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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

December 2023 Supplementary Examinations

Programme: B.E. — **Semester: III**

Branch: AI and ML — **Duration: 3 hrs.**

Course Code: 22MA3BSMML — **Max Marks: 100**

Course: Mathematical Foundation for Machine Learning

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

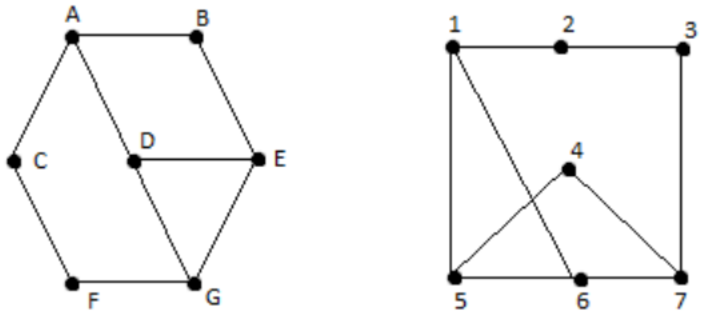
1 a) Find the least positive residue of $3^{999999999}$ modulo 7. **06**

b) There are certain things whose number is unknown. When this number is divided by 3, the remainder is 2, when divided by 5, the remainder is 3 and when divided by 7, the remainder is 2. What is the number of the things? **07**

c) Solve the polynomial congruence $x^3 + x + 3 \equiv 0 \pmod{25}$. **07**

UNIT - II

2 a) Define the graph isomorphism and hence verify whether the following graphs are isomorphic or not. **06**



b) (i) Let G be a graph of order 9 such that each vertex has degree 5 or 6. Prove that at least 5 vertices have degree 6 or at least 6 vertices have degree 5. **07**
(ii) Is there possible to have a set of seven persons such that each person in the set knows exactly three other persons in the set.

c) Let G be a disconnected graph of even order n with two components each of which is complete. Prove that G has a minimum of $\frac{n(n-2)}{2}$ edges. **07**

OR

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

3 a) Obtain the incidence matrix for the graph whose adjacency matrix is **06**

$$X(G) = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

b) Let G be a simple graph with n vertices and m edges where m is at least 3. If $m \geq \frac{1}{2}(n-1)(n-2)+2$ then prove that G is a Hamilton graph. Is the converse true? **07**

c) Prove that a connected graph G remains connected after removing an edge *e* from G if and only if *e* is a part of some cycle in G. **07**

UNIT - III

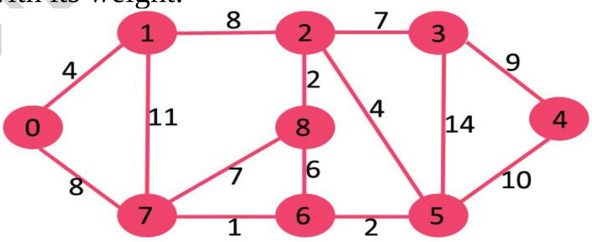
4 a) Eight cities A, B, C, D, E, F, G, H are required to be connected by a new railway network. The possible tracks and the cost of involved to lay them (in crores of rupees) are summarized in the following table. Draw the graph of the given data and determine a railway network of minimal cost that connects all these cities using Kruskal's algorithm. **06**

TRACK	AB	AD	AG	BC	CD	CE	DF	EF	FG	FH	GH
COST	155	145	120	145	150	95	100	150	140	150	160

b) i. Let F be a forest with k components (trees). If n is the number of vertices and m is the number of edges in F, prove that $n = m + k$. **07**

ii. Suppose that a tree T has two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4. Find the number of pendant vertices in T.

c) Apply Dijkstra's algorithm to find the shortest path from A to the remaining vertices along with its weight. **07**



UNIT - IV

5 a) There are n pairs of children's gloves in a box. Each pair is of a different color. Suppose the right gloves are distributed at random to n children, and thereafter the left gloves are also distributed to them at random. Find the probability that (i) no child gets a matching pair, (ii) every child gets a matching pair, (iii) exactly one child gets a matching pair, and (iv) at least 2 children get matching pairs. **06**

b) Determine number of integers between 1 and 300 (inclusive) which are (i) Divisible by exactly two of 5, 6, 8? (ii) Divisible by at least two of 5, 6, 8? **07**

c) Four persons $P1, P2, P3, P4$ who arrive late for a dinner party find the only one chair at each of five tables $T1, T2, T3, T4$ and $T5$ is vacant. $P1$ will **07** not sit at $T1$ or $T2$, $P2$ will not sit at $T2$, $P3$ will not sit at $T3$ or $T4$, and $P4$ will not sit at $T4$ or $T5$. Find the number of ways they can occupy the vacant chairs.

UNIT - V

6 a) Apply mathematical induction to prove that $7^{n+2}+8^{2n+1}$ is divisible by 57 for every non-negative integer n. **06**

b) Solve the recurrence relation $a_n - 3a_{n-1} = 5\left(3^n\right)$, $n \geq 1$ given that $a_0 = 2$. **07**

c) Find the generating function of: **07**

(i) $1^2, 2^2, 3^2, 4^2, \ldots,$ (ii) $0^2, 1^2, 2^2, 3^2, 4^2, \ldots$ (iii) $1^3, 2^3, 3^3, 4^3, \ldots$

OR

7 a) Apply mathematical induction to prove the given result $P(n)$ for all non-negative integers n, $P(n): 3+3\times5+3\times5^2+\ldots\ldots+3\times5^n = 3\left(\frac{5^{n+1}-1}{4}\right)$. **06**

b) There are 3 pegs fixed vertically on a table top, and n circular disks having holes at their centers and having increasing diameters are slipped onto one of these pegs, with the largest disk at the bottom. The disks are to be transformed one at a time on to another peg with the condition that at no time a larger disk is put on a smaller disk. Determine the no of moves for the transfer of all the disks, so that at the end the disks are in the original order. **07**

c) Determine the coefficient of: **07**

(i). x^{10} in the expansion of $\frac{\left(x^3-5x\right)}{\left(1-x\right)^3}$.

(ii). x^{18} in the expansion of $(x+x^2+x^3+x^4+x^5)(x^2+x^3+x^4+\ldots)^5$.

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