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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2024 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: Machine Learning **Duration: 3 hrs.**
Course Code: 23MA3BSMML **Max Marks: 100**
Course: Mathematical Foundation for Machine Learning - 1

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

		UNIT - 1	CO	PO	Marks
1	a)	Determine whether the linear transformation $T:\mathbb{R}^4 \to \mathbb{R}^3$ defined by $T(x, y, z, t) = (x - y + z + t,\ x + 2z + t,\ x + y + 3z - 3t)$ is one-one or on to or bijective. If not one-one then determine the basis of the subspace which are mapped to the zero vector.	CO1	PO1	**06**
	b)	Consider the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -1 & 0 \\ 1 & 4 & -2 \end{bmatrix}$ which defines a linear operator on $\mathbb{R}^2$. Find the matrix of the linear transformation relative to the basis $S = \{u_1, u_2, u_3\} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$.	CO1	PO1	**07**
	c)	Apply the method of cofactors to obtain the determinant of $A = \begin{bmatrix} -2 & 2 & 0 & 1 \\ 2 & -1 & 3 & 0 \\ -1 & 0 & 2 & -4 \\ 0 & -3 & 5 & 3 \end{bmatrix}$. Also determine the number of multiplications and additions used to obtain the same.	CO1	PO1	**07**
		UNIT – 2			
2	a)	The linear transformation $G:\mathbb{R}^3 \to \mathbb{R}^3$ is defined by $G(x, y, z) = (x + y + 3z, x + 5y + z, 3x + y + z)$. Find G^2, G^3 and G^{-2}.	CO1	PO1	**06**
	b)	Determine whether or not the linear maps F, G and H are linearly independent where $F, G, H : R^3 \to R^2$ are defined by $F(a, b, c) = (a + b,\ a + b + c)$, $G(a, b, c) = (2a + c,\ a - b)$ and $H(a, b, c) = (2b + c,\ a + c)$.	CO1	PO1	**07**

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		c)	Derive the linear transformation on $\mathbb{R}^2$ that defines rotation of the vector by an angle θ in the anti-clockwise direction. Construct a single matrix that defines a dilation of factor 2, then a shear of factor 3 in the x direction, then a rotation through an angle $\frac{\pi}{2}$ counterclockwise.	CO1	PO1	**07**
			UNIT – 3			
	3	a)	Consider $f(t)=t+2$, $g(t)=3t-2$ and $h(t)=t^2-2t-3$ in $P(t)$ with the inner product defined as $\langle p,q\rangle=\int_0^1 p(t)q(t)dt$. Find $\langle f,h\rangle$, $\langle g,h\rangle$ and the angle between $f(t)$ and $g(t)$.	CO1	PO1	**06**
		b)	Verify the Cauchy-Schwartz inequality $\langle u,v\rangle^2 \le \langle u,u\rangle\langle v,v\rangle$ if $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & 1\\ 2 & -2\end{bmatrix}$ are vectors in $M_{2\times 2}$ with the inner product $\langle A,B\rangle = Tr\left(B^T A\right)$.	CO1	PO1	**07**
		c)	Show that the ordered set $S=\{u_1,u_2,u_3,u_4\}$ where $u_1=(1,1,1,1)$, $u_2=(1,1,-1,-1)$, $u_3=(1,-1,1,-1)$ and $u_4=(1,-1,-1,1)$ is an orthogonal basis of $\mathbb{R}^4$. Hence express $v=(0,2,0,6)$ as a linear combination of the vectors of S.	CO1	PO1	**07**
			OR			
	4	a)	If $\langle\ ,\ \rangle$ is an inner product on a vector space over a real field, then expand $\langle 3u-2v, 2u+w\rangle$. Prove that the expansion is true for $u=(1,3,-4,2)$, $v=(4,-2,2,1)$ and $w=(5,-1,-2,6)$ in $\mathbb{R}^4$ when $\langle\ ,\ \rangle$ is the usual dot product.	CO1	PO1	**06**
		b)	Write a Pseudo code (or a python code) to determine the condition number of the matrix with respect to the L_2-norm. Calculate the condition number of the coefficient matrix of the system $\begin{bmatrix}1 & 10^4\\ 2 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ 3\end{bmatrix}$ with respect to L_2-norm and hence conclude that the problem is ill-conditioned.	CO2	PO5	**07**
		c)	Prove that the mapping $\left(\mathbb{R}^2,\langle\ ,\ \rangle\right)$ is an inner product space if $\langle u,v\rangle = u_1v_1-2u_1v_2-2u_2v_1+5u_2v_2$ where $u=(u_1,u_2)$ and $v=(v_1,v_2)$. Hence find the norm of $(2,-1)$ with respect to norm induced by the inner product.	CO1	PO1	**07**
			UNIT – 4			
	5	a)	Let W be a subspace of $\mathbb{R}^5$ spanned by $u=(1,2,3,-1,2)$ and $v=(2,4,7,2,-1)$. Find a basis of $W^{\perp}$.	CO1	PO1	**06**

		b)	Obtain the QR decomposition of the matrix $A=\begin{bmatrix}1&0&2\\-1&2&0\\-1&-2&2\end{bmatrix}$.	CO1	PO1	**07**
		c)	A steel producer gathers the following data: <table><tr><td>Year</td><td>1997</td><td>1998</td><td>1999</td><td>2000</td><td>2001</td></tr><tr><td>Annual Sales (million $)</td><td>1.2</td><td>2.3</td><td>3.2</td><td>3.6</td><td>3.8</td></tr></table>Represent the years 1997,…2001 as 0, 1, 2, 3, 4 respectively and let x denote the year. Let y denote the annual sales (in millions of dollars). (i) Write a Pseudo code to determine the least squares line of the form $y=a+bx$. (ii) Determine the least squares line of the form $y=a+bx$.	CO2	PO5	**07**
			UNIT – 5			
	6	a)	Find the characteristic and minimal polynomial of the matrix $\begin{bmatrix}3&1&-1\\2&4&-2\\-1&-1&3\end{bmatrix}$.	CO1	PO1	**06**
		b)	Write a pseudo code to find the spectral norm of a matrix. Apply the same to find the spectral norm of $A=\begin{bmatrix}1&1&2\\0&2&-1\end{bmatrix}$.	CO2	PO5	**07**
		c)	Determine the orthogonal matrix that diagonalizes the matrix $A=\begin{bmatrix}2&2&4\\2&5&8\\4&8&17\end{bmatrix}$.	CO1	PO1	**07**
			OR			
	7	a)	Obtain the eigen space of the linear transformation $T:P_2(t)\rightarrow P_2(t)$ given by $T\left(at^2+bt+c\right)=(3a+2b+c)t^2+(a+4b+c)t+a+2b+3c$.	CO1	PO1	**06**
		b)	Determine A^5 if $A=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}$ using diagonal decomposition of matrices.	CO1	PO1	**07**
		c)	The eigenvalues of $A=\begin{bmatrix}3&1&1\\1&3&-1\\1&-1&3\end{bmatrix}$ are 1 and 4. Determine the eigenvectors. Write a pseudo code to obtain orthonormal eigenvectors and implement the same to obtain orthonormal eigenvectors.	CO2	PO5	**07**

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