U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: Civil Engineering **Duration: 3 hrs.**
Course Code: 23MA3BSMCV **Max Marks: 100**
Course: Mathematics for Civil Engineering- 3

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	CO	PO	Marks
1	a)	If P is the pull required to lift a load W by means of a pulley block, find a linear law of the form $P = a + bW$ connecting P and W using the following data. Estimate the value of P when $W = 20$. P: 12, 15, 21, 25 W: 50, 70, 100, 120	1	1	**6**
	b)	Find the correlation co-efficient and hence find the two regression lines of the following data. x: 1, 2, 3, 4, 5 y: 10, 12, 16, 28, 25	1	1	**7**
	c)	Show that the angle between the lines of regression is $\tan\theta = \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\left(\frac{1-r^2}{r}\right)$ with usual notations.	1	1	**7**
		OR			
2	a)	In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate (i) the mean of x and y (ii) the co-efficient of correlation between x and y.	1	1	**6**
	b)	The following table gives the results of the measurements of train resistances, V is the velocity in miles per hour, R is the resistance in pounds per ton. V: 20, 40, 60, 80, 100, 120 R: 5.5, 9.1, 14.9, 22.8, 33.3, 46.0 If R is related to V by the relation $R = a + bV + cV^2$, find the values of a, b and c using least square method.	1	1	**7**
	c)	The following results were obtained from record of age (x) and blood pressure (y) of a group of 10 men. (Mean: x = 53, y = 142; Variance: x = 130, y = 165) Find the appropriate equation of regression lines when $\sum(x-\bar{x})(y-\bar{y}) = 1220$ and use it to estimate the blood pressure of a man whose age is 45.	1	1	**7**

Table for 1 a):

P	12	15	21	25
W	50	70	100	120

Table for 1 b):

x	1	2	3	4	5
y	10	12	16	28	25

Table for 2 b):

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46.0

Table for 2 c):

