B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2023 Supplementary Examinations

Programme: B.E.   Semester: III
Branch: Artificial Intelligence and Machine Learning   Duration: 3 hrs.
Course Code: 21MA3BSMAI   Max Marks: 100
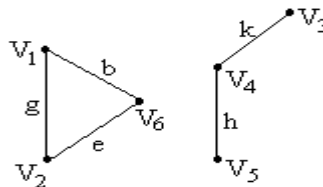Course: Mathematical Foundation For AI and ML   Date: 12.09.2023

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
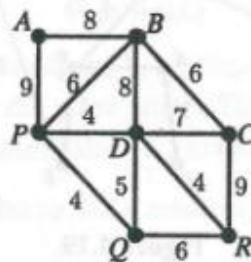2. Missing data, if any, may be suitably assumed.

UNIT - I

1 a) Determine the order $|V|$ of the graph $G(V, E)$ in the following cases: **6**

   (i)   G is a cubic graph with 9 edges
   (ii) G is regular with 15 edges
   (iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.

   b) Define isomorphism of graphs and construct five non-Isomorphic spanning subgraphs of the graph K_4.   **7**

   c) Define incidence matrix of a graph. Construct the incidence matrix for the following disconnected graph and write any two important observations on it.   **7**
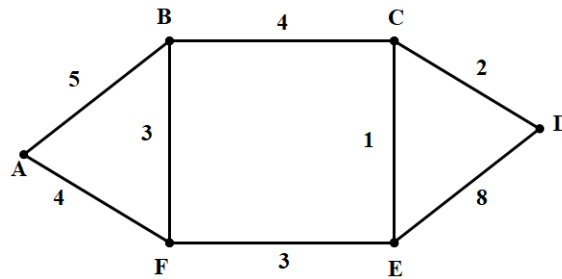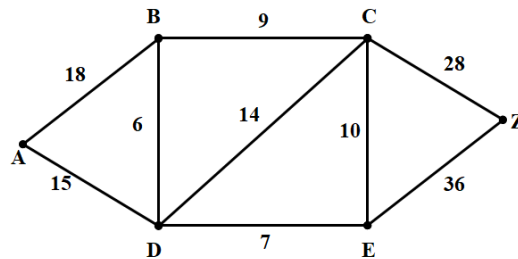

UNIT - II

2 a) Apply Kruskal's algorithm to find the minimal spanning tree and its weight for the weighted graph shown below: **6**

b) Determine the maximum flow between the vertices A and D by identifying the cut-set minimum capacity for the network shown below. **7**

c) Apply the Dijkstra's algorithm to find the shortest path and its weight from the vertex A to each of the vertices in the weighted directed graph shown below. **6**

UNIT - III

3 a) Find the coefficient of: **6**

(i) $x^9 y^3$ in the expansion of $(2x - 3y)^{12}$.

(ii) xyz^2 in the expansion of $(2x - y - z)^4$.

b) In how many ways can the integers 1, 2, 3,..,10 be arranged in a line so that no even integer is in its natural place? **7**

c) A girl student has sarees of 5 different colours: blue, green, red, white and yellow. On Mondays, she does not wear green; on Tuesdays, blue or red; on Wednesdays, blue or green; on Thursdays red or yellow; on Fridays red. In how many ways can she dress without repeating a colour during a week (from Monday to Friday)? **7**

OR

4 a) Using the moves $R : (x, y) \rightarrow (x+1, y)$ and $U : (x, y) \rightarrow (x, y+1)$, find in how many ways can one go **6**

(i) from the point $(2, 5)$ to the point $(8, 11)$ in the xy-plane by moves and not rise above the line $y = x + 3$?

(ii) from $(4,6)$ to $(12,14)$ and not rise above the line $y = x + 2$?

b) In how many ways 5 number of a's, 4 number of b's and 3 number of c's can be arranged so that all the identical letters are not in a single block? **7**

c) Applying generating functions to determine the coefficient of x^{18} in the expansion of $(x + x^3 + x^5 + x^7 + x^9)(x^3 + 2x^4 + 3x^5 + ...)^3$. **7**

UNIT - IV

5   a)   Apply mathematical induction to show that if S is a finite set with n elements, where n is a nonnegative integer, then S has 2^n subsets.   **6**

     b)   Consider a game in which two players take turns removing any positive number of matches they want from one of the two piles of matches. The player who removes the last match wins the game. Show that if the two piles contain the same number of matches initially, the second player can always guarantee a win.   **7**

     c)   Solve the recurrence relation $a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$, subject to initial conditions $a_0 = 0$, $a_1 = 1$ and $a_2 = 2$.   **7**

OR

6   a)   Show that if n is an integer greater than 1, then n can be written as the product of primes by using strong induction.   **6**

     b)   A bank pays a certain percentage of annual interest on deposits, compounding the interest once in 3 months. If a deposit doubles in 6 years and 6 months, what is the annual percentage of interest paid by the bank?   **7**

     c)   Let a_n denote the number of n-letter sequences that can be formed using the letters A, B and C such that any nonterminal A has to be immediately followed by B. Find the recurrence relation for a_n and hence solve.   **7**

UNIT – V

7   a)   i.   Find the remainder when 24^{1947} is divided by 17 using Fermat's little theorem.   **6**
         ii.   Find the remainder when $1! + 2! + \ldots\ldots + 1000!$ is divided by 8.

     b)   A child has some marbles in a box. If the marbles are grouped in sevens, there will be five left over; if they are grouped in elevens, there will be six left over; if they are grouped in thirteens, eight will be left over. Determine the least number of marbles in the box.   **7**

     c)   Solve the polynomial congruence: $x^3 + x + 3 \equiv 0 \pmod{25}$.   **7**

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