U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

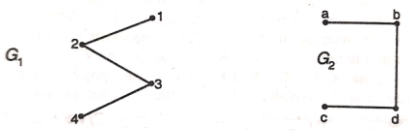
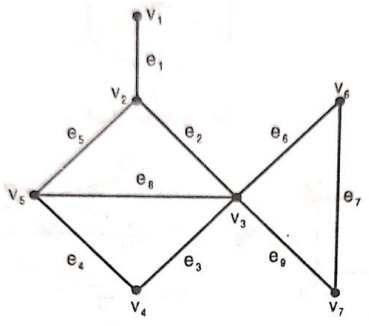
Autonomous Institute Affiliated to VTU

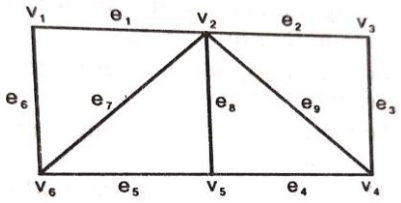
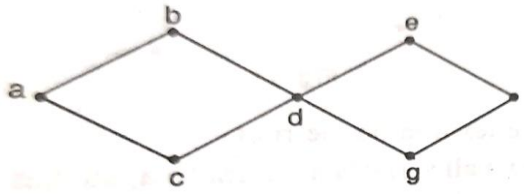
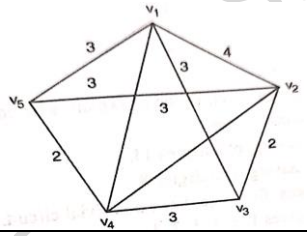
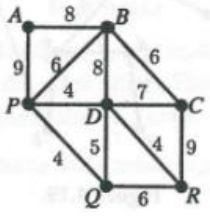
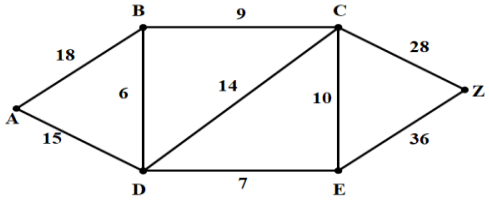
January / February 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: Machine Learning **Duration: 3 hrs.**
Course Code: 21MA3BSMAI **Max Marks: 100**
Course: Mathematical Foundations for AI & ML

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			UNIT - 1	CO	PO	Marks
1	a)		(i) Define degree of a vertex. (ii) How many vertices are needed to construct a graph with 7 edges in which each vertex is of degree 2?	1	1	**6**
	b)		(i) Define Isomorphic graph. (ii) Verify that the following graphs are isomorphic or not? 	1	1	**7**
	c)		(i) Define Incidence matrix. (ii) Draw the undirected graph corresponding to the adjacency matrix $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$.	1	1	**7**
			OR			
2	a)		Given a graph G,  find (i) A walk of length 8 starting with vertex v_1 and ending with vertex v_6. (ii) A path of length 4 starting with vertex v_1 and ending with vertex v_4. (iii) A cycle of length 4 starting with vertex v_2.	1	1	**6**

		b)	Define (i) Euler Graph (ii) Hamiltonian path (iii) verify the following graph whether it is Eulerian or Hamiltonian:  	*1*	*1*	**7**
		c)	Let G be a disconnected graph of even order n with two components each of which is complete then prove that G has a minimum of $\frac{n(n-2)}{4}$ edges.	*1*	*1*	**7**
			UNIT - 2			
	3	a)	Prove that there is one and only one path between every pair of vertices in a tree T.	*1*	*1*	**6**
		b)	Apply BFS algorithm to find a spanning tree of a graph G given below.  	*1*	*1*	**7**
		c)	Apply Prim's algorithm to find the minimal spanning tree of the weighted graph given below.  	*1*	*1*	**7**
			OR			
	4	a)	Prove that any connected graph G with n vertices and $(n-1)$ edges is a tree.	*1*	*1*	**6**
		b)	Apply Kruskal's algorithm to find the minimal spanning tree and its weight for the weighted graph shown below.  	*1*	*1*	**7**
		c)	Apply the Dijkstra's algorithm to find the shortest path and its weight from the vertex **1** to each of the vertices **2**, **3**, **4**, **5**, **6** in the weighted directed graph shown below.  	*1*	*1*	**7**

			UNIT - 3			
	5	a)	Find the coefficients of i. x^0 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^{15}$ and ii. x^{12} in the expansion of $x^3(1-2x)^{10}$.	*2*	*1*	**6**
		b)	Consider the 6 digits' number 1, 3, 4, 5, 6 and 8 and repetitions of digits are allowed. (i) How many 3-digit numbers can be found. (ii) How many 3-digit numbers must contain the digit 5.	*2*	*1*	**7**
		c)	A man has 7 relatives, 4 of them are ladies and 3 gentlemen, his wife has 7 relatives and 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives and 3 of wife's relatives?	*2*	*1*	**7**
			OR			
	6	a)	In how many ways can one select two books from different subjects from among 6 distinct Computer science books, 3 distinct Mathematics books and 2 distinct Chemistry books?	*2*	*1*	**6**
		b)	In how many ways can the letters of the word **COMPUTER** be arranged? How many of them begin with **C** and end with **R**? How many of them do not begin with **C** but end with **R**?	*2*	*1*	**7**
		c)	If 10 persons were invited for a party, in how many ways can they and the host be seated at a circular table? In how many of these ways will 2 particular persons on either side of the host?	*2*	*1*	**7**
			UNIT - 4			
	7	a)	Solve the recurrence relation $a_n = a_{n-1} + 2,\ n \geq 2$ subject to the initial condition $a_1 = 3$.	*2*	*1*	**6**
		b)	Solve the recurrence relation $a_n = 4(a_{n-1} - a_{n-2})$ subject to the initial conditions $a_0 = a_1 = 1$.	*2*	*1*	**7**
		c)	Apply mathematical induction to prove that $n^3 + 2n$ is divisible by 3 for any positive integer n.	*2*	*1*	**7**
			OR			
	8	a)	Apply the mathematical induction to prove that $3^n - 1$ is divisible by 2.	*2*	*1*	**6**
		b)	Solve the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$ subject to the initial conditions $a_0 = 2,\ a_1 = 3$.	*2*	*1*	**7**
		c)	Solve the recurrence relation $a_n = a_{n-1} + 3^n$ subject to the initial condition $a_0 = 1$.	*2*	*1*	**7**
			UNIT - 5			
	9	a)	Find the remainder when 72^{1001} is divisible by 31.	*3*	*1*	**6**
		b)	Apply Chinese remainder theorem to solve $x \equiv 2 (\text{mod}\, 5)$, $x \equiv 4 (\text{mod}\, 7)$ and $x \equiv 5 (\text{mod}\, 11)$.	*3*	*1*	**7**
		c)	Apply RSA algorithm to encrypt the message $m = 2$ using prime numbers $p = 3$ and $q = 11$ by finding the encryption key.	*3*	*1*	**7**

			OR			
	10	a)	Apply the Fermat's little theorem to show that $8^{30}-1$ is divisible by 31.	*3*	*1*	**6**
		b)	Solve the linear Diophantine equation $15x+21y=39$.	*3*	*1*	**7**
		c)	Solve the polynomial congruence: $x^3+3x+5\equiv 0(\text{mod}9)$.	*3*	*1*	**7**

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