U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January / February 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: AI and ML **Duration: 3 hrs.**
Course Code: 23MA3BSMML **Max Marks: 100**
Course: Mathematical Foundation for Machine Learning - 1

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - I	CO	PO	Marks
1	a)	Find the matrix of linear map T on R^3 defined by $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+2y-3z \\ 4x-5y-6z \\ 7x+8y+9z \end{bmatrix}$ with respect to basis $S=\{(1,1,1),\ (0,1,1),\ (1,2,3)\}$.	1	1	**6**
	b)	Check whether the linear map $T: R^4 \to R^3$ where $T = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$ is one-one and onto. Also, if not one-one then find the non-zero vector whose image is a zero vector.	1	1	**7**
	c)	Apply Gaussian elimination approach to find the determinant of the matrix $A = \begin{bmatrix} -2 & 2 & 0 & 1 \\ 2 & -1 & 3 & 0 \\ -1 & 0 & 2 & -4 \\ 0 & -3 & 5 & 3 \end{bmatrix}$. Hence write the number of additions and multiplications you have used to get the result.	1	1	**7**
		OR			
2	a)	Given that $G: R^2 \to R^2$ and $S=\{u_1, u_2\}=\{(1,3),(2,5)\}$ is the basis of R^2, $G(x, y)=(2x-7y, 4x+3y)$ and $v=(4,-3)$ in R^2. Verify that $[G]_s.[v]_s=[G(v)]_s$.	1	1	**6**
	b)	Let $G: R^3 \to R^3$ defined by $G(x, y, z)= (2x, 4x-y, 2x+3y-z)$. Is G singular or non-singular? Is G is invertible? Justify. If G is invertible then find G^{-1}.	1	1	**7**
	c)	Derive the recursive formula for number of additions and multiplication in finding determinant of a matrix using cofactor expansion method. Also write the pseudocode to count the number of operations.	1	1	**7**

			UNIT - II			
	3	a)	Determine whether or not the set of linear maps $\{F, G, H\}$ from $R^3 \rightarrow R^2$ defined by $F(a, b, c) = (a+b+c,\ a+b)$, $G(a, b, c) = (2a+c,\ a+b)$, and $H(a, b, c) = (a-b, 0)$ are linearly independent.	1	1	**6**
		b)	Discuss the following maps on R^2 graphically with matrix representation i. Reflection through x-axis, ii. Rotation, iii. Horizontal shear.	1	1	**7**
		c)	Determine the image of the triangle with vertices $(1, 2)$, $(2,8)$ and $(3,2)$ when transformed by the affine transformation $T(x, y) = (x+4, y+2)$. Plot the region and its image.	1	1	**7**
			OR			
	4	a)	Derive the matrix of the linear transformation $T: R^2 \rightarrow R^2$, which rotates a vector $v \in R^2$ by an angle θ in an anti-clockwise direction. Find if there exist: i. a preimage of $(1, -3)$. ii. an image of $(3, -1)$ when $\theta = \frac{\pi}{3}$.	1	1	**6**
		b)	Let the linear maps $F: R^3 \rightarrow R^2$ and $G: R^2 \rightarrow R^2$ be defined by $F(x, y, z) = (2x, y+z)$ and $G(x, y) = (y, x)$. Check whether $G \circ F$, $F \circ G$, F^2 and G^{-2} exists, if so find the composite linear maps.	1	1	**7**
		c)	Find the image of the triangle having vertices $A(1,6,1)$, $B(3,0,1)$ and $C(4,16,1)$ in homogeneous coordinates under the sequence of transformations T followed by R, followed by S when $T = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$, $R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $S = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Sketch the original and the final triangle.	1	1	**7**
			UNIT – III			
	5	a)	If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 5 & 5 \end{bmatrix}$ in a vector space of matrices $M_{2\times 2}$ with an inner product $\langle A, B \rangle = tr(B^T A)$ then find $\langle A, B \rangle$, $\lVert A \rVert$ and $\lVert B \rVert$.	1	1	**6**
		b)	If $u = (1,3,-4,2)$, $v = (4,-2,2,1)$ and $w = (5,-1,-2,6)$ are in R^4 then prove that $\langle 3u - 2v, w \rangle = 3\langle u, w \rangle - 2\langle v, w \rangle$, where $\langle\ ,\ \rangle$ is a standard inner product.	1	1	**7**

