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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2023 Semester End Main Examinations

Programme: B.E. **Semester: III**

Branch: AI and ML **Duration: 3 hrs.**

Course Code: 22MA3BSMML **Max Marks: 100**

Course: Mathematical Foundation for Machine Learning **Date: 10.04.2023**

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

UNIT - I

1 a) Using Fermat's Little theorem to prove that 42 divides $n^7 - n$. **06**

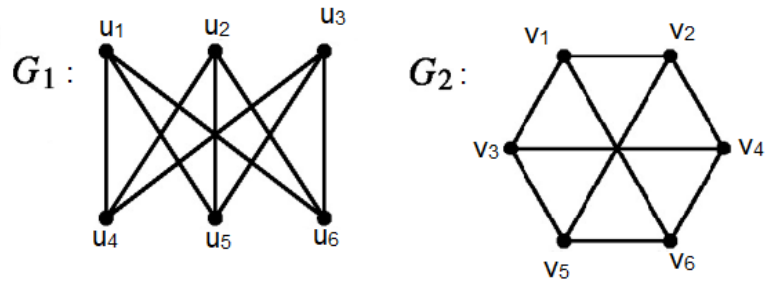
b) Apply Chinese remainder theorem to solve the system of linear congruence's $x \equiv 2\ (mod\ 5)$, $x \equiv 3\ (mod\ 7)$ and $x \equiv 1\ (mod\ 8)$. **07**

c) In an RSA crypto system, a person uses two prime numbers, 13 and 17, to generate the public and private keys. If the public key generated by the person is 35 then determine the private key he should generate to decrypt the message? **07**

UNIT - II

2 a) Given that $G\ (V, E)$ be a simple graph of order $|V| = n$ and size $|E| = m$. If G is a bipartite graph, then prove that $4m \leq n^2$. **06**

b) Define the graph isomorphism and hence verify whether the given graphs are isomorphic or not. **07**



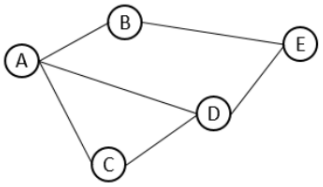
c) Define Euler trail and Euler circuit with an example. Prove that G is a connected graph and every vertex has even degree if and only if there is a Euler circuit in G. **07**

OR

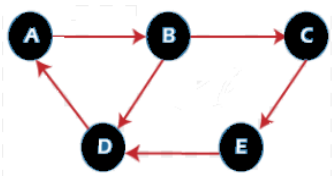
3 a) If $G\ (V, E)$ is a simple graph then prove that $2|E| \leq |V|^2 - |V|$. Also show that for complete graph with $'n'$ vertices $|E| = \frac{1}{2}n(n-1)$. **06**

b) Define directed graph and undirected graph. Also write an adjacency matrix for the following graphs. **07**

i.



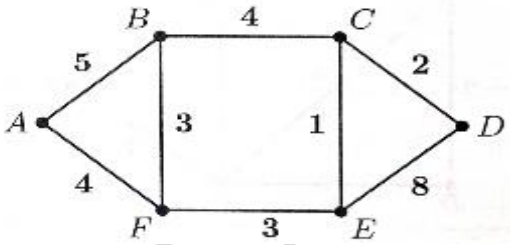
ii.



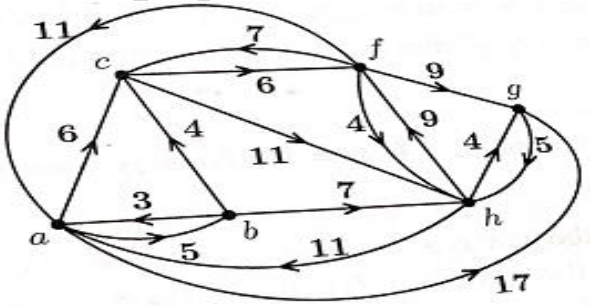
c) Let G be a simple graph with $'n'$ vertices and $'m'$ edges where $'m'$ is at least 3. If $m \geq \frac{1}{2}(n-1)(n-2)$ then prove that G is Hamiltonian graph. Is converse true? **07**

UNIT - III

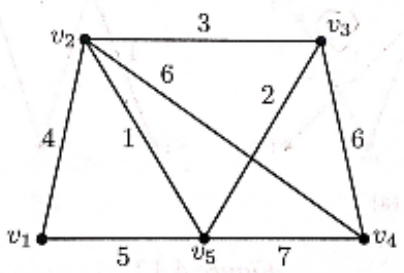
4 a) Determine the maximum flow between the vertices A and D in the network shown below by identifying the cut-set minimum capacity. **06**



b) Apply Dijkstra's algorithm to the weighted digraph shown below to find the shortest distance from the vertex $'c'$ to all other vertices. **07**



c) Apply Prim's algorithm to find a minimal spanning tree for the following weighted graph **07**



UNIT - IV

5 a) Determine the coefficient of xyz^2 in the expansion of $(2x-y-z)^4$. **06**

b) Among the first 500 positive integers **07**

i. Find the integers which are not divisible by 2, nor by 3 nor by 5.

ii. Find the integers are exactly divisible by one of them.

c) Find the Rook polynomial for the labeled board given below. **07**

	1	2		
3		4		
	5		6	7
			8	

UNIT - V

6 a) Use mathematical induction to prove that $4^{n+1} + 5^{2n-1}$ is divisible by 21 for every non-negative integer $'n'$. **06**

b) Suppose that there are $n \geq 2$ persons at a party and that each of these persons shakes hands (exactly once) with all of the other persons present. Using a recurrence relation, find the number of handshakes. **07**

c) Use generating function to find the coefficient of x^{20} in the expansion of $(x^2 + x^3 + x^4 + x^5 + x^6)^5$. **07**

OR

7 a) Use strong induction to prove that $\sqrt{2}$ is irrational. **06**

b) Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$, given that $F_0 = 0$ and $F_1 = 1$. **07**

c) Find the generating function for the number of integer solutions to the equation $c_1 + c_2 + c_3 + c_4 = 20$, where $-3 \leq c_1$, $-3 \leq c_2$, $-5 \leq c_3 \leq 5$ and $0 \leq c_4$. Also find the number of such solutions. **07**

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