right) = \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{L} + \phi_n\right)$$

$$a_n^2 + b_n^2 = A_n$$

$$a_n = A_n \cos\phi_n \qquad b_n = A_n \sin(\phi_n)$$

Let $L \to \infty$

$$f(x) = \lim_{N\to\infty} \sum_{n=0}^{N} A_n \cos\left(\frac{n\pi x}{L} + \phi_n\right)$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} \left\{ \int_0^{2L} f(x) \cos\left(\frac{n\pi x}{L} + \phi_n\right) dx \right\} \cos\left(\frac{n\pi y}{L} + \phi_n\right)$$

Fourier integral

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx \right] e^{-isx}\, ds$$

$$F[f(x)] = \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx = F(s)$$

$F[f(x)] = F(s)$ → Fourier transform.
↳ operator → function

$\frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{-isx} ds = f(x)$ → Inverse fourier transform.

Fourier series decomposes a periodic function into a discrete set of contributions in terms of one fundamental frequency. Fourier transform provides a continuous frequency resolution of a possibly non-periodic function.

Q. Find the fourier transform of $f(x) = \begin{cases} xe^{-x} & x > 0 \\ 0 & x < 0 \end{cases}$

$$F[f(x)] = \int_{-\infty}^{\infty} f(x) e^{isx} dx = \int_{-\infty}^{0} f(x) e^{isx} dx + \int_{0}^{\infty} f(x) e^{isx} dx$$

$$F[f(x)] = 0 + \int_{0}^{\infty} x e^{-x} e^{isx} dx = \int_{0}^{\infty} x e^{-(1-is)x} dx.$$

$$= \left[\frac{xe^{-(1-is)x}}{-(1-is)} - \frac{(1)\, e^{-(1-is)x}}{(1-is)^2} \right]_0^{\infty}$$

$$= \left[\frac{0-0}{-(1-is)} - \left(\frac{0 - e^0}{(1-is)^2} \right) \right]$$

$$= \frac{1}{(1-is)^2}$$

Q. Obtain fourier transform of

$$f(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

$$F[f(x)] = \int_{-\infty}^{\infty} f(x) e^{isx} dx = \int_{-\infty}^{0} f(x) e^{isx} dx + \int_{0}^{\pi} f(x) e^{isx} dx + \int_{\pi}^{\infty} f(x) e^{isx} dx$$