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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2024 Supplementary Examinations

Programme: B.E. **Semester: III**

Branch: AI and ML **Duration: 3 hrs.**

Course Code: 22MA3BSMML **Max Marks: 100**

Course: Mathematical Foundation for Machine Learning

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

1 a) Solve the linear congruence $9x \equiv 21(mod\ 30)$. **06**

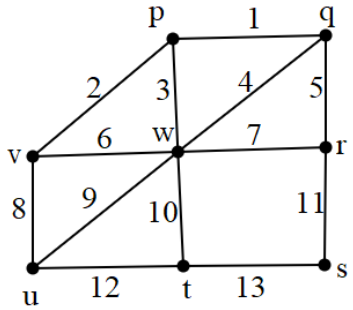
b) Determine the remainder when $6^{32} + 7^{58}$ is divided by 43. **07**

c) Given the public key $(e, n) = (7, 55)$, encrypt the plain text P, L where the alphabets $A, B, \ldots, Y, Z$ are assigned the numbers $2, 3, \ldots, 26, 27$. Find the cipher text and also determine the private key. **07**

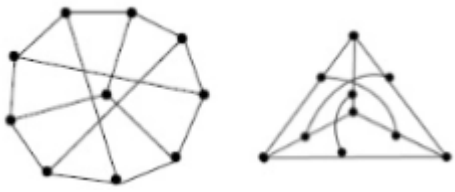
UNIT - II

2 a) Draw a simple graph of order 6, which is (i) Eulerian but not Hamiltonian, (ii) Hamiltonian but not Eulerian, (iii) Both Eulerian and Hamiltonian, (iv) Neither Eulerian nor Hamiltonian. Also indicate the circuit/cycle in each case. **06**

b) For the following graph, give the order, size, degree of each vertex and construct the adjacency matrix and incidence matrix. **07**

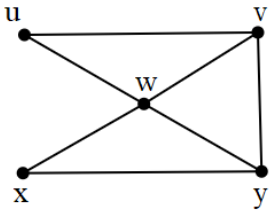


c) Verify whether the following graphs are isomorphic by checking for the one-one correspondence between their vertices. **07**



OR

3 a) In the given graph, draw **06**
(i) a u-v trail of length 5,
(ii) a u-v path of length 3,
(iii) a u-u circuit of length 6,
(iv) a u-u cycle of length 4.
Also verify the graph is Eulerian or Hamiltonian.



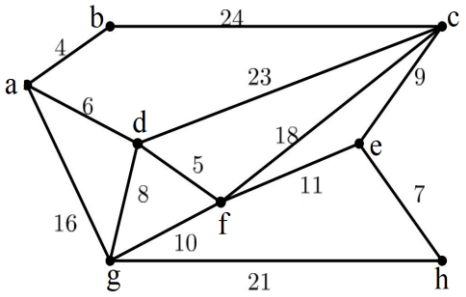
b) Define an incidence matrix. Obtain the incidence matrix for the graph whose adjacency matrix is **07**

$$X(G) = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

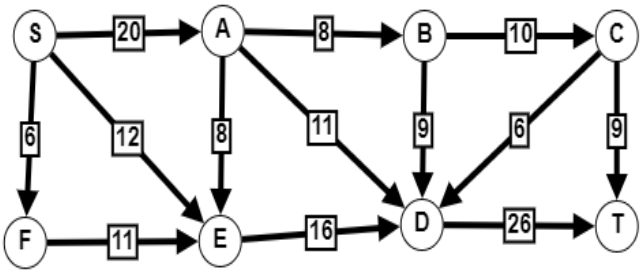
c) Prove that a connected graph G remains connected after removing an edge *e* from G if and only if *e* is a part of some cycle in G. **07**

UNIT - III

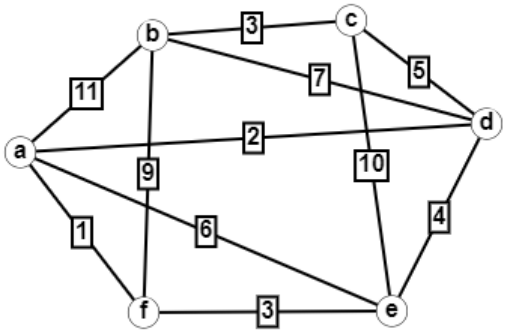
4 a) Apply Prim's algorithm to find the minimum spanning trees of the graph G given below. **06**



b) Determine the maximum flow between the source(**S**) and Sink(**T**) by identifying the cut-set minimum capacity for the network shown below. **07**



c) Apply Dijkstra's algorithm to find the shortest path between the vertices 'a' and 'b' in the following graph. **07**

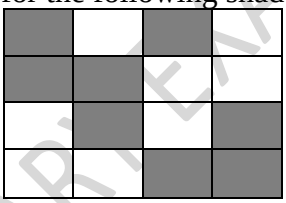


UNIT - IV

5 a) (i) Find the number of derangements of $3, 4, 5, 6$ distinct objects. **06**
(ii) Assign 7 books to 7 reviewers two times such that everyone gets a different book the second time.

b) Find the number of positive integers $n, 1 \leq n \leq 2000$ that are **07**
(i) divisible by $2, 3$ or 5, (ii) not divisible by $2, 3$ or 5.

c) Obtain the Rook polynomial for the following shaded chessboard. **07**



UNIT - V

6 a) Prove by mathematical induction that for all $n \in N$, $\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$. **06**

b) Solve the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$, where $a_0 = 2, a_1 = 3$. **07**

c) Determine the coefficient of x^{18} in the expansion of **07**
$(x + x^3 + x^5 + x^7 + x^9)(x^3 + 2x^4 + 3x^5 + \ldots)^3$.

OR

7 a) Apply mathematical induction to prove that the number of diagonals of a convex polygon with n vertices is $\frac{1}{2}n(n-3)$, for $n \geq 4$. **06**

b) Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$, subject to conditions $a_0 = 1$ and $a_1 = 4$. **07**

c) Find a recurrence relation, initial conditions and the next two terms of the sequence $2, 3, 8, 19, 46, \ldots$. **07**

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