	x	y
Mean	53	142
Variance	130	165

<table>
<tr><td></td><td></td><td></td><td>UNIT - 2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>3</td><td>a)</td><td>In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing
i. no defective,
ii. one defective,
iii. two defective blades,
in a consignment of 10,000 packets.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>The mean height of 500 students is 151 cm and the standard deviation is 15 cm. Assuming that the heights are normally distributed, find how many students height lies between 120 cm and 155 cm.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>The joint probability function for two discrete random variables x and y is given by the following table.
<table>
<tr><td>x \ y</td><td>-3</td><td>2</td><td>4</td></tr>
<tr><td>1</td><td>0.1</td><td>0.2</td><td>0.2</td></tr>
<tr><td>3</td><td>0.3</td><td>0.1</td><td>0.1</td></tr>
</table>
i. Determine the marginal distributions of x and y,
ii. Determine covariance of x and y.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>If X is a Poisson variate and it is found that the probability $P(X=2)=\frac{2}{3}P(X=1)$. Find $P(X=0),\ P(X=3)$ and $P(X>3)$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>In an examination taken by 500 candidates, the average and standard deviation of the marks obtained (normally distributed) are 40% and 10%. Find approximately
i. How many will pass, if 50% is fixed as a minimum?
ii. How many have scored marks above 60%?</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>If X and Y are independent random variables, X takes values 2, 5, 7 with probability 1/2, 1/4, 1/4 respectively and Y takes values 3, 4, 5 with probability 1/3, 1/3, 1/3 respectively.
i. Find the joint probability distribution of X and Y.
ii. Show that the covariance of X and Y is equal to zero.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Determine the Laplace transform of a periodic function $f(t)=\begin{cases}1, & 0\le t\le \frac{a}{2}\\ -1, & \frac{a}{2}\le t\le a\end{cases}$ with period $T=a$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the inverse Laplace transform of $F(s)=\frac{2s+5}{(s-3)^2}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Laplace transform technique to determine the displacement $x(t)$ of a mass spring damper system described by the below equation $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=e^t$, with $x=0$, $\frac{dx}{dt}=-1$ at $t=0$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
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<table>
<tr><td></td><td>6</td><td>a)</td><td>i. Evaluate $L^{-1}\left[\frac{4}{(s-4)^5}\right]$.
ii. Find $L[\sin 2t \cos 3t\,]$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Express the function $f(t)=\begin{cases} t^2 & 0<t<2 \\ 4t & 2<t<4 \\ 8 & t>4 \end{cases}$ in terms of unit step function and hence find its Laplace transform.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Laplace transform techniques to solve the boundary value problem $\frac{d^2y}{dx^2}+y=0,\ \ y(0)=2\,, y\left(\frac{\pi}{2}\right)=1$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Determine the Fourier series expansion of the function $f(x)=\begin{cases} 2-x, & 0<x<1 \\ x, & 1<x<2 \end{cases}$, given $f(x+2)=f(x)$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Obtain the Fourier series expansion of $f(x)=\frac{x+1}{2}$ in $(0,2\pi)$ and $f(x+2\pi)=f(x)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>The turning moment y units of a crank shaft of a steam engine are given for a series of values of the crank angle x in degrees. Obtain first harmonic of a Fourier series to represent y.
<table>
<tr><td>$x°$</td><td>0</td><td>60</td><td>120</td><td>180</td><td>240</td><td>300</td></tr>
<tr><td>y</td><td>0</td><td>298</td><td>254</td><td>254</td><td>60</td><td>147</td></tr>
</table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>8</td><td>a)</td><td>Find the Fourier coefficients of the periodic function $f(x)$ given by $f(x)=\begin{cases} -k & \text{in} \quad -\pi<x<0 \\ k & \text{in} \quad 0<x<\pi \end{cases}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>If $f(x)=\begin{cases} \pi-x, & 0\le x\le\pi \\ x-\pi, & \pi\le x\le 2\pi \end{cases}$ then express $f(x)$ as a Fourier series.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>A mechanical engineer is analysing the motion of a machine part connected to a flywheel. The displacement of the part, represented by y, varies with the rotation angle x of the flywheel. The engineer collects the following periodic data points for y corresponding to various values of x:
<table>
<tr><td>$x°$</td><td>0</td><td>60</td><td>120</td><td>180</td><td>240</td><td>300</td><td>360</td></tr>
<tr><td>y</td><td>1.98</td><td>2.15</td><td>2.77</td><td>-0.22</td><td>-0.31</td><td>1.43</td><td>1.98</td></tr>
</table>
Express y as Fourier series up to first harmonic.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>9</td><td>a)</td><td>Derive the finite difference formula to solve numerically the one-dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the numerical solution of the heat equation $u_t=0.5\,u_{xx}$ subject to the conditions $u(0,t)=u(4,t)=0$, and $u(x,0)=\sin(\pi x)$ by taking $h=1=k,\ \ 0\le t\le 3$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Solve $u_{tt}=0.25\,u_{xx}$ at the pivotal points for $0\le t\le 0.4$ subject to the conditions $u(0,t)=0, u(4,t)=0$ and $u_t(x,0)=0$,
$u(x,0)=5x(x-4)$ by taking $h=1, k=1/4$.</td><td>1</td><td>1</td><td>7</td></tr>
</table>

			OR			
	10	a)	Derive the finite difference formula to solve numerically the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.	*1*	*1*	**6**
		b)	Solve $u_{xx} = 32u_t$ subject to the conditions $u(0,t) = 0,\ u(1,t) = 0$ and $u(x,0) = x(1-x)$. Find the values of u for four-time level by Bender-Schmidt process taking $h = 1/4$.	*1*	*1*	**7**
		c)	Find the solution of the initial boundary value problem $u_{tt} = u_{xx}$ subject to the initial conditions $u(x,0) = \sin(\pi x)$ and $u_t(x,0) = 0$, $0 \leq x \leq 1$ and the boundary conditions $u(0,t) = u(1,t) = 0$, $t \geq 0$ by taking $h = k = 0.2$.	*1*	*1*	**7**

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