U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

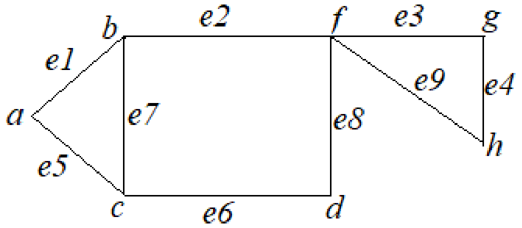
Autonomous Institute Affiliated to VTU

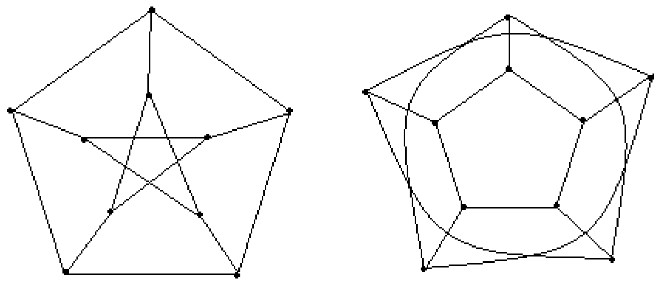
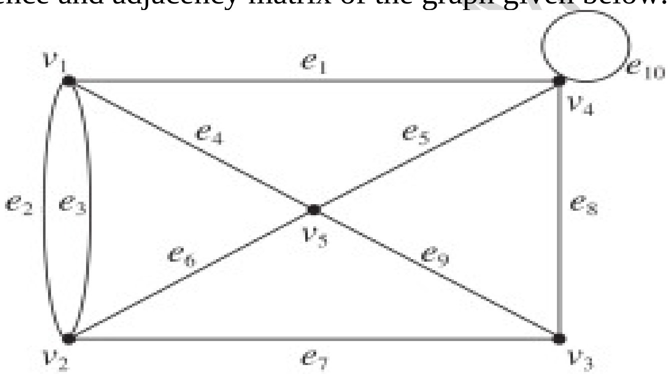
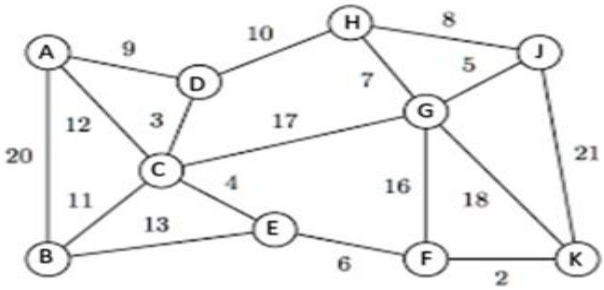
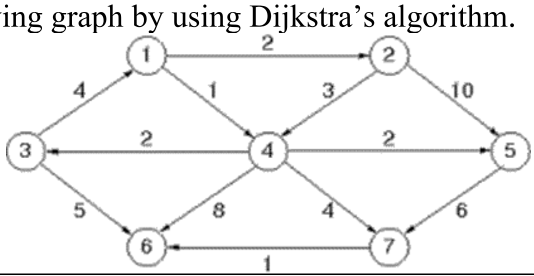
January / February 2025 Semester End Main Examinations

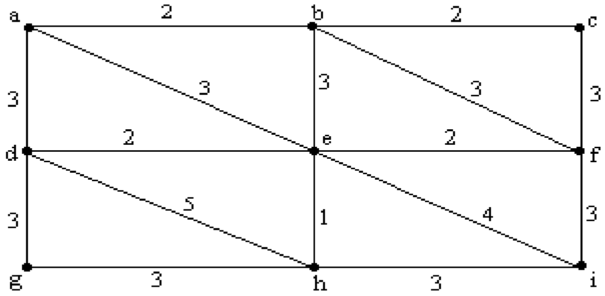
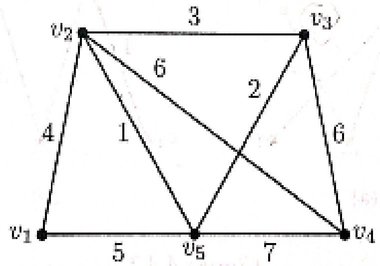
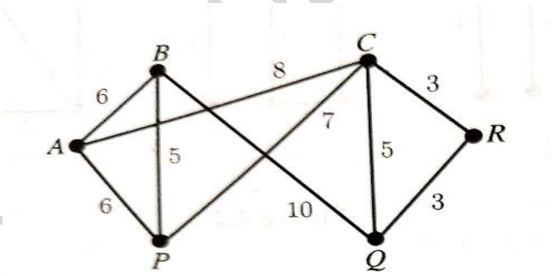
Programme: B.E.
Semester: III
Branch: AI and ML
Duration: 3 hrs.
Course Code: 22MA3BSMML
Max Marks: 100
Course: Mathematical Foundations for Machine Learning

