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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2025 Semester End Make-Up Examinations

Programme: B.E. — **Semester: III**

Branch: AI and ML — **Duration: 3 hrs.**

Course Code: 23MA3BSMML — **Max Marks: 100**

Course: Mathematical Foundations for Machine Learning-1

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

		UNIT - 1	*CO*	*PO*	Marks
1	a)	Prove that the linear transformation $T:V\to W$ is one-to-one if and only if the equation $Tv=0$ has only a trivial solution. Give an example of one-to-one and an example of not one-to-one linear transformations.	*1*	*1*	**6**
	b)	Find the determinant of the matrix $A=\begin{bmatrix}-1 & 2 & 1 & -6\\ 7 & -2 & -4 & -2\\ -5 & 3 & 3 & -10\\ 4 & -5 & -7 & 6\end{bmatrix}$ by using elementary row transformations. Hence write the number of additions and multiplications you have used to get the result.	*2*	*1*	**7**
	c)	Let $G:\mathbb{R}^3\to\mathbb{R}^3$ be the linear transformation defined by $G(x,y,z)=(4x+y+z,\ x-2y,\ 2x-y-3z)$. Is G invertible? If yes, then find G^{-1}.	*1*	*1*	**7**
		OR			
2.	a)	Check whether the linear transformation $T:\mathbb{R}^5\to\mathbb{R}^4$ where $T=\begin{bmatrix}2 & -1 & 1 & -6 & 8\\ 1 & -2 & -4 & 3 & -2\\ -7 & 8 & 10 & 3 & -10\\ 4 & -5 & -7 & 0 & 4\end{bmatrix}$ is one-one and on to. Also, if not one-one find the non-zero vector whose image is zero vector.	*1*	*1*	**6**
	b)	Derive the number of additions and multiplications required in finding the determinant of the $n\times n$ matrix using cofactors method. Also, write a pseudocode to give this count for $n=3$, 4, 5 and 6.	*2*	*1*	**7**
	c)	Let $F(x,y,z)=(3x+2y-4z,\ x-5y+3z)$ be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ and let the basis for $\mathbb{R}^3$ and $\mathbb{R}^2$ be $S=\{(1,1,1),(1,1,0),(1,0,0)\}$ and $S'=\{(1,3),(2,5)\}$ respectively. Verify $[F]_{S,S'}[v]_S=[F(v)]_{S'}$ for $v=(1,-3,2)$ in $\mathbb{R}^3$.	*1*	*1*	**7**

			UNIT - 2			
	3.	a)	Determine whether the linear maps $F:\mathbb{R}^3 \to \mathbb{R}^2$, $G:\mathbb{R}^3 \to \mathbb{R}^2$ and $H:\mathbb{R}^3 \to \mathbb{R}^2$ defined by $F(x,y,z)=(2y+z,\ x+z)$, $G(x,y,z)=(2x+z,\ x-y)$ and $H(x,y,z)=(x+y-z,\ x+y+z)$ respectively are linearly independent or not.	*1*	*1*	**6**
		b)	Let $F:R^3 \to R^3$ and $G:R^3 \to R^2$ defined by $F(x,\ y,\ z)=(2x, y+z, y)$ and $G(x,y,z)=(x+y, x-z)$. Determine the compositions $G \circ F$, $F \circ G$, F^2 and G^2 if they exists.	*1*	*1*	**7**
		c)	Determine the image of the triangle with vertices $(1,2)$, $(2,8)$ and $(3,2)$ when transformed by the affine transformation $T(x,y)=(x+4, y+2)$. Plot the region and its image.	*1*	*1*	**7**
			OR			
	4.	a)	Let $G:R^3 \to R^3$ be a linear transformation defined by $G(x,\ y,\ z)=(2x-3y, x-2y+z, 3y-5z)$. Find G^2, G^3, $G^2(2,-3,1)$ and $G^3(2,-3,1)$.	*1*	*1*	**6**
		b)	Determine the matrix that describes a reflection about x-axis, followed by rotation through $\frac{\pi}{3}$, followed by a dilation of factor 3. Find the image of the point $(4,1)$ under this sequence of mappings.	*1*	*1*	**7**
		c)	Find the image of the triangle having vertices $A(1,\ -3,\ 1)$, $B(2,\ -3,\ 1)$ and $C(5,\ 6,\ 1)$ in homogeneous coordinates under the sequence of transformations $T=\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$ followed by $R=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ followed by $S=\begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Sketch the original and the final triangle.	*1*	*1*	**7**
			UNIT - 3			
	5.	a)	Design and implement a pseudocode to find the condition number of the matrix $A=\begin{bmatrix} 5 & 7.5 & -1.75 \\ 2.1 & -3.25 & 6.125 \end{bmatrix}$ using Frobenious norm.	*2*	*1*	**6**
		b)	Prove or disprove that vector space $\mathbb{R}^2$ is an inner product space with respect to the function $\langle u,v \rangle = u_1v_1 + 3u_2v_2 - u_1v_2 + u_2v_1$, for $u,\ v \in \mathbb{R}^2$. Is this a normed vector space?	*1*	*1*	**7**
		c)	Show that $S=\{u_1,u_2,u_3,u_4\}$ is an orthogonal and a basis of $\mathbb{R}^4$, where $u_1=(1,\ 2, 1, -2)$, $u_2=(2,\ -1, 2, 1)$, $u_3=(0,\ 3, 0, 3)$ and $u_4=(2,\ 0, -2,\ 0)$. Also express $v=(1,3,-5,6)$ as a linear combination of the vectors of S and hence write its coordinate vector.	*1*	*1*	**7**
			OR			