<table>
<tr><td></td><td></td><td>c)</td><td>If u and v are any two vectors in an inner product space V then prove the following:
(i) if $\|u\|=\|v\|$ then prove that $\langle u+v,u-v\rangle=0$,
(ii) $u=v$ if and only if $\langle u,w\rangle=\langle v,w\rangle, w\in V$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Design and implement the pseudocode to find the Frobenious norm for the matrix $A=\begin{bmatrix}9 & 8 & 7\\ 6 & 5 & 4\end{bmatrix}$. Write all computational steps.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Given $f(t)=t+2$, $g(t)=3t-2$ and $h(t)=t^2-2t-3$ in $P(t)$ with the inner product defined as $\langle p,q\rangle=\int_0^1 p(t)q(t)dt$. Find $\langle f,g\rangle$, $\langle f,h\rangle$, $\|f\|$ and $\|g\|$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Prove that vector space $V=\mathbb{R}^3$ is an inner product space with respect to inner product $\langle u,v\rangle=u_1v_1-u_2v_1+u_1v_2+4u_2v_2$. Is this a normed vector space?</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - IV</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Show that $(\mathbb{R}, d)$ forms a metric space when $d:\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ is defined as $d(x,y)=|x-y|,\ x,y\in\mathbb{R}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the projection of the vector $v=(1,2,3,4,6)$ along $W=\text{span}\{(1,2,1,2,1),(1,-1,2,-1,1)\}$ in R^5.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find an orthogonal basis and hence an orthonormal basis of the subspace W spanned by the following vectors $v_1=(1,1,1,1)$, $v_2=(1,-1,2,2)$ and $v_3=(1,2,-3,-4)$ of R^4.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>8</td><td>a)</td><td>Let W be a subspace of R^5 spanned by $u=(1,2,3,-1,2)$ and $v=(2,4,7,2,-1)$. Find a basis of $W^{\perp}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find an orthogonal matrix P whose first row is $u_1=\left(\frac{1}{3},\frac{2}{3},\frac{2}{3}\right)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>A sales organization obtains the following data relating the number of salespersons to annual sales.
<table><tr><td>Number of salespersons (x)</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>Annual Sales (millions of dollars)</td><td>2.3</td><td>3.2</td><td>4.1</td><td>5.0</td><td>6.1</td><td>7.2</td></tr></table>Let x denote the number of salespersons and let (y) denote the annual sales (in millions of dollars). Find the least squares line of the form $y=a+bx$ and hence estimate the annual sales when there are 13 salespersons.</td><td>1</td><td>1</td><td>7</td></tr>
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			UNIT - V			
	9	a)	Find the eigen spaces of the linear transformation $T: R^3 \rightarrow R^3$ defined by $T\left(x, y, z\right) = (3x + 2y + z,\ x + 4y + z,\ x + 2y + 3z)$.	1	1	**6**
		b)	Find the characteristic and minimal polynomial of the matrix $\begin{bmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix}$.	1	1	**7**
		c)	Find the eigenvalue decomposition of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.	1	1	**7**
			OR			
	10	a)	Find the minimal polynomial of the matrix $A = \begin{bmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$.	1	1	**6**
		b)	Design and implement the pseudocode to find the spectral radius of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -1 \end{bmatrix}$.	1	1	**7**
		c)	Orthogonally diagonalize the matrix $A = \begin{bmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{bmatrix}$.	1	1	**7**

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