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	*PO*	Marks
		UNIT – 1			
1	a)	Apply Wilson's theorem to find the remainder when $18!$ is divided by 437.	*1*	*1*	**6**
	b)	Apply Fermat's Little theorem to show that 42 divides $n^7 - n$.	*1*	*1*	**7**
	c)	Determine the decryption key by applying RSA algorithm given $(2537, 13)$ using the prime number 43 and 59.	*2*	*1*	**7**
		OR			
2	a)	Find the remainder when 72^{1001} is divide by 31.	*1*	*1*	**6**
	b)	Solve the system of linear congruences $x \equiv 2(\bmod 3)$, $x \equiv 3(\bmod 5)$ and $x \equiv 2(\bmod 7)$ using Chinese remainder theorem.	*1*	*1*	**7**
	c)	Solve the polynomial congruence: $x^3 + 3x + 5 \equiv 0(\bmod 27)$.	*1*	*1*	**7**
		UNIT – 2			
3	a)	Determine the order $\lvert V \rvert$ of the graph $G(V,E)$ in the following cases: i). G is a cubic graph with 9 edges ii). G is regular with 15 edges iii). G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.	*1*	*1*	**7**
	b)	For the graph, determine a) A walk from *b* to *d* which is not a trail b) *b-d* trail which is not a path c) A closed walk from *b* to *b* which is not a circuit. d) A circuit from *b* to *b* which is not a cycle e) The number of paths from *b* to *h* 	*1*	*1*	**6**

<table>
<tr><td></td><td></td><td>c)</td><td>Prove that there are $\frac{1}{2}(n-1)$ edge-disjoint Hamilton cycles in the complete graph with n vertices, where n is odd number ≥ 3.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Define the following:
(i) Complete graph
(ii) Spanning subgraph
(iii) Euler circuit
(iv) Bipartite graph</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Determine whether the two graphs given below are isomorphic.
</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Define the incidence and adjacency matrix of a graph and hence find the incidence and adjacency matrix of the graph given below.

</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT – 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Apply Prim's Algorithm to find a minimal spanning tree for the weighted graph.

</td><td>2</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the shortest path and shortest distance from the vertex 1 to vertex 6 in the following graph by using Dijkstra's algorithm.

</td><td>2</td><td>1</td><td>7</td></tr>
</table>

	c)	Define cut set of a graph and hence find the maximum flow between the vertices ‘ a ’ and ‘ i ’ by identifying the cut set of minimum capacity.  	*2*	*1*	**7**
		OR			
6	a)	Prove that a tree T with n vertices has n-1 edges.	*1*	*1*	**6**
	b)	Apply Kruskal’s algorithm to find a minimal spanning tree for the weighted graph given below  	*2*	*1*	**7**
	c)	Apply Dijkstra’s algorithm to find the shortest path from **A** to **R** in the weighted graph shown below:  	*2*	*1*	**7**
		UNIT – 4			
7	a)	Find the coefficients of (i) x^9y^3 in the expansion of $(2x-3y)^{12}$, and (ii) x^0 in the expansion of $\left(3x^2-\frac{2}{x}\right)^{15}$.	*1*	*1*	**6**
	b)	In how many ways can one arrange the letters in the word CORRESPONDENTS so that (i) there is no pair of consecutive identical letters? (ii) there are exactly two pairs of consecutive identical letters (iii) there are at least three pairs of consecutive identical letters?	*1*	*1*	**7**
	c)	A girl student has sarees of 5 different colours: blue, green, red, white and yellow. On Mondays she does not wear green; on Tuesdays blue or red; on Wednesdays blue or green; on Thursdays red or yellow; on Fridays red. In how many ways can she dress without repeating a colour during a week (from Monday to Friday)?	*1*	*1*	**7**

<table>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>8</td><td>a)</td><td>Determine the coefficients of
(i) $x^{11}y^4$ in the expansion of $\left(2x^3-3xy^2+z^2\right)^6$ and
(ii) $a^2b^3c^2d^5$ in the expansion of $\left(a+2b-3c+2d+5\right)^{16}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>In how many ways can the integers $1,\ 2,\ 3,\ldots\ldots, 10$ be arranged in a line so that no even integer is in its natural place.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply product and expansion formula to find the Rook polynomial for the board ‘C’ shown below.
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<tr><td>1</td><td>2</td><td bgcolor="gray"></td><td bgcolor="gray"></td><td bgcolor="gray"></td></tr>
<tr><td>3</td><td>4</td><td bgcolor="gray"></td><td bgcolor="gray"></td><td bgcolor="gray"></td></tr>
<tr><td bgcolor="gray"></td><td bgcolor="gray"></td><td bgcolor="gray"></td><td>5</td><td>6</td></tr>
<tr><td bgcolor="gray"></td><td bgcolor="gray"></td><td bgcolor="gray"></td><td>7</td><td>8</td></tr>
<tr><td bgcolor="gray"></td><td bgcolor="gray"></td><td>9</td><td>10</td><td>11</td></tr>
</table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>9</td><td>a)</td><td>Apply induction principle to prove that $6^{n+2}+7^{2n+1}$ is divisible by 43, $\forall\, n\geq 1$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Suppose that there are $n\geq 2$ persons at a party and that each of these persons shakes hands (exactly once) with all of the other persons present. Derive the recurrence relation and hence find the number of handshakes.</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Solve the recurrence relation $F_{n+2}=F_{n+1}+F_n$ for $n\geq 0$, given that $F_0=0$ and $F_1=1$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>10</td><td>a)</td><td>Apply strong induction principle to prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply structural induction to prove that if T is a full binary tree, then $n(T)\leq 2^{h(T)+1}-1$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>c)</td><td>Solve the recurrence relation $a_n-3a_{n-1}=5\times 7^n$ for $n\geq 1$, given that $a_0=2$.</td><td>1</td><td>1</td><td>7</td></tr>
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