U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January / February 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: Common to all Branches **Duration: 3 hrs.**
Course Code: 15MA3GCMAT **Max Marks: 100**
Course: Engineering Mathematics-3

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

		UNIT - 1	CO	PO	Marks
1	a)	Discuss the consistency and hence solve the system of equations $x_1 + 2x_2 + 2x_3 = 4,\ x_2 - x_3 = 1,\ x_1 + 3x_2 = 0$.	1	1	6
	b)	Apply Gauss-Seidel iterative method to find the approximate solution of the system of equations $27x_1 + 6x_2 - x_3 = 85$, $6x_1 + 15x_2 + 2x_3 = 72$, $x_1 + x_2 + 54x_3 = 110$ with initial solution as (0, 0, 0). Perform 3 iterations.	1	1	7
	c)	Determine the eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$.	1	1	7
		OR			
2	a)	Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.	1	1	6
	b)	Apply Gauss-Seidel iterative method to solve the system of equations: $10x + y + z = 12,\ x + 10y + z = 12,\ x + y + 10z = 12$. with initial solution as (0, 0, 0). Perform 3 iterations.	1	1	7
	c)	Find the eigen values and the corresponding eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.	1	1	7
		UNIT - 2			
3	a)	Obtain the Fourier series expansion of the periodic function $f(x) = e^{-2x}$ in $(0, 2\pi)$.	2	1	6
	b)	Find the Fourier series of the triangular wave given by the function $f(x) = \begin{cases} 1 + \frac{4x}{3} & -\frac{3}{2} < x < 0 \\ 1 - \frac{4x}{3} & 0 < x < \frac{3}{2} \end{cases}$	2	1	7

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

<table>
<tr><td></td><td></td><td>c)</td><td>Express y as a Fourier series in the interval $(0, 2l)$ up to the first harmonic term for the following data:<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>4</td><td>8</td><td>15</td><td>7</td><td>6</td><td>2</td></tr></table></td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Obtain the Fourier series expansion of $f(x)=x$ in the interval $(0, 2l)$.</td><td>2</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Obtain the Fourier series expansion of $f(x)=x\cos x$ in the interval $(-\pi, \pi)$.</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Obtain the Fourier series of y up to the first harmonic term from the following data:<table><tr><td>x</td><td>0^0</td><td>60^0</td><td>120^0</td><td>180^0</td><td>240^0</td><td>300^0</td></tr><tr><td>y</td><td>7.9</td><td>7.2</td><td>3.6</td><td>0.5</td><td>0.9</td><td>6.8</td></tr></table></td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Form the partial differential equation by eliminating the arbitrary functions from $z = f(x+at) + g(x-at)$.</td><td>4</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Solve: $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.</td><td>4</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Obtain the various possible solutions of one-dimensional heat equation $u_t = c^2 u_{xx}$ by the method of separation of variables.</td><td>3</td><td>1</td><td>7</td></tr>
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<tr><td></td><td>6</td><td>a)</td><td>Form the partial differential equation by eliminating the arbitrary function F from $F(x^2+y^2, z-xy) = 0$.</td><td>4</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Solve: $(mz-ny)p + (nx-lz)q = ly - mx$.</td><td>4</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Obtain the various possible solutions of one-dimensional wave equation $u_{tt} = c^2 u_{xx}$ by the method of separation of variables</td><td>3</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Find the inverse Fourier transform of the function $e^{-|s|a}$.</td><td>2</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the Fourier Transform of the function $f(x) = \begin{cases} 1 & for\ |x| \le 1 \\ 0 & for\ |x| > 1 \end{cases}$.</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Solve the integral equation $\int_0^\infty f(x)\, sintx\, dx = \begin{cases} 1, & 0 \le t \le 1 \\ 2, & 1 \le t \le 2 \\ 0, & t \ge 2 \end{cases}$</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>8</td><td>a)</td><td>Find the inverse Fourier transform of $F[s] = e^{-s^2}$.</td><td>2</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2 & |x| \le a \\ 0 & |x| > a \end{cases}$.</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the Fourier cosine transform of the function $f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$.</td><td>2</td><td>1</td><td>7</td></tr>
</table>

			UNIT - 5			
	9	a)	Find the extremal of the functional $\int_0^{\frac{\pi}{2}} \left(y'^2 - y^2 + 4y\cos x\right)dx$, given $y(0) = y\left(\frac{\pi}{2}\right) = 0$	*5*	*1*	**6**
		b)	Show that the curve joining the points $(1,0)$ and $(2,1)$ for which $I = \int_1^2 \frac{1}{x}\sqrt{1+y'^2}\,dx$ is an extremum is a circle	*5*	*1*	**7**
		c)	Find the path on which a particle in the absence of friction will slide from one point to another in shortest time under the action of gravity.	*5*	*1*	**7**
			OR			
	10	a)	Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.	*5*	*1*	**6**
		b)	Solve the variational problem $\delta\left(\int_0^{\pi/2} \left(y^2 - y'^2\right)dx\right) = 0$ under the conditions $y(0) = 0$ and $y\left(\pi/2\right) = 2$.	*5*	*1*	**7**
		c)	A heavy cable hangs freely under the gravity between two fixed points. Show that the shape of the cable is Catenary.	*5*	*1*	**7**

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