	6.	a)	Find the value of α such that the matrices $A=\begin{bmatrix}\alpha & 8 & -7\\ 6 & 5\alpha & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & 3\\ -4 & 5 & 6\alpha\end{bmatrix}$ are orthogonal with respect to an inner product $\langle A,B\rangle = Tr\left(B^T A\right)$. Hence find $\lVert A\rVert$ and $\lVert B\rVert$.	1	1	**6**
		b)	If u and v are any two vectors in an inner product space V then prove the following: (i) if $\lVert u\rVert = \lVert v\rVert$ then prove that $\langle u+v, u-v\rangle = 0$, (ii) $u=v$ if and only if $\langle u,w\rangle = \langle v,w\rangle, w\in V$.	1	1	**7**
		c)	For the functions $f(t)=t^2-1$ and $g(t)=3t$ in polynomial space $P(t)$ with an inner product $\langle f,g\rangle = \int_{-1}^{1} f(t)g(t)\,dt$, verify the following: i) $\lVert f+g\rVert \le \lVert f\rVert + \lVert g\rVert$ ii) $\lVert f+g\rVert^2 + \lVert f-g\rVert^2 = 2\left(\lVert f\rVert^2 + \lVert g\rVert^2\right)$.	1	1	**7**
			UNIT – 4			
	7.	a)	Let $u,v\in\mathbb{R}^n$ define a function $f(u,v)=\sum_{i=1}^{n}\lvert u_i - v_i\rvert$. Prove or disprove whether $f(u,v)$ satisfies all the properties of a distance(metric) function.	1	1	**6**
		b)	Let $P_2(t)$ be the vector space of polynomials of degree ≤ 2 with $\langle f,\ g\rangle = \int_0^1 f(t)g(t)\,dt$. Find a basis of the subspace W orthogonal to $h(t)=2t+1$.	1	1	**7**
		c)	Find an orthogonal matrix P whose rows form an orthonormal basis of $\mathbb{R}^3$ if the first row of P is $\left(\frac{1}{3},\frac{2}{3},\frac{2}{3}\right)$.	1	1	**7**
			OR			
	8	a)	Let $v=(1,2,3,4,6)\in\mathbb{R}^5$ and W be the subspace of $\mathbb{R}^5$ spanned by the vectors $(1,2,1,2,1)$ and $(1,-1,2,-1,1)$. Determine the projection of the vector v along W.	1	1	**6**
		b)	Construct an orthogonal basis and hence an orthonormal basis of the subspace W spanned by the vectors $v_1=(1,1,1,1)$, $v_2=(1,2,4,5)$ and $v_3=(1,-3,-4,-2)$ of $\mathbb{R}^3$.	1	1	**7**
		c)	Design and implement pseudocode to find the linear relationship of the form $y=ax+b$ using the least squares method for the following data x: 2.5, 2.7, 3.1, 3.3 y: 1.2, 2.3, 3.2, 3.6 Hence determine the value of y when $x=3.8$.	2	1	**7**

x	2.5	2.7	3.1	3.3
y	1.2	2.3	3.2	3.6

			UNIT - V			
	9.	a)	Design and implement the algorithm to find the spectral norm of a matrix $A=\begin{bmatrix} 1 & 7 \\ -2 & 4 \\ 8 & 6 \end{bmatrix}$.	*2*	*1*	**6**
		b)	Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x,y,z)=(3x+2y+z\ ,\ x+4y+z,\ x+2y+3z)$. Find the eigenspace of the T .	*1*	*1*	**7**
		c)	Check whether the matrix $A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is diagonalizable. If so, find the modal matrix which diagonalizes A.	*1*	*1*	**7**
			OR			
	10.	a)	Compute A^9 by eigenvalue decomposition if $A=\begin{bmatrix} 3 & 1 \\ 1 & -3 \end{bmatrix}$.	*1*	*1*	**6**
		b)	Given $A=\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$, orthogonally diagonalize $A^T A$.	*1*	*1*	**7**
		c)	Find the characteristic and minimal polynomial of the matrix $A=\begin{bmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix}$.	*1*	*1*	**7**

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