U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January / February 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: Civil Engineering **Duration: 3 hrs.**
Course Code: 23MA3BSMCV **Max Marks: 100**
Course: Mathematics for Civil Engineering- 3

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

<table>
<tr><th></th><th></th><th>UNIT - 1</th><th>CO</th><th>PO</th><th>Marks</th></tr>
<tr><td>1</td><td>a)</td><td>Fit a straight line $y = a\,x + b$ to the following data:
<table><tr><td>x</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td></tr><tr><td>y</td><td>16</td><td>19</td><td>23</td><td>26</td><td>30</td></tr></table></td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>Obtain the lines of regression and hence find the co-efficient of correlation for the data:
<table><tr><td>x</td><td>1</td><td>3</td><td>4</td><td>2</td><td>5</td></tr><tr><td>y</td><td>8</td><td>6</td><td>10</td><td>8</td><td>12</td></tr></table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Obtain the rank correlation for the following data:
<table><tr><td>x</td><td>78</td><td>89</td><td>97</td><td>69</td><td>59</td><td>79</td><td>68</td><td>57</td></tr><tr><td>y</td><td>125</td><td>137</td><td>156</td><td>112</td><td>107</td><td>138</td><td>123</td><td>108</td></tr></table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>a)</td><td>Establish the formula $r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_z^2}{2\ \sigma_x \sigma_y}$ where $z = x - y$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>Fit a second-degree parabola $y = ax^2 + b\,x + c$ by the least square method for the data:
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>10</td><td>12</td><td>13</td><td>16</td><td>19</td></tr></table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Find the coefficient of correlation between two variables x and y given $\sigma_y = 2\sigma_x$ and the acute angle between their lines of regression is $\tan^{-1}\left(\frac{3}{5}\right)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>UNIT - 2</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>a)</td><td>The random variable X has the following probability mass function determine (i) K (ii) $P(X < 3)$ (iii) $P\ (3 < X \leq 5)$
<table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$P(X)$</td><td>K</td><td>$3K$</td><td>$5K$</td><td>$7K$</td><td>$9K$</td><td>$11K$</td></tr></table></td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>The mean weight of 1,000 students during medical examination was found to be 70kg and standard deviation 6. Assume that the weight is normally distributed, find the number of students having weight
(i) more than 75kg (ii) less than 65kg.</td><td>1</td><td>1</td><td>7</td></tr>
</table>

<table>
<tr><td></td><td></td><td>c)</td><td>The joint probability distribution of two random variables X and Y are given as:
<table>
<tr><td>Y \ X</td><td>-4</td><td>2</td><td>7</td></tr>
<tr><td>1</td><td>1/8</td><td>1/4</td><td>1/8</td></tr>
<tr><td>5</td><td>1/4</td><td>1/8</td><td>1/8</td></tr>
</table>
Compute the following (i) Marginal distribution of X and Y (ii) $E(X)$ and $E(Y)$ (iii) $E(XY)$ (iv) $COV(X,Y)$</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>The mean height of 500 students is 151 cm and the standard deviation is 15 cm. Assuming that the heights are normally distributed, find how many students height lies between 120 cm and 155 cm.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains
(i) no defective fuse (ii) 3 or more defective fuses (iii) at least one defective fuse.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>If X and Y are independent random variables with the following respective distribution. Find the joint distribution of X and Y. Also verify the $COV(X,Y)=0$.
<table>
<tr><td>x_i</td><td>1</td><td>2</td></tr>
<tr><td>$f(x_i)$</td><td>0.6</td><td>0.4</td></tr>
</table>
<table>
<tr><td>y_j</td><td>5</td><td>10</td><td>15</td></tr>
<tr><td>$g(y_j)$</td><td>0.2</td><td>0.5</td><td>0.3</td></tr>
</table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Find the Laplace transform of the function $f(t)=\frac{\sin 3t \cos t}{t}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the Laplace transform of a periodic function of period a defined by $f(t)=\begin{cases} E\ ; & 0 \le t < \frac{a}{2} \\ -E\ ; & \frac{a}{2} \le t \le a \end{cases}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the inverse Laplace transform of $\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Find the Laplace transform of the function $f(t)=te^{-2t}\sin 4t$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Express the following function in terms of unit step function and hence find its Laplace transform, where $f(t)=\begin{cases} t^2\ ; & 0 < t \le 2 \\ 4t\ ; & t > 2 \end{cases}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Laplace transform technique to solve the differential equation $\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y=5e^{2t}$ given $y=2, \frac{dy}{dt}=1$ at $t=0$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Find a Fourier series of the periodic function $f(x)=\frac{\pi-x}{2}$ in $(0,2\pi)$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the Fourier series expansion of the function $f(x)=1-x^2$ in the interval $(-1,1)$.</td><td>1</td><td>1</td><td>7</td></tr>
</table>

<table>
<tr><td></td><td></td><td>c)</td><td>Obtain the Fourier series of y up to the first harmonic from the following data:
<table><tr><td>x</td><td>0</td><td>60°</td><td>120°</td><td>180°</td><td>240°</td><td>300°</td></tr><tr><td>y</td><td>7.9</td><td>7.2</td><td>3.6</td><td>0.5</td><td>0.9</td><td>6.8</td></tr></table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>8</td><td>a)</td><td>Obtain the Fourier series expansion of the periodic function $f(x) = \begin{cases} 8\ ; & 0 < x < 2 \\ -8\ ; & 2 < x < 4 \end{cases}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Obtain the Fourier series expansion of the periodic function $f(x) = x^2$ in $(-\pi, \pi)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Obtain the Fourier series of y up to the first harmonic.
<table><tr><td>x</td><td>0</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>y</td><td>9.0</td><td>18.2</td><td>24.4</td><td>27.8</td><td>27.5</td><td>22.0</td></tr></table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>9</td><td>a)</td><td>Derive the finite difference formula to solve the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t) = 0$, $u(4,t) = 0, u_t(x,0) = 0$ and $u(x,0) = x(4 - x)$ by taking $h = 1$, $k = 0.5$ up to two time-levels.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Solve $u_{xx} = 32u_t$ subject to the conditions $u(0,t) = 0$, $u(1,t) = 0$ and $u(x,0) = x(1 - x)$. Find the values of u for four time-levels by Bender-Schmidt process taking $h = 1/4$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>10</td><td>a)</td><td>Derive the finite difference formula to solve the one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Solve numerically $16\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ subject to the boundary conditions $u(0,t) = 0$, $u(5,t) = 0$, the initial conditions $u_t(x,0) = 0$ and $u(x,0) = x^2(x - 5)$ by taking $h = 1$ and $k = \frac{1}{8}$ up to two time-levels.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Solve $u_t = u_{xx}$ subject to the conditions $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = \sin \pi x$ for $0 \le t \le 0.1$ by taking $h = 0.2$ and $k = 0.02$. Carry out computations up to two time-levels.</td><td>1</td><td>1</td><td>7</td></tr>
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